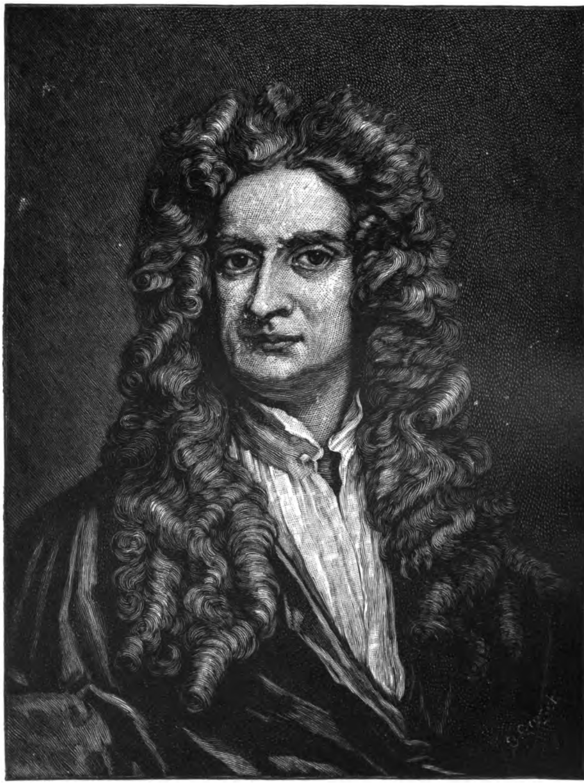

SIR ISAAC NEWTON

A TEXT-BOOK OF PHYSICS

LARGELY EXPERIMENTAL

INCLUDING THE HARVARD COLLEGE "DESCRIPTIVE LIST OF ELEMENTARY EXERCISES IN PHYSICS"

BY

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INTRODUCTION.

THIS book is intended as a text-book and laboratory manual for beginners in the systematic study of physics. The course of work herein described is designed to correspond to the following statement, recently adopted as a definition of the requirement in elementary experimental physics for admission to Harvard College and the Lawrence Scientific School :

"A course of study dealing with the leading elementary facts and principles of physics, with quantitative laboratory work by the pupil.

"The instruction given in this course should include qualitative lecture-room experiments, and should direct especial attention to the illustrations and applications of physical laws to be found in every-day life. The candidate will be required to pass a written examination, the main object of which will be to determine how much he has profited by such instruction. This examination may include numerical problems. It will contain more questions than any one candidate is expected to answer, in order to make allowance for a considerable diversity of instruction in different schools.

"The pupil's laboratory work should give practice in the observation and explanation of physical phenomena, some familiarity with methods of measurement, and some training of the hand and the eye in the direction of precision and skill. It should also be regarded as a means of fixing in the mind of the pupil a considerable variety of facts and principles. The candidate will be required to pass a laboratory examination, the main object of which will be to determine how much he has profited by such a laboratory course.

"The candidate must name as the basis for his laboratory examination at least thirty-five exercises selected from a list of about sixty described in a publication issued by the University under the title "Descriptive List of Elementary Exercises in Physics." In this list

the divisions are mechanics (including hydrostatics), light, heat, sound, and electricity (with magnetism). At least ten of the exercises selected must be in mechanics. Any one of the four other divisions may be omitted altogether, but each of the three remaining divisions must be represented by at least three exercises.

"The candidate will be required to present a note-book in which he has recorded the steps and the results of his laboratory exercises, and this note-book must bear the indorsement of his teacher, certifying that the notes are a true record of the pupil's work. It should contain an index of the exercises which it describes. These exercises need not be the same as those upon which the candidate presents himself for the laboratory examination, but should be equivalent to the latter in amount and grade of quantitative work.

"The note-book is required as proof that the candidate has formed the habit of keeping a full and intelligible record of laboratory work through an extended course of experiments, and that his work has been of such a character as to raise a presumption in favor of his preparation for the examination. But much greater weight will be given to the laboratory examination than to the note-book in determining the candidate's attainments in physics. Experience has shown that pupils can make the original record of their observations entirely presentable, so that copying will be unnecessary, and they should in general be required to do so.

"This course, if taken in the last year of the candidate's preparation, is expected to occupy in laboratory work, recitations, and lectures, five of the ordinary school periods, about fifty minutes in length, per week for the whole year. With few exceptions exercises like those in the Descriptive List already mentioned can be performed in a single school period, but for satisfactory results it will often be necessary to repeat an exercise. Two periods per week for the year should be sufficient for the laboratory work proper. If the course is begun much earlier than the last year of the candidate's preparation, as it well may be, it will require more time."

The *Exercises* of this book are identical with those given in the *Descriptive List* referred to in the statement just quoted.

Those who are familiar with the first edition of the book will observe in this revision much new matter and a great

alteration of form, or, rather, of arrangement. The object of these changes is to adapt the work better to the capacity of young students, to the restrictions of school-programs, and to the needs of those who are to go no farther with the study of physics. A comparison of the Tables of Contents of the two editions will show that the new features are too numerous to be named here.

The division of the book into a First Part and a Second Part is intended to facilitate and encourage beginning the study of physics very early in the school course. Most of the Exercises in the First Part are equivalent to those contained in Hall's *Elementary Lessons in Physics*, which have proved to be quite within the power of boys fourteen or fifteen years old. The cost of apparatus for these exercises is very small, and it is to be hoped that schools which are unable to provide laboratory equipment for the whole course will find it possible to complete the First Part in accordance with the directions of the book.

Lists of apparatus for the whole course have been made with much care, and well-known manufacturers[1] in Boston and elsewhere will undertake to furnish everything at a reasonable price and nearly everything at very short notice.

Mr. Bergen's engagements have been such as to give him little time for work upon this second edition, and his contributions have therefore been limited to problems. Numerous problems, too, have been taken from a collection made by Mr. W. H. Snyder, A.M., of the Worcester Academy, and generously placed by him at our disposal.

There are many others, teachers, students, and manufac-

[1] The Franklin Educational Company, Harcourt St., Boston, Mass.
The Knott Company, 14 Ashburton Place, Boston, Mass.
The Ziegler Company, 141 Franklin St., Boston, Mass.
The Ritchie Company, Brookline, Mass.
The Olmsted Scientific Company, 215 Wabash Avenue, Chicago.

turers, who have contributed to the form of the course or the form of the apparatus, as these now appear. Some of these contributors are mentioned by name in the following pages and some are not. A few acknowledgments made in the first edition it has seemed unnecessary to repeat.

E. H. H.

August 27, 1897.

NOTE.

THE following estimates of cost for apparatus and materials are only approximate. It is hardly possible to make an accurate estimate, as prices will vary from time to time, and different dealers have somewhat different grades of apparatus. The cheapest is not necessarily the best to buy.

FOR THE FIRST PART.

Teacher's apparatus and supplies, pp. 174–178	$78
Student's apparatus, pp. 170–173, for each member of laboratory squad	6
Table, accommodating six workers, p. 178	25
Total for *all* Exercises and Experiments of the First Part, with laboratory squads limited to twelve	200

FOR THE SECOND PART.

Teacher's apparatus, pp. 581–586	$350
By omitting the thermopile and accompanying apparatus and the Roentgen-ray apparatus this expense can be reduced about $100.	
Student's apparatus, pp. 572–581, for a single experimenter	90
Student's apparatus for a squad of twelve	750
Total for *all* Exercises and Experiments of the Second Part, with laboratory sections limited to twelve	1100
Total for all Exercises and Experiments of the book	1300
Total for all Experiments and for thirty-five Exercises of average expense	950

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CHAPTER I.

INTRODUCTORY.

1. Definition of Physics. — Physics is the science of *mechanics*, *heat*, *sound*, *light*, *electricity*, and *magnetism*. Everybody knows something about these things before he begins to study them in a regular way, but sometimes he does not know them by the names which are given to them in books.

2. Use of Physics.—In sailing boats or flying kites, in walking or swimming, in almost any kind of bodily work or play, we have to do with physics, that part of physics which is called *mechanics*. We learn to do many mechanical acts very well indeed by observation and experience, without thinking very much about them or knowing exactly how we do them; but when we have to do something that we have never done before and have never seen any one else do, something, perhaps, that nobody ever did before, we must think and study.

3. Illustrations.—Thus no man who has practiced swimming need study mechanics to improve himself in that art; but if he would build a ship and make it swim through all kinds of weather and water, he must study mechanics a good deal in order to know what size and shape to give the various parts, how best to put them together, and how to

balance the whole. If it is to be a steamship, some one must know a good deal about heat, to make the furnaces and boilers right. We must know about magnetism to make and use the ship's compass. We may use electricity to furnish light on board at night. We must study sound in order to make the best fog-signals to guard against collisions and shipwreck in thick weather.

In short, the science of physics in all its main divisions is not only a very interesting study to many minds, but it is of great use to civilized mankind. Man has become civilized, indeed, not by merely imitating what his fathers have done, but by *studying*, that is, *observing* and *thinking*, and gradually improving upon the work of those who have gone before him.

4. Qualitative Knowledge.—Everybody knows that a piece of wood will float in water, and that a stone will sink. Everybody knows that if a stick and a stone are tied together and put into water, the stone tends to sink the stick, and the stick tends to float the stone. This kind of knowledge is called *qualitative*. It tells in a general way how the stick and the stone act toward each other.

5. Quantitative Knowledge.—Some people know enough about the laws of flotation to calculate with accuracy how large a stick of a known kind of wood will be needed to float a stone of known size and weight. They have what is called *quantitative* knowledge of the matter. They can tell *how much* the stone will pull down on the wood, and the wood pull up on the stone, when the two are together in water.

Everybody knows that a beam has a quality which we call strength—it can bear a load. This is qualitative knowledge. Everybody knows that a thick beam is stronger than a thin beam. This is quantitative knowledge of a kind, a rather indefinite kind. Some people know how much

stronger the thick beam is than the thin beam. They have a more complete quantitative knowledge.

6. Comparison of the Two Kinds of Knowledge.—It is evident that quantitative knowledge is more useful than mere qualitative knowledge. The former includes the latter. The qualitative man says, "I want to build a house. I shall need some land to put it on, and some beams and boards and bricks," etc. The quantitative man says, "Yes, you will need all these things, but if you don't make your ideas more precise you will have a lumber-yard when you have done, not a house."

7. Object of this Course in Physics.—The knowledge of physics which children and older people get by merely knocking about in the world is mostly qualitative. The course of work laid out in this book is intended to add greatly to the pupil's stock of this kind of knowledge, and to do something more. It aims to make the pupil familiar with quantitative work, and to give him a considerable amount of quantitative knowledge.

We shall begin at once with simple introductory measurements. All the exercises of this chapter are called *Preliminary Exercises.*

Measurement of Distance.

8. The Straight Line.—The line to be measured may be along the edge of a table (or sheet of paper) from one fine scratch to another, a distance of about 15 inches. It is a great convenience to have all the pupils measure equal distances; accordingly, the teacher is advised to lay off these distances by some method like the following: A carpenter's square is placed along the edge of the table as in Fig. 1, and while it is held firmly in place a fine light scratch is

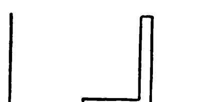

Fig. 1.

made with the point of a sharp knife-blade at right angles with the edge of the table at the points *a* and *b*. The distance from *a* to *b* is the one to be measured by the pupil. The first-described method of using the measuring-stick in the following Exercise is not a good method, but it is one that many will use if they are not properly instructed. The second method is a good one, and the two are here brought together in order that the pupil may see at once the right way and the wrong way to use such an instrument.

Much of the interest and profit of this Exercise will come from the opportunity given each pupil to compare his own work with that of others.

EXERCISE A.

MEASUREMENT OF A STRAIGHT LINE.

Apparatus: A short measuring-stick (No. 1) * and a meter-rod (No. 2).

To each pupil is given a measuring-stick about one-fourth as long as the distance from *a* to *b*. We will suppose that these sticks are made by sawing a meter-rod, graduated to millimeters, into ten equal parts. The saw-cut will usually leave the divisions at the very ends of the sticks imperfect, and these divisions should not be used in the measurements.

Let each pupil measure his distance at least twice carefully, with his measuring-stick laid flat upon the table, the marks upon the stick being thus *horizontal*, and let him write upon the blackboard the results of his two measurements.

Then let each pupil measure his distance twice again, this time placing his measuring-stick upon its edge, so that the marks upon it will be vertical, making a light, fine mark upon the table with a sharp pencil to *set* the stick by, whenever it is moved forward a length. These new measurements are also to be placed upon the blackboard under the first ones.

Finally let each pupil measure his whole distance at once with his meter-rod and write this last measurement with the others.

* Any piece of apparatus to be used in the Exercises will usually be referred to by the number it bears in the list of apparatus given at the end of the book.

9. Errors.—To judge of the accuracy of a set of measurements it is not enough to know how much these differ among themselves, for the importance of the difference usually depends upon the ratio which the difference bears to the whole quantity measured. A thousandth part of an inch might be a very serious difference to a watchmaker in the measurement of some small cylinder, while a difference of several inches in the measurement from one mile-post to another would be of little consequence. The pupil should therefore form the habit of comparing his errors, or the differences of his measurements, with the whole quantity that he had to measure.

Let us suppose, for instance, that in Exercise A the measurements made by one pupil are 37.30 cm., 37.00 cm., and 37.10 cm. The greatest difference is found between the first and second. It is 0.3 cm., and its ratio to 37.15 cm., which is midway between 37.30 cm. and 37.00 cm., is 0.0081—. We see, then, that the difference between the two measurements of the line is about *eight one-thousandths*, not quite one per cent, of the length of the line.

Each pupil should make a similar calculation from his own measurements in Exercise A.

10. Units and Standards of Measurement.—The importance of having definite units of length, of weight, etc., so that any man in dealing with his neighbor may know just how much is meant by the words *foot*, *pound*, and the like, is so great that in all civilized countries the exact meaning of such words is fixed by law, and very great care is taken to make and preserve government *standards*, as they are called, standard yard-sticks, standard pound-weights, for instance, with which as patterns the measuring instruments used in business are compared and tested.

Interesting accounts of the foot, the yard, the meter, etc., can be found in almost any encyclopedia.

Meter-rods for school use are in many cases marked off in inches on one side. With the information given by such a rod, the class can find how many centimeters are equal to one inch. This number carried to two places of decimals is accurate enough for most purposes.

11. The Right Triangle.—Right triangles, that is, triangles having one right angle (see Fig. 2), are much used in the study and application of physics. In such triangles

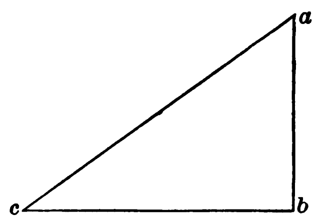

Fig. 2.

there is a simple and important relation between the length of the longest side and the length of the other two sides. Part 1 of the following Exercise B is intended to show this relation and at the same time to give practice in measurement.

12. Circles.—The relation between the length of the diameter of a circle and the length of its circumference is also very frequently used in physics. Part 2 of Exercise B has to do with this relation.

EXERCISE B.

THE LINES OF THE RIGHT TRIANGLE AND THE CIRCLE.

Apparatus: A 30-cm. measuring-stick (No. 3). A sheet of paper upon which is drawn carefully a right triangle no side of which is less than 10 cm. long. (No two pupils should use exactly similar

triangles.) A cylinder of wood 4 or 5 cm. in diameter (No. 4). A narrow straight-edge strip of thin paper.

PART 1. MEASUREMENT OF THE SIDES OF A RIGHT TRIANGLE. —Let each pupil measure very carefully all the sides of his triangle, not being content to read to the nearest 0.1 cm., but striving to note and measure 0.05 cm. distances, if he can do so without hurting his eyes.

After the measurements are made, square the length of each side and compare the greatest square with the sum of the other two squares. The conclusion drawn from this comparison must not be extended to triangles which are not right-angled.

PART 2. MEASUREMENT OF THE CIRCUMFERENCE AND DIAMETER OF A CIRCLE.—Measure carefully the diameter of one end of the cylinder. Then wrap the strip of paper around the curved surface of the cylinder at the same end, and mark upon the edge of the strip the point where the second winding of the paper begins to overlap the first. Then unfold the paper and measure upon it that distance which extended once around the cylinder. Then divide this distance, which of course is equal to the circumference of the circle, by the length of the diameter. The ratio thus obtained is one which it is important to know, although we shall not have much occasion to use it in this book. Mathematicians, physicists, and engineers use it so much that thay have a particular sign, π, to denote it.

This sign is a Greek letter and is called *pē* by students of Greek, but when used as just described it is often called *pī* to distinguish it from *p*.

13. Discussion of Exercise B.—The measurements of Exercise B may be discussed somewhat as follows: The square of the longest side of the triangle is found by one pupil to be 404.01, and the sum of the squares of the other two sides 406.05. If the two short sides were measured correctly, how large an error in the measurement of the longest side would cause the disagreement here found? The long side was measured as 20.10 cm. If it had been called 20.20 cm., its square would have been 408.04, which is about as much too large as the square actually found is too small. If the distance had been measured as 20.15 cm., the square would have been 406.02, a quantity very close

indeed to the sum of the other two squares. If, therefore, the original error lay entirely in the measurement of the longest side, this error must have been very nearly 0.05 cm. Of course the error may have been made in measuring the other sides, or in drawing the triangle, or in all parts of the work. An error which mistakes 20.15 for 20.10, or 201.5 for 201.0, or 2015 for 2010, is called in each case an error of 5 parts in 2015, or 1 part in 403, or an error of about $\frac{1}{4}$ per cent (see remarks following Exercise A).

QUESTION.

In the case of the circle, which would make the greater difference in the result (circumference ÷ diameter), an error of 0.05 cm. in the measurement of the diameter or an error of 0.10 cm. in the measurement of the circumference?

Measurement of Area.

14. Unit of Area.—Thus far we have been measuring lines. To measure a line, as we see, is merely to find out by trial that it is so many centimeters or inches long. A line 10.6 cm. long is one that could be divided into ten full centimeters and six tenths of another centimeter. We here call the centimeter our *unit* of length.

If we have to measure a *surface*, the whole table-top, for instance, our task is to find the number of square centimeters, or square inches, or square feet, that would be required to cover it, or that it would make if it were cut up without waste into squares. In this case the square centimeter, or square inch, or whatever square we choose to take, is the *unit* of area. We might set about to measure surfaces by actually placing a little square, a square centimeter, for instance, on the given surface, marking a line close around it, then moving it to a new place, marking around it, and so on till we had marked off the whole surface into little squares, with perhaps some fractions of

squares. But this is not the common or the best way of measuring surfaces. The common way is to measure the length of certain lines on the surface and from the lengths of these lines to *calculate* the extent of the surface.

15. Measurement of Rectangles.—If the surface is in the form of a rectangle, like Fig. 3, it is plain that we have merely to multiply the number of units, centimeters let us say, in the length by the number of centimeters in the width, and the result, $8 \times 4 = 32$ in this figure, is the number of square centimeters into which the surface can be divided. This is called the *extent* or *area* of the surface.

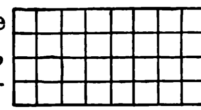

FIG. 3.

In the next Exercise we shall undertake to find rules for the measurement of surfaces not quite so simple in shape as the rectangle shown in Fig. 3. These will be of the class called parallelograms.

16. Parallelograms.—A *parallelogram* is a flat figure bounded by four straight lines, each line being parallel to the line opposite. Thus A and B in Fig. 4 are parallelograms.

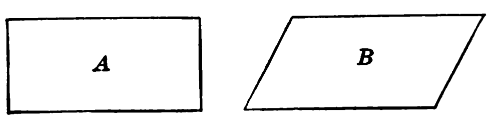

FIG. 4.

A is what we have just called a rectangle, and we have seen how to find the area of any rectangle, but B is not quite so simple at first sight. A parallelogram like B, which contains no right angle, is called an *oblique* parallelogram.

EXERCISE C.

AREA OF AN OBLIQUE PARALLELOGRAM.

Apparatus: The 30-cm. measuring-stick (No. 3). An oblique parallelogram of paper about 20 cm. long and 10 cm. wide. (One of the straight-edged rulers (No. 24) may prove useful in this Exercise.)

Draw upon the paper figure a line like *c* in Fig. 5, taking care to make a right angle with the top line and the bottom line, and then cut or tear the paper along the line *c*. Take the small piece thus removed and join it to the larger piece, in such a way as to make a figure that you know how to measure. Measure the length and width of the figure thus formed and calculate the extent of its surface.

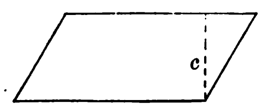

FIG. 5.

Then put the two pieces together as they were at first and ask yourself whether you could not, if another oblique parallelogram were given you, find the extent of its surface without cutting it.

For the Class-room.

Estimate without measurement the length and width of some visible and convenient rectangles, a book-cover, a table-top, a window, etc., and calculate the areas from these estimated dimensions. Then take the true dimensions and calculate the true areas.

Measurement of Volume.

17. Unit of Volume.—We have now to speak of the measurement of *volume*. The *unit* of volume may be the cubic centimeter, or the cubic inch, or the cubic foot, etc. We shall generally use the cubic centimeter as our unit.

We mean, then, by the *volume of a body* the number of cubic centimeters that could be made of that body if it were cut up without waste, as one might cut up a large piece of clay or putty.

18. Rectangular Bodies.—In the case of a body whose surface is made up of rectangles, a brick, for instance, it is

easy to see how the volume may be calculated, if we know the length and the width and the thickness. We have *volume* = *length* × *width* × *thickness.*

19. Irregular Bodies.—If the body is of less regular shape, like an ordinary stone or a lump of coal, it is not so easy to calculate its volume from measurements of length, width, and thickness. There is, however, a very easy way of finding the volume of such a body by the use of water, as will presently be seen.

20. Volume of Water.—It is easy to find the volume of a quantity of water in several ways. One way is to pour the water into a rectangular box. Then we can measure its length and width and depth and calculate its volume. Another way is to pour it into a glass measuring-dish having marks upon it to tell the number of cubic centimeters required to fill it to certain depths. Another method is to weigh the water, for it is known that one cubic centimeter of water weighs one *gram.* Indeed this is the *definition* of one gram, *the weight of a cubic centimeter of water.** If the balance which we use for weighing reads in ounces instead of grams, we shall have to remember that *1 oz.* = *about 28.3 gm.*, so that 1 oz. of water will be 28.3 cubic centimeters. We shall commonly find the volume of a body of water by *weighing.*

21. The Water Method.—We will now try the water method of finding the volume of a body, a rectangular solid. We shall find its volume by the water method and also by direct measurement and calculation, and then see how well the two results agree. This will test the water method, and if we find it to work well, we can use it with irregular solids which we cannot measure directly.

* To be exact one must add *at 4° of the centigrade scale of temperature.* For the purpose of this book such exactness is unnecessary.

EXERCISE D.

VOLUME OF A RECTANGULAR BODY BY DISPLACEMENT OF WATER.

Apparatus: A brass can (No. 5) called C in Fig. 6 A small catch-bucket (No. 6) called p in Fig. 6. A spring-balance (No. 7). A rectangular block of wood (No. 8) so loaded as to sink in water.

Closing the overflow tube t of the can C, pour water into C until it is filled nearly to the brim. Then open the tube and let all the water flow out that will do so, catching it in the small can p. The large can should rest steadily upon the table, but the small one is better held in the hand when the flow begins, otherwise some water may be spilled. The flow should stop rather suddenly at last, with little or no drip.

Throw away all the water thus caught in p and then weigh p on the spring-balance to the nearest gram or the nearest twentieth of an ounce, according to the graduation of the balance.* Then, closing the tube t as before, lower into the can C the wooden block until it rests upon the bottom. Then, or sooner if the can C seems

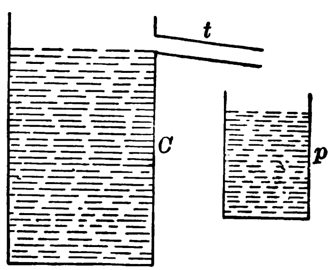

FIG. 6.

likely to be overflowed, open the tube t, and as before catch the water that runs out in the small can p. The water, Fig. 7, now

* Ordinary small spring-balances now in the market are often marked off in half-ounce divisions, which are about $\frac{1}{8}$ inch long. The pupil will learn to *estimate* the position of the pointer when it falls between two lines, so as to read to about $\frac{1}{20}$ of an ounce.

stands just as high in C as it did just before the block was put into it. The block has crowded out into the can p just its own bulk of

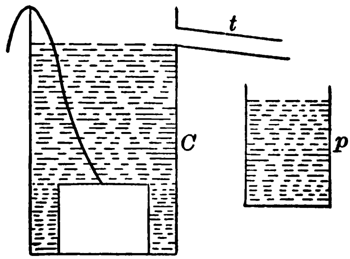

FIG. 7.

water. If, then, we can find the volume of the water that the block drove over into p, we shall have the volume of the block itself.

Weigh p and the water it contains.

Weight of small can and water =........
" " " " empty =........
" " water alone =........

If the weight as thus found is in grams, it is equal to the number of cubic centimeters in the block. If the weight as thus found is in ounces, we must multiply the number of ounces by 28.3 in order to find the number of cubic centimeters in the block.

Now measure carefully the length, width, and thickness of the block and calculate the number of cubic centimeters it contains from these measurements.

(Experiments for finding the volumes of irregular bodies by the water method may well be postponed till the next Exercise, which would otherwise be a very brief one. Potatoes, stones, lumps of coal, etc., of suitable size may be used for these further experiments.

Practice for the Eye.

A line 10 inches long is drawn on a blackboard with a cross-line at any point, and the members of the class *estimate* the distance from either end to the cross-line. Practice like this helps toward accurate reading of the spring-balance.

QUESTIONS.

(1) The true length of a certain line is 16.4 cm. One person measures it as 16.6 cm., another as 16.3 cm. How great is the error of each in per cents of the true length?

(2) A certain rectangle is 50 cm. long and 20 cm. wide. It is measured by one person as 50 cm. long and 20.2 cm. wide, and by another person as 50.2 cm. long and 20 cm. wide. If the *area* is calculated from each set of measurements, how great (in per cents) will the error be in each case?

(3) A certain rectangle has a base 100 cm. long and an altitude of 40 cm. Which will cause the greater error in the estimated area, an error of 2 cm. in the base or an error of 1 cm. in the altitude?

(4) A rectangular solid is 40 cm. long, 30 cm. wide, and 20 cm. thick. How great (in per cents) is the error made by calculating the volume from measurements which give 41 cm. for the length, 31 cm. for the width, and 19 cm. for the thickness?

CHAPTER II.

DENSITY AND SPECIFIC GRAVITY.

22. Definition of Density.—The weight of unit volume of a substance is called the *density* of the substance. If we know the density of a substance we can calculate the weight of any volume of that substance. Engineers and other scientific men often have to find by this method the weight of objects which it would be inconvenient to weigh. The weights of buildings and bridges, for instance, are found in this way. Books used by scientific men contain tables giving the densities of many different substances.

The density of a substance may be expressed as the weight in grams of one cubic centimeter, or as the weight in pounds of one cubic foot, or in any one of many other ways. For brevity, we call the first method of expression just given the *density in grams and cubic centimeters*, and the second, the *density in pounds and cubic feet.* The following Exercise will make the matter plainer, and will give good practice in measuring and weighing.

EXERCISE I.

WEIGHT OF UNIT VOLUME OF A SUBSTANCE.

Apparatus: A block of wood (No. 9). A spring-balance (No. 7). A measuring-stick (No. 3). Thread for suspending the block.

Find the weight of the block in grams and also in ounces.

Measure the length of each of the four edges which are parallel to the grain of the wood, take the average of these measurements and call it the *length* of the block.

Measure the length of each of the four long edges which are crosswise to the grain of the wood, and call the average of these four measurements the *width* of the block.

Measure the length of each of the four short edges and call the average of these four measurements the *thickness* of the block.

The weight in ounces is to be turned into pounds.

From the length, width, and thickness in centimeters the length, width, and thickness in *feet* may be found by the rule that 1 ft. = 30.5 cm., but it is shorter to find the volume in feet from the volume in cubic centimeters by the rule that 1 cu. ft. = 28300 cu. cm.

Calculate, 1st, how many grams, or what part of a gram, 1 cu. cm. of the block weighs; 2d, how many pounds, or what part of a pound, 1 cu. ft. of such wood weighs.

23. Density of Water.—The density of water in grams and cubic centimeters is 1; that is, 1 cu. cm. of water weighs 1 gm. (see § 20). The density of water in pounds and cubic feet is very nearly 62.4; that is, 1 cu. ft. of water weighs 62.4 lbs. These numbers for water should be committed to memory.

QUESTIONS.

1. What ratio is found from the results* of Exercise 1 between the density of wood in grams and cubic centimeters and its density in pounds and cubic feet?

2. How does this compare with the ratio of the two densities of water, as given above?

3. If the ratio is the same for the wood as for water, is this a mere coincidence, or is the same thing true in the case of other substances?

PROBLEMS.

(1) If a piece of iron 10 cm. long, 8 cm. wide, and 7 cm. thick weighs 4000 gm., what is its density in gm. and cu. cm.? What is its density in lbs. and cu. ft.?

(2) The density of mercury in gm. and cu. cm. is about 13.6. How many lbs. would 1 cu. ft. of it weigh?

* It is well to take the average of the results found by the various members of the class.

24. Weight.—Before going farther we need to think carefully about the meaning of the word *weight*, which we have already used a number of times, and shall have to use very often. The word has two meanings.

Sometimes when we speak of the weight of a body we mean the *amount* of the body, as when we speak of 10 lbs. of butter or 100 lbs. of iron.

At other times we mean by the weight of a body the amount of the earth's downward pull upon that body, as shown by the spring-balance, for instance.

It is somewhat hard to remember this distinction, because the *units* in which we tell the *amount* of a body have the same name as the units in which we tell the *pull* which the earth exerts upon the body. For instance, we say that the earth exerts a pull, or *force*, of 5 lbs. upon 5 lbs. of wood, or 5 lbs. of coal, or anything which consists of, or *is*, 5 lbs. of substance.

Often when we use the word *weight* it makes no difference which of its two meanings we have in mind, but sometimes it does make a difference. Thus, when we put a body under water, as we shall do in the next Exercise, and say that it appears to lose *weight* in going from air to water, we do not mean that there appears to be any less of the *body* in water than there was in air. We mean that it requires a smaller pull of the spring-balance to keep the body from sinking in water than it does to keep it from sinking in air.

25. Mass.—In strict scientific language, the word *mass* is commonly used in speaking of the *amount* of a substance, and the word *weight* in speaking of the earth's pull upon that substance. For example, a piece of iron the *mass* of which is 50 lbs. is subject to a *weight* * of 50 lbs. exerted by the earth.

* Such distinctions, which use words in a *scientific* sense different from the popular every-day sense, are often necessary in science, but it would be rather absurd to try to make the popular use of the words agree with the scientific use in all cases.

Specific Gravity.

26. Definition.—It is often convenient to know the *ratio which the weight of a body bears to the weight of an equal bulk of water.* This ratio is called the *specific gravity* of the body. *Gravity* comes from a Latin word *gravis*, meaning *heavy*. *Specific* here means *distinctive*, or *particular*. The *specific gravity* of a body is its *particular heaviness*—the degree of heaviness which distinguishes this body from other bodies of the same size but different weight.

27. Loss of Weight in Water.—In finding specific gravities it is a common practice to weigh bodies under water. The use of this practice will be made plain by Exercise 3. The loss of apparent weight suffered by a body in going from air to water is shown in Exercise 2.

EXERCISE 2.

LIFTING EFFECT OF WATER UPON A BODY ENTIRELY IMMERSED IN IT.

Apparatus: Overflow-can (No. 5). Catch-bucket (No. 6) Spring-balance (No. 7). Loaded block (No. 8). Thread.

Fill the can and let it overflow and drip as in Exercise D. Catch this overflow in the small bucket and throw it away. Then weigh the empty bucket in grams.

Weigh the block in grams before immersing it in the water.

Lower the block, still suspended from the balance, into the overflow-can till it is entirely covered, catching the overflow and saving it.

Weigh the block in the water, the balance being entirely above the water.

Weigh the bucket with the overflowed water.

Subtract the (apparent) weight of the block in water from its weight in air, and call the difference the *loss of weight of the block in water*, or the *buoyant force exerted upon the block by the water.*

Find weight of the water in the small bucket, and compare this with the loss of weight of the block in water.

If there is time, make a similar experiment with other bodies.

The law illustrated in this Exercise is called from its discoverer the *law*, or *principle*, of *Archimedes.* (See any encyclopedia for an account of Archimedes.)

PROBLEMS.

(1) A certain body weighs 100 gm. out of water and 50 gm. in water. How great is the volume of the body?

(2) A certain body 5 cm. long, 3 cm. wide, and 2 cm. thick weighs 200 gm. in water. How much does it weigh out of water? 30

230

EXERCISE 3.

SPECIFIC GRAVITY OF A SOLID BODY THAT WILL SINK IN WATER.

Apparatus: The spring-balance (No. 7). The gallon jar (No. 10) nearly filled with water. A lump of sulphur (No. 11). Thread.

Weigh the sulphur out of water; then in water.

We know from Exercise 2 that a body immersed in water loses in apparent weight an amount equal to the weight of the water whose place it has taken. It is easy, therefore, to get from the two weighings just made the ratio which we have undertaken to find in this Exercise.

If time permits, find in this Exercise, by the same method that is used for the sulphur, the specific gravity of other solids that will sink in water—such as glass, coal, etc.

QUESTIONS.

1. If the specific gravity of 1 cu. cm. of iron is 7, what is the specific gravity of 50 cu. cm. of the same kind of iron? Of 1 cu. ft. of the same kind of iron?

2. If the sp. gr. of lead is 11.3, what is the weight in grams of 1 cu. cm. of lead? What, then, is the density of lead in grams and cubic centimeters (see § 22)?

3. If the sp. gr. of a certain kind of wood is 0.7, what is the weight in lbs. of 1 cu. ft. of this wood? What, then, is its density in lbs. and cu. ft.?

4. A certain body weighs 7 lbs. out of water and 4 lbs. in water. What is its specific gravity?

28. Various Expressions for Specific Gravity.—By definition we have

$$Sp.\ grav.\ of\ a\ body = \frac{Wt.\ of\ the\ body}{Wt.\ of\ an\ equal\ volume\ of\ water}.$$

It is evident that the quantity written below the line in this definition may be expressed in other ways. We may write

$$Sp.\ grav.\ of\ a\ body = \frac{Wt.\ of\ the\ body}{Wt.\ of\ water\ displaced\ by\ the\ body\ when\ immersed},$$

or

$$Sp.\ grav. = \frac{Wt.\ of\ the\ body}{Loss\ of\ weight\ of\ the\ body\ when\ immersed},$$

or

$$Sp.\ grav. = \frac{Wt.\ of\ the\ body}{Lifting\ effect\ of\ water\ upon\ the\ body\ when\ immersed}.$$

These expressions all mean the same thing, but sometimes one of them is more convenient than the others. In the Exercise next before us we shall use the last form.

EXERCISE 4.

SPECIFIC GRAVITY OF A BLOCK OF WOOD BY USE OF A SINKER.

Apparatus: A rectangular block of wood (No. 9). The spring-balance (No. 7). The gallon jar (No. 10) nearly filled with water. A lead sinker (No. 12). Thread.

We have to find two quantities, by experiment: 1st, the *weight of the body*; 2d, the *lifting effect of water upon it when immersed.*

Weigh the wood in air and record its weight.

Now put the block into water. You see that it floats. To make it

stay under water you must ***hold it down.*** Try this, putting your fingers on the block. In this case, you see, the lifting effect of the water, when the block is wholly beneath its surface, is greater than the weight of the block. We must find out how much it is.

We shall use the lead sinker to hold the block under water, and we need to know the weight of the sinker alone under water. Weigh it in this position and record the weight.

Now suspend the block from the balance * and the lead sinker from the thread under the block, and consider how much the two, block and sinker, would weigh in the position shown by Fig. 8, the block out of water and the sinker in water. You can tell this from the weighings already made. Write it down.

Wt. of block in air + Wt. of sinker in water = ... + ...

Now lower the block and sinker till both are cov-

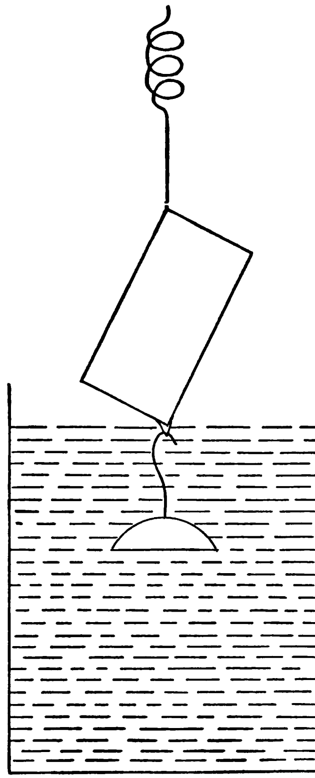

Fig. 8.

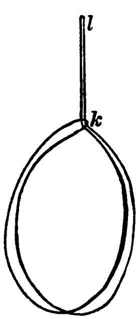

Fig. 9.

* The success of a difficult experiment like this depends greatly upon the care with which the details of the work are thought out by the teacher. The following method of attaching the block to the balance is recommended: Take a thread two feet long and tie the ends together. Then make of it a slip-noose by passing one end, *l* (Fig. 9), through the other end, *k*. The block may then be placed in the noose and the loop *l* slipped upon the hook of the balance, but to prevent slipping when the lead weight is to be suspended from the loop below the block it is well to pass the loop *l* *twice* through at *k*.

ered by the water, and weigh the two together in this position and record:

Wt. of block and sinker together in water = ...

Just before the *block* entered the water, the sinker being already in, the weight was . . . . Just as soon as the block also was covered the weight was only . . . . The difference is the *lifting effect of the water upon the block.* We have now all that we need for calculating the specific gravity of the block by means of the formula,

$$Sp.\ grav. = \frac{Wt.\ of\ block}{Lifting\ effect\ of\ water\ upon\ block\ immersed}.$$

QUESTIONS.

(1) A brick-shaped body 20 cm. long, 10 cm. wide, and 5 cm. thick weighs 1500 grams. What is its *density* in gram and centimeter units?

What would be the weight of an equal bulk of water?

What, then, is the *specific gravity* of this body?

(2) A body whose volume is 700 cu. cm. has the density 8 in gram and centimeter units. How much does it weigh? What is its specific gravity?

(3) A body 20 ft. long, 10 ft. wide, and 5 ft. thick weighs 93,600 lbs. What is its density in pound and foot units?

What would be weight of an equal bulk of water, one cu. ft. of water weighing 62.4 lbs.? What, then, is the specific gravity of the body?

(4) A body whose volume is 700 cu. ft. has the density 499.2 in pound and foot units. How much does it weigh? What is its specific gravity?

(5) What numerical relation do we find in these problems, and in those of page 19, between density in gram and centimeter units and specific gravity?

(6) What relation do we find in the same problems between density, in pound and foot units, and specific gravity?

29. Flotation.—Thus far we have been considering the action of water upon bodies entirely immersed in it. We shall now have to do with floating bodies.

EXERCISE 5.

WEIGHT OF WATER DISPLACED BY A FLOATING BODY.

Apparatus: The same as in Exercise 2, with the exception of the sinking body, which is here replaced by one that floats (No. 4).

Weigh the cylinder, in grams, in air. Find, in grams, the weight of water which it displaces from the overflow-can. Compare these two weights.

It will be well to repeat the overflow operation carefully a number of times.

The fact shown in this Exercise concerning the relation between the weight of a floating body and the weight of water displaced by it should be firmly fixed in the experimenter's mind. It leads to a method of finding the specific gravity of floating bodies.

EXERCISE 6.

SPECIFIC GRAVITY BY FLOATING METHOD.

Apparatus: The gallon jar (No. 10) nearly filled with water. A slender wooden cylinder (No. 13). A support for holding this cylinder upright in water (No. 14). A measuring-stick (No. 3).

If a cylinder floated upright with its top just level with the top of the water, we should at once know its specific gravity to be 1. If it floated just half in and half out of water, we should know its specific gravity to be 0.5. The cylinder that we have to use will not float all in water or exactly half in water, but if we float it, and find the length of the part then in the water, we shall, by comparing this with the length of the whole cylinder, find some way of ascertaining the specific gravity of the cylinder,

Measure the length of the whole cylinder.

Float the cylinder in the jar (Fig. 10), keeping it upright by

means of the holder, which is attached to the side of the jar. Jeggle the cylinder to make sure that it is free to take its proper position. After each joggling it should come to rest at the same depth as before. The rings of the holder must not *grip* the cylinder at all. When sure that the cylinder floats as it should, measure the length of the submerged part, from the bottom of the cylinder up to the *flat* surface of the water.

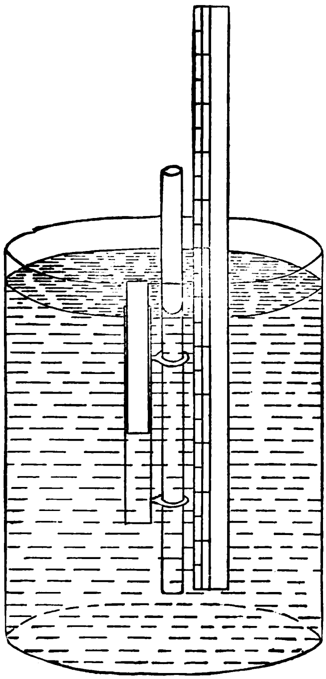

FIG. 10.

To find the specific gravity from the two measurements now made, begin by recalling the fact (see Exercise 5) that the water displaced by the floating cylinder weighs just as much as the cylinder itself.

How many times is the length of the submerged part of the cylinder contained in the whole length?

How many times the weight of the cylinder would be the weight of a like cylinder of water?

How great, then, do you find the specific gravity of the wooden cylinder to be?

PROBLEMS AND QUESTIONS.

(1) A block whose specific gravity is 0.6 floats in water. How much of it is below the surface?

(2) A block whose volume is 1000 cu. cm., and whose specific gravity is 0.4, floats in water. How many cu. cm. of the block are below the surface?

(3) A block that weighs 4 oz. in air is fastened to a sinker that weighs 6 oz. in water, and the two together weigh 3 oz. in water. What is the specific gravity of the block?

(4) A block whose specific gravity is 0.5, and which weighs 100 gm. alone in air, is fastened to a sinker that weighs 150 gm. alone in water. How much will both together weigh in water?

(5) A certain body has the density 187.2 in pound and foot units. What is its specific gravity?

(6) Is the specific gravity of the human body much greater or much less than 1?

(7) Why does filling the lungs with air help one to *float* in water?

EXERCISE 7.

SPECIFIC GRAVITY OF A LIQUID: TWO METHODS.

Apparatus: The gallon jar (No. 10) nearly filled with water, and the smaller jar (No. 15) nearly filled with a solution of sulphate of copper.* The small glass bottle (No. 16). The spring-balance (No. 7). Thread.

FIRST METHOD.

Weigh the bottle empty. Dip the bottle into the jar of sulphate of copper and let it fill with the liquid. Holding the bottle over the jar, put the stopper in place, thus crowding out the excess of liquid, then wipe the outside of the bottle and weigh it carefully with its contents.

Pour the sulphate of copper back into its jar, then fill the bottle with water, just as it was before filled with the other liquid, and again weigh the bottle and its contents.

From the three weighings now made the specific gravity of sulphate of copper can easily be found.

SECOND METHOD.

We found in Exercise 2 that a body going from air into water lost in apparent weight an amount equal to the weight of its own bulk of water. So a body going from air into a solution of sulphate of copper will lose in apparent weight an amount equal to the weight of its own bulk of the solution. This gives a method of finding the specific gravity of the solution. As a *body* to be weighed first in air, then in water, then in the solution, we will use the bottle with enough water

* This solution may be made by putting 2 lbs. of sulphate of copper crystals into about 3 qts. of *warm* water in a glass vessel and stirring occasionally till the crystals are dissolved.

in it to make it sink in either liquid. We may, indeed, use the bottle *full* of water, just as it was left at the end of the first part of this Exercise.

EXPERIMENTS.

1. Exhibit and show in operation two graduated glass hydrometers —one for determining the specific gravity of liquids less dense than water (App. No. XI.), the other for use with liquids more dense than water (App. No. XII.).

2. Show in a bottle together several liquids of different specific gravities that do not tend to mix with each other; for instance, mercury, chloroform, water, and kerosene.

3. Take a small tumbler containing some mercury and drop into it a piece of iron. Do not put into it gold or silver, as mercury attacks these metals.

4. Place a dry sponge on water. It floats lightly, but is the specific gravity of the *fibres* of the sponge greater or less than that of water? To answer this question push the sponge beneath the surface. What rises from it? Squeeze the sponge very hard till nothing more seems to come from it. Now will it rise to the surface when released?

QUESTIONS.

1. A glass sphere which weighs 100 gm. in air weighs 60 gm. in water and 40 gm. in sulphuric acid of a certain strength. What is the specific gravity of the glass?

What is the specific gravity of the sulphuric acid?

2. A vessel contains a layer of water 10 cm. deep and above this a layer of kerosene (sp. gr. 0.8) 10 cm. deep. What is the weight of a cube, each edge of which is 10 cm. long, that, if placed in this vessel, will sink till one-half its volume is in the water and one-half in the kerosene? *Ans.* 900 gm. What is its specific gravity? *Ans.* 0.9.

3. A certain ship weighs with its cargo 10,000 tons.

(*a*) How many cubic feet of fresh water would it displace?

(*b*) How many cubic feet of sea-water of specific gravity 1.026 would it displace?

4. If 1 cu. cm. of mercury weighs 13.6 gm. and 1 cu. cm. of cork weighs 0.25 gm., how deep will a cylinder of cork 20 cm. long sink, when placed on end in mercury?

5. Which has the greater specific gravity, cream or skimmed milk?

6. A piece of cloth thrown upon water will float at first and afterward sink. Why does it not sink at once?

7. Can the pupil tell from his own observation which of the following substances are more dense and which are less dense than water: kerosene-oil, ordinary lubricating-oil, butter, cheese, potatoes, eggs, meat, ice, india-rubber?

CHAPTER III.

FLUID-PRESSURE.

30. Fluids.—Water and air readily *flow* from one position or shape to another. They are examples of that class of substances called *fluids*. Fine sand and other like substances flow in a certain way, but examination shows them to consist of little hard or tough particles very different from equally small particles of water.

Fluids are divided into *liquids* and *gases*. Water is an example of the liquids; air an example of the gases.

31. Fluid-pressure.—Fluids settle snugly around solid, that is, *non-fluid*, bodies placed in them and act upon these bodies with a peculiarly even pressure. We shall now make some experiments with liquid-pressure and later with gas-pressure.

EXPERIMENTS WITH PRESSURE-GAUGE.

Fill the gallon glass jar (No. 10) with water to a level about one inch from the top. Close the smaller end of a student-lamp chimney tight with a good cork stopper. Make the pressure-gauge (No. I.) ready for use by the following operation, having first put on a fresh rubber diaphragm if necessary: Release the glass tube from the rubber tube and wet the whole length of the glass tube inside with water, leaving within it a column of water about one-half inch long to serve as an index.* Hold the gauge itself under water for a little time before reconnecting the glass tube with the rubber tube,

* It may be necessary to use water colored by some aniline dye before a large class.

in order to allow the air within the gauge to come to the temperature of the water. On reconnecting the glass tube leave the water-index near the rubber tube.

DIFFERENT LEVELS.—Now push the gauge down into the jar and raise and lower it repeatedly in the water, keeping the glass tube with the water-index horizontal, and let the class determine from the movements of this index whether the pressure of the water against the rubber diaphragm increases or decreases when the gauge is pushed deeper in the water.

DIFFERENT DIRECTIONS.—Rest the bottom of the supporting pillar of the gauge upon the bottom of the jar, and, still keeping the glass tube horizontal, turn the upper pulley so that by means of the rubber band the lower pulley will be turned and the rubber diaphragm will face downward, sidewise, and upward in succession, its centre remaining practically unchanged in position. Let the class determine by watching the water-index whether the pressure upon the rubber diaphragm is any greater when it faces upward than when it faces downward or sidewise.

DIFFERENT POINTS ON THE SAME LEVEL.—Push the closed end of the lamp-chimney down into the water till it is near the bottom of the jar. Move the gauge-face about, without changing its level, so as to bring it under this closed end. Move it now out of and now into this position, thus changing the depth of water immediately above it from one-half inch or less to several inches. Let the class determine by watching the index whether such changes of position, without change of *level*, make any difference in the pressure against the gauge-face.

We shall make considerable use farther on of the facts brought out by these experiments. Just here we can see that they explain, at least in a general way, why a body immersed in water weighs, or appears to weigh, less than when in the air. For we see that there is an upward pressure of the water against the under side of the body, and that this upward pressure is greater than the downward pressure against the upper side of the body.

32. Slight Effect of Pressure upon the Density of Water. —Having seen that there is greater pressure on low levels

than on high levels in water, we may well ask whether this greater pressure crowds the particles of water closer together on the low levels, thus making the water *denser* than on high levels. In fact there is an effect of this kind, but it is so slight that we need take no account of it in any ordinary case. It is very difficult to compress water much.

EXPERIMENT.

Fill a bottle with water and close it with a rubber stopper having one hole through it. Then, holding the stopper firmly in place, push down into the hole a solid brass rod of a size to fit rather closely. The bottle will probably be broken by this effort to compress the water within it. (App. No. II.)

33. Uniform Increase of Pressure with Depth.—We have not made, and cannot well make with the *gauge* used, any accurate measurement of the rate at which pressure changes with change of level in water. The fact is, however, that if we place a surface of 1 sq. cm. horizontal at any depth in water the column of water just above it is resting upon the given surface.* If we carry the given surface down 1 cm. farther, we now have resting upon it a load somewhat greater than before, greater by the weight of the additional 1 cu. cm. of water which is now above it. As 1 cu. cm. of water weighs 1 gm., the pressure upon a surface of 1 sq. cm. changes by 1 gm. for each 1 cm. change of level in the water.

QUESTIONS.

A cubical box, 10 cm. along each edge, has extending from its top, as in Fig. 11, a tube 15 cm. tall and 1 sq. cm. in cross-section (inside).

(1) If the box, but not the tube, is full of water, how great is the water-pressure on the whole of the bottom?

* The pressure upon the given surface may be greater than the weight of the column of water resting upon it, for there may be, and usually is, a downward pressure of air or something else upon the top of the water-column.

(2) If the tube as well as the box is full of water, how great is the pressure upon that one sq. cm. of the bottom which lies just beneath the tube?

(3) Is the pressure equally great per sq. cm. at other parts of the bottom?

(4) How much is the total pressure now on the bottom of the box?

(5) How great is the pressure per sq. cm. at the top of the box just at the bottom of the tube?

(6) How great is the total *upward* pressure of the water against the top of the box?

(Disregard the atmospheric pressure upon the top of the water-column in all these questions at first. Afterward call this atmospheric pressure 1000 gm. per sq. cm., and ask the same questions as before.)

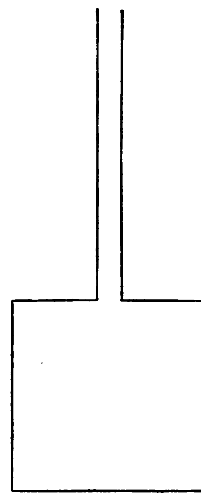

FIG. 11.

34. Gas-pressure.—We have made some experiments with liquid-pressure. We must now begin to learn something about air-pressure, which in many practical matters of every-day life has a very important connection with water-pressure. We will at the start repeat in a slightly varied form a famous experiment first made by Torricelli, an Italian, about the middle of the seventeenth century. It is intended to show the pressure of the air about us, which is called *atmospheric pressure.*

EXPERIMENT.*

Take two pieces of strong glass tubing about 0.7 cm. in inside diameter, one of them, about 1 m. long, closed at one end, and the other, about 20 cm. long, open at both ends, and connect them by means of a thick-walled piece of rubber tubing about 25 cm. long. The rubber tube should fit tight upon the glass tubes, and for greater security should be fastened on by means of wire or string.

* This experiment can be more conveniently performed with a single straight glass tube if a mercury-well is available.

Holding the tubes thus connected (App. No. III.) by the free end of the short glass tube, the closed end of the long glass tube hanging down, pour mercury by means of a small funnel of glass or paper into the tubes, tapping or shaking them occasionally to dislodge air-bubbles, until the top of the mercury-column reaches the rubber tube. Then gently raise the closed end of the long glass tube until this tube points straight upward (Fig. 12), meanwhile holding the other glass tube upright and taking care that no mercury is spilled.

During the latter part of this operation it will be noticed that the mercury begins to fall away from the closed end of the long glass tube, and finally several inches of this tube will be apparently empty.* But the mercury continues to stand very much higher in the long glass tube than in the short one.

FIG. 12.

35. Explanation.—It was known before the time of Torricelli that if air was drawn from the upper part of a tube the lower end of which rested in water the water would rise in the tube, but the true reason for this was not known. Torricelli maintained, and Pascal, a Frenchman, showed by experimenting at different heights in the air, that the pressure of the atmosphere, due to its weight, accounted for the rise of liquids in a vacuum. We have only to think of the fact that the air, although its density is very small compared with that of water, has, because of its great quantity, a great weight, and we see that the air, pressing upon the mercury-surface in the shorter tube, balances the column of mercury in the long tube.

36. Amount of the Atmospheric Pressure—Barometer.—By measuring the difference in height of the two mercury-

* Really this space contains a very little air, from the bubbles that were in the mercury-column before it was inverted, but so little that we may at present disregard it and consider the space above the mercury as empty. Such a space is called a *vacuum*, from a Latin word meaning *empty*.

surfaces we can get a measure of the atmospheric pressure. We find that the atmospheric pressure is about as great upon the surface of the earth as would be the pressure of a layer of mercury 76 cm. deep, or a layer of water about 10.3 m. deep, over the whole earth. The pressure per square centimeter at any given part of the earth's surface varies somewhat from day to day, and even from hour to hour.

If we fasten the apparatus that has just been used to a suitable support, it will serve permanently as a rude *barometer*, indicating the variations of the atmospheric pressure.

37. Pressure in Different Directions.—Air-pressure, like liquid-pressure, is at any given point equal in all directions, if the air is at rest.

EXPERIMENT.

Take a strong thistle-tube (No. IV.) of the shape shown in Fig. 13 and tie a piece of thick sheet rubber across the mouth, which may be about 1 inch in diameter. Make the covering air-tight by means of some cement, melted beeswax and rosin, for instance, poured in at the point *J*. Connect this thistle-tube by means of a thick-walled rubber tube to an air-pump (No. V.), and exhaust the air. The rubber cap, not being supported by air-pressure beneath, will now be pushed down by the atmospheric pressure into a deep cup-shape Pinch the rubber tube so that no air shall leak back into the thistle-tube, and then turn the mouth of the latter in all directions, sidewise, downward, and oblique. Observe whether the depth of the rubber cup changes during this operation, as it would do if the pressure upon it changed.

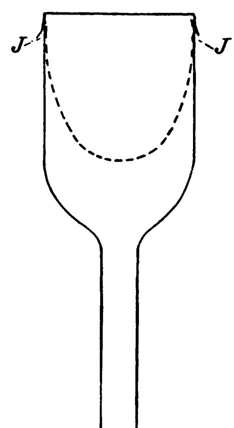

Fig. 13.

38. Air-pressure at Different Levels.—We should find by proper experiments that in air at rest, as in water at rest, pressure is equally great at all points on the same level. We should find, also, that the air-pressure diminishes with increase of height from the earth's surface, but, as the density of air is very little compared with that of water, it requires a considerable change of level to make much difference in the air-pressure.

The rate at which atmospheric pressure decreases with increase of height being well known, it is a common practice to estimate the height of mountains by noting the difference of atmospheric pressure at the summit and base. "Aneroid" barometers are frequently used for such work. *Aneroid* means *without liquid.* An aneroid barometer contains no mercury nor other liquid. It is an air-tight metal box with a flexible metal cover. The middle of the cover moves in or out slightly with changes of pressure, and its slight motions are magnified to the eye by various mechanical contrivances. Some aneroid barometers are about as large as ordinary watches and look much like them.

39. Difference between Liquids and Gases.—Liquids are much heavier than gases, in most cases. Most liquids are easily *seen.* Most gases are practically invisible. But perhaps the most striking difference between liquids and gases is a difference in compressibility. We have seen that it is difficult to compress water much, but it is very easy to compress air.

EXPERIMENT.

Take the bent glass tube (No. VI.), closed at one end, and pour into it a little mercury, enough to fill the bend. At first the mercury will stand a little higher in the long arm, but by tipping the tube and letting out a little of the air imprisoned in the short arm the level can be made nearly the same in both arms, as in Fig. 14. Now

measure the length of the imprisoned air-column, and write it under the letter V* on the blackboard.

V.	P.	$V \times P$.
....	....	....
....	....	....
....	....	....
....	....	....


FIG. 14.

The pressure upon this air is now, if the mercury-level is the same in both arms, equal to that upon the unimprisoned air. It is as great a pressure as would be exerted by the weight of a column of mercury as tall as that in the barometer (Fig. 12). Take, then, a reading of this barometer and record this reading under the letter P.

Pour in more mercury till the difference of level in the two arms is about 20 cm., then measure again the length of the inclosed air-column. Record this length under V, and record under P the present difference of mercury level *plus* the height of the barometer column.

Proceed by stages in this way till the volume of the inclosed air-column is about one-half what it was at first. Multiply each number under V by the corresponding number under P, and write the products in the column headed $V \times P$.

40. Boyle's Law.—An examination of the last column in the table of the preceding section will probably indicate a very simple law connecting pressure and volume in the case of a given body of air. This law is important, and should be remembered by the pupil. It is sometimes called Boyle's law and sometimes Mariotte's law. We shall call it by the shorter name, *Boyle's law*.

Illustrations and Applications of Fluid-pressure.

41. Principle of the Hydraulic Press.—The questions on pp. 30 and 31 have brought out the fact that pressure trans-

* The *length* of the air-column is the same as its *volume*, if we take for our unit of volume the space contained in unit length of the tube.

mitted through a small tube may extend to a broad surface beyond the tube so as to make the total pressure on this surface very great. The following experiment will show that similar effects can be produced with air-pressure.

EXPERIMENT.*

Take a common rubber football and blow air into it till it is about half filled, connecting a rubber tube with the key for greater convenience in blowing (App. No. VII). Then rest one end of a board,

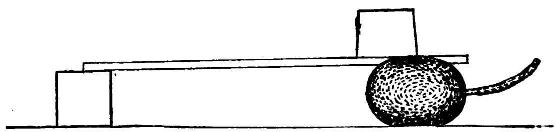

FIG. 15.

as in Fig. 15, on the football and the other end upon a box or block of about the same height. Then place a weight of 25 lbs. or more on the board nearly over the ball, holding the rubber tube attached to the key in such a way that the air cannot escape from the ball. Then blow through the tube into the ball and observe that you can in this way lift the weight.

42. Hydrostatic Press. — The preceding experiment illustrates the operation of the *hydrostatic press*, a machine in which a very great force is obtained, for lifting or compressing bodies, by pumping water through a small tube into a large cylinder, one end of which is closed by a movable stopper called a *piston*. (See § 204.)

EXPERIMENTS.

1. Take again the pressure-gauge and the accompanying apparatus, § 31. Fill the lamp-chimney with water, and then, holding a card across the open end, invert the chimney, lower the end covered by the card into the water, and then remove the card. Most of the water will now remain in the chimney, although its upper end is nine or ten inches above the surface of the water in the jar.

* An experiment with Gage's piston and cylinder apparatus may be substituted for this to show the same effect.

How does the pressure per sq. cm. inside the chimney on a level with the outside water-surface compare with the pressure per sq. cm. at this outer surface, that is, the atmospheric pressure?

How, then, will the pressure per sq. cm. at points higher in the chimney compare with the atmospheric pressure?

After these questions have been answered by the aid of what the class already knows about liquid-pressure, test the correctness of the answer by means of the gauge.

2. Take a long narrow glass tube open at both ends, and dip one end into a vessel of water. Apply the lips to the other end and draw the water up till the tube is filled.

In what sense is the water *drawn* up?

(The operation begins with an expansion of the lungs which lessens the air-pressure within them. Then air runs from the place of high pressure, the tube, to the place of low pressure, the lungs. So the air-pressure within the tube is lessened.)

3. After nearly filling the tube as in Experiment 2 quickly close the top with a finger and then lift the lower end from the water. Uncover the top of the tube for an instant, then cover it again.

Explain the behavior of the water during these operations.

4. Fill or nearly fill a tumbler or broad-mouthed bottle with water and then cover it with a sheet of thick paper. Hold the paper firmly in place with the hand and invert the tumbler; then take away the hand that holds the paper. (As accidents may happen, the tumbler should be held over some large dish.)

In this experiment it should be noticed that the paper does not press close against the rim of the tumbler after the inversion. It hangs rather loose, having dropped down or sagged a little, thus allowing the air above the water to expand a trifle, decreasing in pressure.

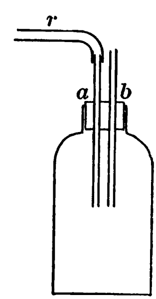

FIG. 16.

5. Fig. 16 (App. No. VIII) shows a bottle closed with a rubber stopper through which two glass tubes, *a* and *b*, open at both ends, extend. To one of the tubes, *a*, is attached a rubber tube, *r*. The bottle and the two glass tubes are full of water.

By applying the lips to the outer end of the tube *r* water can be "drawn" into the mouth. Can this be done when the tube *b* is closed by a finger at the top?

6. Show some form of the Cartesian *diver* (No. XIV), explaining why it sinks when greater pressure is put upon the water in which it is placed.

43. Pumps.—Many contrivances for making fluids run from one place to another are called *pumps*. A flow may be caused by decreasing the pressure at the place where the fluid is to be delivered or by increasing the pressure at the place from which it is to be removed.

EXPERIMENT.

Show in operation glass models of the "lifting-pump" (App. No. IX, Fig. 17) and "force-pump" (App. No. X, Fig. 18), discussing their action.

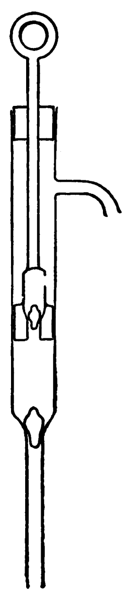

FIG. 17.

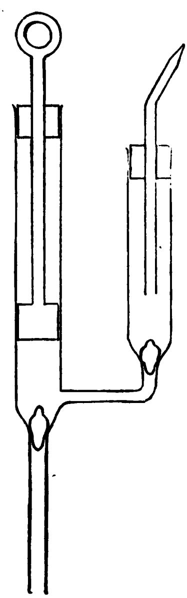

FIG. 18.

44. The Siphon.—The apparatus illustrated in the following experiment is called the *siphon*. It is found in a great variety of forms and is of much use.

EXPERIMENT.

Take two glass tubes, each about 6 in. long, connected by a rubber tube about 1 ft. long. Fill the whole with water, then close each end with a finger. Hold one end beneath the surface of the water in the gallon jar (Fig. 19); remove the finger from that end, and bring the other end, still closed, down outside the jar to a level lower than the water surface.

Is the water-pressure against the finger that closes the tube now greater or less than the atmospheric pressure upon an equally large surface? If greater, the water will run out when the finger is removed. If less, the air will run in and drive the water up in the tube when the finger is removed. Try the experiment.

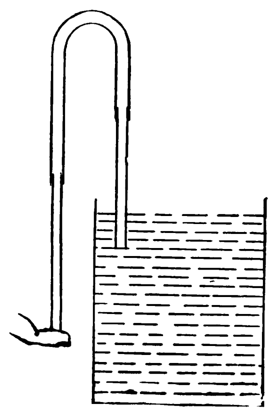

FIG. 19.

Repeat the experiment, but now hold the outer end of the tube, before opening it, higher than the level of the water in the jar.

45. Balancing Columns.—The method of finding specific gravities that is suggested by the following experiment is called the method of *balancing columns*. In the form here shown it cannot well be used with liquids that naturally mix with each other, as alcohol and water do. Later this general method will be used in a form that does not bring the two liquids into contact with each other.

EXPERIMENT.

Take a bent glass tube (App. No. XIII, Fig. 20) each arm of which is about one foot long, and pour water into it till both arms are

about half full; then pour kerosene into one arm till it is nearly full. Does the water now stand as high in the other arm as the kerosene does in the first arm? Can you from this experiment see a third method for finding the specific gravity of a liquid?

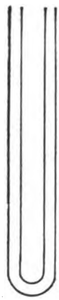

FIG. 20.

QUESTIONS.

(1) Does water stand at the same level in the spout as in the main part of a watering-pot?

(2) If one branch of a U tube (see Fig. 20) were larger than the other, would water stand at the same level in both?

(3) Is it necessary in finding the specific gravity of a liquid by the method indicated in Art. 45 to have the two branches of the tube equally large?

(4) Does the height of mercury in the tube of a barometer depend upon the size of the tube? (We neglect at present what is called the capillary effect. See Second Part.)

(5) If the height of the barometer mercury-column, of specific gravity 13.6, is 76 cm., how tall a column of water could be sustained by the atmospheric pressure if there were a vacuum above the water? (Give the answer in ft. as well as in cm.)

(6) A water-tank 10 ft. deep is to be emptied by means of a tube used as a siphon. What is the least length the tube can have?

(7) With ordinary atmospheric pressure what is the greatest height to which water may be raised by means of a pump working *above* it?

(8) Do you understand the operation of the "trap" which allows water to flow from a sink to a sewer, but does not allow gas to come from the sewer to the sink?

CHAPTER IV.

THE LEVER.

46. Definition and Illustration.—Civilized men do most of their work with tools or machines. Many tools and many parts of machines consist of a piece of iron or wood or other material movable to a certain extent upon a support called a pivot, or axis, or fulcrum, by means of which a force applied in one direction at a certain spot may produce another force different in direction or in magnitude, or in both, at another spot. *Such a tool or part of a machine is called a lever.*

One of the most familiar examples of the lever is a crowbar. A hammer, as used to *draw out* a nail from a board, is another example. Each half of a pair of scissors is a lever. We shall study some very simple forms of the lever to find out what relations hold between the forces exerted at different points.

EXERCISE 8.

THE STRAIGHT LEVER: FIRST CLASS.

Apparatus: The lever and supporting bar (No. 17) fastened to the long horizontal bar that reaches above the table from end to end. Two scale-pans (Nos. 18A and 18B). A set of weights (No. 19).

Hang one scale-pan carrying a load of 8 oz. on the right-hand end of the lever at a distance of 14 cm. from the middle, as in Fig. 21.

Hang the other pan, with an equal load, on the left-hand end of the lever, at such a distance from the middle that the lever will *balance*, that is, stay horizontal when once placed so, even when the ap-

paratus is jarred somewhat by tapping the short bar to which the lever is attached. Then make a record like this:

Left wt.	Left dist. fr. centre	Right wt.	Right dist. fr. centre.
$(1+8)=9$ oz.	....	$(1+8)=9$ oz.	14.0 cm.

(The space here left blank (in the record) is to be filled by the left-hand distance which the student finds necessary to make the apparatus balance.)

Change the right-hand weight to 7 oz., keeping its *place* unchanged, and move the left-hand weight, still 9 oz., to some new position which will make the whole balance, in spite of jarring as before. Make a record, as before, of the weights and distances, putting it just beneath the record for the first arrangement.

Change the right-hand weight to 5 oz. without changing its place, and find what position the left-hand weight, still remaining 9 oz., must have in order that the lever may balance. Record the distances

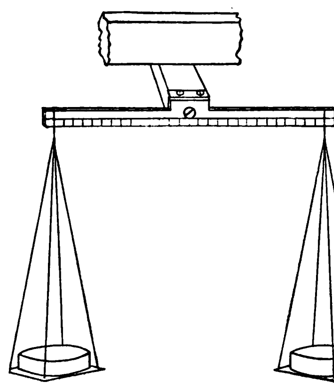

FIG. 21.

and weights for this case under the records already made for the first and second cases.

One more case may be taken, in which the right-hand weight becomes 4 oz., still at 14 cm., which will give a fourth line in the record table. More observations with different arrangements might be made, but it is better to make a moderate number of good observations than a large number of hasty or careless ones.

By studying the record table now made the student should find a rule by which, when the two weights and one distance are given, the other distance can be found by calculation; or when the two distances and one weight are given, the other weight can be found by calculation.

QUESTIONS.

(1) If a mass of 6 oz. is suspended from a point 4 cm. to the left of the centre of the lever in Exercise 8, how great a load placed at a distance of 10 cm. to the right of the centre will make equilibrium?

(2) A mass of 8 oz. is suspended from a point 5 cm. from the centre of the lever and is balanced by a mass of 10 oz. How far from the centre is the latter placed?

(3) Two masses, 4 oz. and 12 oz. respectively, are to be suspended from a lever. Describe three possible arrangements of the masses, any one of which will cause them to balance.

(4) A boy pushing down at one end of a lever 6 ft. long pries up a stone weighing 100 lbs. at the other end. The fulcrum is 2 ft. from the stone. The weight of the lever itself is neglected. How great is the force exerted by the boy?

47. More than Two Weights.—In the preceding Exercise the class found out how to make the two weights hung from the lever balance each other. Let us ask now what the rule for balancing would be if there were more than two weights in use, as in Fig. 22, for instance.

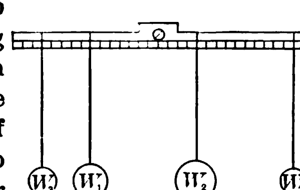

FIG. 22.

EXPERIMENTS.

We will make the apparatus balance with four weights, two on each side. We will call the weight nearest the centre on the left hand weight No. 1, which we will write W_1, for short. The other weight on the left-hand side we will call No. 3, or W_3. The two weights on the right hand we will call W_2 and W_4.

When the whole balances, we will call

The distance of W_1 from the middle, D_1,
" " " W_2 " " " D_2,
" " " W_3 " " " D_3,
" " " W_4 " " " D_4.

Now if we go back for a moment to the case of two weights, which the class has studied, and if we call these P_1 and P_2, and their distances from the middle d_1 and d_2, we can state the rule for balancing in this way:

$$P_1 \times d_1 \text{ must equal } P_2 \times d_2.$$

In the new case, where we have four weights, we may *guess** that the rule is

$$(W_1 \times D_1) + (W_3 \times D_3) = (W_2 \times D_2) + (W_4 \times D_4),$$

and then test the truth of our guess by trial.

Try other like cases.

48. Circular Lever.—In the experiments with which we have just been engaged the weights have been suspended from the top of the lever on a level with that part of the pivot upon which the lever rests. In other experiments which are to follow we shall not always be able to keep this arrangement, and we have now to find out what would be the effect of hanging one or more of the weights from points higher or lower than the point of support of the lever. For this purpose we shall use No. XV, the piece of apparatus shown in Fig. 23, in which the straight lever thus far used is replaced by a circle of wood about 8 inches in diameter, supported by a screw passing horizontally through the centre. Such a circle, or disk, of wood comes under the general definition of a lever.

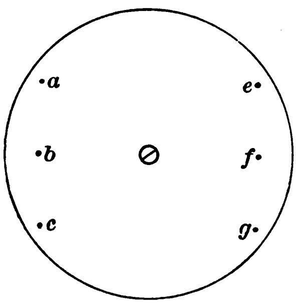

FIG. 23.

EXPERIMENTS.

We will hang at *b* and *f* such weights as will balance each

* Shrewd guessing, *followed by a test*, should be encouraged by the teacher as a means of extending knowledge. In fact, it is the constant resource of the *investigator*.

other, leaving the disk in equilibrium, and will then move one of the weights to a point vertically above or vertically below its present place ; that is, from *f* to *e* or *g*, or from *b* to *a* or *c*. Shall we still have equilibrium? Try.

We will now turn the disk a little, so that the lines *a b c* and *e f g* will be no longer quite vertical, and will see whether now a weight at *e* or at *g* has just the same effect as if at *f*. Try.

A careful note should be made of conclusions for future use.

49. Centre of Gravity.—In the experiments upon the lever thus far, the lever itself, whether a bar or a disk, has *balanced*, when left to itself without load. We have, therefore, not had to consider the weight of the lever itself. But many levers are used in such a way that their own weight helps or hinders the operation to be performed with them. To understand such cases we must learn something about what is called the *centre of gravity* of a body.

EXPERIMENT.

Take a board, cut in any irregular shape, like Fig. 24, for

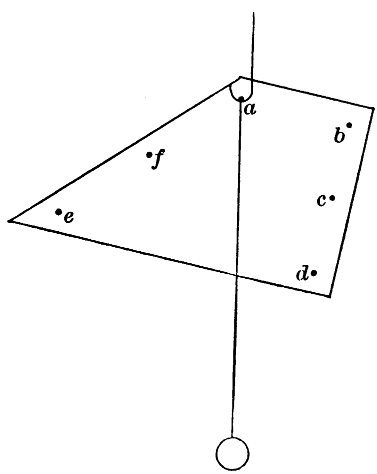

Fig. 24.

instance. Bore several small holes *straight* through the board,

and put into each hole a wire nail that will fit close, long enough to project about half an inch on each side of the board. Tie a bullet at one end of a thread and make a loop in the other end. Put this loop over one hook of a piece of wire bent into the shape shown in Fig. 25, and then rest the nail *a*, Fig. 24, in the hooks of the same wire, so that the board and the string carrying the bullet will both hang free, the string near the face of the board.* Mark with a pencil the course of the string downward across the board.

FIG. 25.

Then suspend the board by the nail *b* and mark the new course of the string. Proceed in this way with all the nails and note the point where the various pencil-marks cross each other.

Finally, place the board horizontal and balance it upon the flat head of a lead-pencil, noting how near the head of the pencil comes to the crossing of the lines marked on the board.

Definition.—By such experiments as this we come to see that there is within the board a certain point which always hangs just beneath the support when the board comes to rest suspended from any one of the nails. We see that the same point has to be just *above* the support when the board rests upon the pencil-top. In short, the board *acts* in these experiments as it would if all its weight were concentrated at this particular point. This point might be called the centre of weight or centre of heaviness of the board, but it is commonly called the *centre of gravity*.

50. Weight of the Lever.—The following Exercise is intended to make the pupil more familiar with the idea of centre of gravity, and to show how it may be taken account of in the use of the lever.

EXERCISE 9.

CENTRE OF GRAVITY AND WEIGHT OF A LEVER.

Apparatus: The lever of No. 17, detached from its supporting bar, and a small block (No 21), the two being fastened together, as in

* The whole apparatus as shown in Fig. 24 will be called No. XVI.)

Fig. 26, so as to make one body, the whole of which will be called

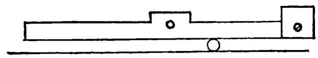

FIG. 26.

the lever in this Exercise. A slender wooden cylinder (No. 13). A 1-oz. scale-pan (18 A or 18 B). A 1-oz. wt. from No. 19.

To find the centre of gravity of the lever, balance it as nearly as you can, bar and block fastened together, in a horizontal position on the cylinder laid on the table (see Fig. 26), the cylinder being kept at right angles with the lever. Find in this way at what particular mark of the bar the centre of gravity is, and record this mark—for example, 9.1 cm.

Then suspend the 1-oz. scale-pan carrying a 1-oz. wt., 2 oz. in all, from any convenient point near the free end of the bar, and letting this end project beyond the edge of the table-top, balance the whole, as now arranged, as nearly as you can, on the cylinder laid on the table as before (see Fig. 27).

Now record the mark from which the scale-pan hangs, 33.4 cm., we may suppose, and the mark which is just over the middle of the cylinder when the whole balances, 21.6 cm., let us say.

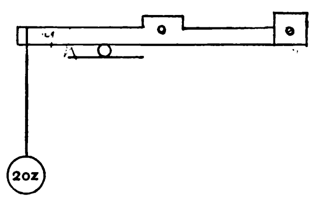

FIG. 27.

This case is like that of the lever studied in Exercise 8. The cylinder now taking the place of the screw as a support, we see that

the left-hand weight is 2 oz ,
" " " distance is $33.4 - 21.6 = 11.8$ cm.,
" right-hand weight is the weight of the lever,
" " " distance is $21.6 - 9.1 = 12.5$ cm.,

that is, the distance from the support in Fig. 27 to the centre of gravity of the bar and block.

We do not as yet know the weight of the lever, but we will call it W_2. and see whether we can find its amount by calculation. If we apply the same rule that was found to hold true in Exercise 8, we shall have

$$2 \times 11.8 = W_2 \times 12.5,$$

which gives for the weight of the bar and block

$$W_2 = \frac{2 \times 11.8}{12.5} = 1.89 \text{ oz., nearly.}$$

The value of W_2 obtained in this way by the pupil should be compared with the weight of the bar and block as found by the teacher with some balance, e.g. No. XVII, much more sensitive than the spring-balance used by the class; for if the method of this Exercise is carefully followed it will give the weight of the lever more accurately than the spring-balance is likely to do.

QUESTIONS.

(1) An oar, the centre of gravity of which is 3 ft. from the end of the handle, weighs 4 lbs. It rests in the rowlock at a point 2 ft. from the end of the handle. How great a force applied at the end of the handle will keep it balanced?

(2) A boy weighing 100 lbs. is see-sawing alone on a plank 20 ft. long weighing 50 lbs. The boy's centre of gravity is 1 ft. from one end of the plank. How far from the same end of the plank is the fulcrum? (The centre of gravity of the plank is at its middle).

(3) If the plank mentioned in the preceding problem is to balance on a fulcrum 8 ft. from one end, with a 100 lb. boy 1 ft. from this end and another boy 1 ft. from the other end, how much must the second boy weigh? (See Art. 47). *Ans.* $54\frac{6}{11}$ lbs.

51. Remarks.—We have now found out how to take account of the weight of the lever itself, when we need to do so. We know that all its weight may be regarded as concentrated at a certain point, which we call the centre of gravity, and we have tried one case in which the weight of the lever itself, acting at the centre of gravity, balanced a certain weight suspended from the bar. In common levers, like the crowbar, the weight of the bar itself is sometimes very important, when the fulcrum is a long distance from the centre of gravity of the bar.

Centre of gravity will be taken up again, and the different kinds of *equilibrium*, stable, unstable, and neutral, will be considered in the Second Part.

We will now return for the present to cases of the lever

where the centre of gravity lies, as in Exercise 8, just under the point of support of the bar. In such cases the weight of the lever itself does not tend to make the bar tip in either direction from its horizontal position.

Classes of Levers.

52. Lever of the First Class.—In the levers which we have studied thus far the support, or *fulcrum* as it is often called, lies between the lines of suspension of two weights. This kind of lever, whether it is a simple bar or a disk or an object of irregular shape, whether its centre of gravity lies at the point of support or not, is called a lever of the *First Class.*

53. The Power, Power-arm, etc.—To take a simple and convenient case, we will consider in Fig. 28 a circle supported at its centre, *F*. We will suppose that this lever is used for the purpose of supporting a weight *W*, and the force used for this purpose, whether it is applied by means of another weight, as in the figure, or by means of the hand, or in any other way, we will call the *power*.

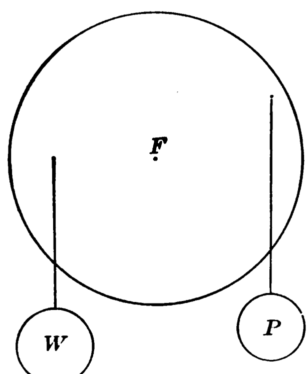

Fig. 28.

We have seen in the experiments of § 48 that, as the lever now stands, it makes no difference whether *W* is suspended from the point which now carries it or from some point higher or lower in the same vertical line, which is called the *line of action* of *W*. A like statement can be made for *P*. We shall call the shortest distance from *P*'s line of action to the fulcrum the *power-arm*, and the short-

est distance from W's line of action to the fulcrum the *weight-arm*.

54. Law for First Class.—In order that P and W may just balance each other we must have, as can be seen from Exercise 8,

$$power \times power\text{-}arm = weight \times weight\text{-}arm.$$

This is the *law* for a lever of the first class.

55. Levers of the Second and Third Classes.—But we may have a case, like that shown in Fig. 29, in which the line of action of the weight lies between the fulcrum and the line of action of the power. This arrangement gives us what is called a lever of the *second class*.

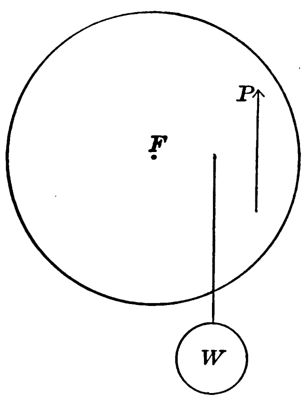

FIG. 29.

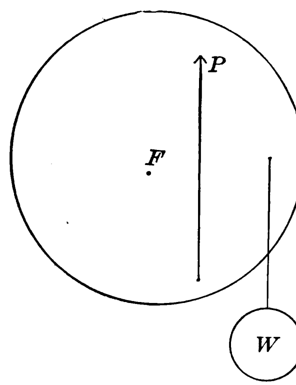

FIG. 30.

There is still a different case, shown in Fig. 30, where the line of action of the power lies between the fulcrum and the line of action of the weight. This is called a lever of the *third class*.

In the second and third classes of levers, as in the first class, the shortest distance from the fulcrum to the line of

action of the weight is called the *weight-arm*, and the shortest distance from the fulcrum to the line of action of the power is called the *power-arm*.

The pupil is to find out by means of the following Exercise whether the laws of the second and third classes of levers are as simple as the law of the first class.

EXERCISE 10.

LEVERS OF THE SECOND AND THIRD CLASSES.

Apparatus: The lever (No. 17) supported as in Exercise 8. A scale pan (No. 18). A set of weights (No. 19). A spring-balance (No. 7).

Suspend the pan with a load of 8 oz. at a point 5 cm. from the middle of the lever, and, on the same arm of the lever, at a distance of 10 cm. from the middle, pull upward with a spring-balance, connected with the lever by means of a loop of thread, until the weight is balanced and the lever becomes horizontal. You have here a lever of the second class. Read the spring-balance and record as follows:

LEVER OF SECOND CLASS.

Weight.	Weight-arm.	Power.	Power-arm.
9 oz.	5 cm.	....	10 cm.

Try other similar cases, and study them all until you are able to write down the law for this class of levers.

Then with the same apparatus place the spring balance between the fulcrum and the line of the weight. You will now have a lever of the third class. Try various cases and record as before

LEVER OF THIRD CLASS.

Weight.	Weight-arm.	Power.	Power-arm.
....	....	....	....
....	....	....	....

Law........ ....

QUESTIONS.

(1) A lever supported at its centre of gravity is used to lift a weight of 100 lbs. applied at a distance of 1 ft. from the fulcrum. The

power is applied 5 ft. from the fulcrum and on the opposite side from the weight. How great must the power be? Must the power be applied upward or downward?

(2) If the power were placed on the same side of the fulcrum as the weight, everything else being as described in the preceding problem, how great would the power have to be? Would it be applied upward or downward?

(3) If the power were 50 lbs. applied at a point 2 ft. toward the right from the fulcrum, and if the weight were applied 8 ft. toward the right from the fulcrum, how great could the weight be?

(4) If a weight of 5 lbs. were placed 4 ft. toward the right from the fulcrum, and a weight of 7 lbs. 6 ft. toward the right from the fulcrum, how far from the fulcrum toward the left must a force of 10 lbs. be applied in order to make the whole balance? *Ans.* 6.2 ft.

In the four preceding problems the weight of the lever has not been considered, because the centre of gravity has been supposed to be at the point of support. Suppose now that the lever weighs 4 lbs. and that its centre of gravity is 3 ft. to the right from the fulcrum, and with this new condition go over each of the four problems again.

56. Force at the Fulcrum.—If we take a case like that shown in Fig. 31, it is plain that 4 oz. applied 7 cm. from the centre will balance 2 oz. applied 14 cm. from the centre, but it may not be perfectly plain how great the pull on the fulcrum itself is. We will, therefore, in the next Exercise try the experiment in one or two simple cases and see what the result will be.

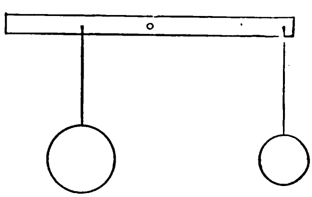

FIG. 31.

EXERCISE II.

FORCE EXERTED AT THE FULCRUM OF A LEVER.

Apparatus: The lever of No. 17 freed from its support. Two scale-pans (Nos. 18A and 18B). Two 1-oz. wts. and one 2-oz. wt. from No. 19. The spring-balance (No. 7). A piece of copper wire about 1 mm. in diameter bent into the form of a hook (*h* in Fig. 32). A piece of thread about 15 cm. long.

Suspend the bar from the balance in the manner indicated by Fig. 32. Note and record the weight of the bar alone. Then suspend one scale-pan with a 1-oz. weight from one arm of the bar, and the other scale-pan with a 2-oz. weight from the other arm in such a way as to balance, taking care not to let the pans and weights spill. Note and record the reading of the balance. Then make the loads (pan and weight) 2 oz. on one side and 4 oz. on the other, and read and record. Try any other experiments that you can with the weights furnished, until you feel reasonably sure that you know the relation between the weights applied and the pull on the balance. Then state what this relation is.

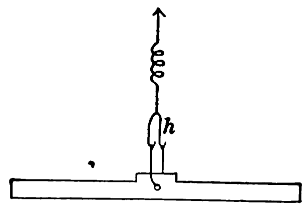

FIG. 32.

57. Laws of the Lever.—In each of the cases in Exercise 11 we have applied two downward forces to the bar in suspending the two scale-pans with their loads, and have found these two forces to be balanced by another force exerted upward by the spring-balance. It will be well for us to study such cases very carefully, for similar ones are often found.

Suppose we are to make three parallel forces, A, B, and C, just balance each other when all are applied to the same body. Can we from what we have now learned tell anything about the relative magnitude and the arrangement of these forces?

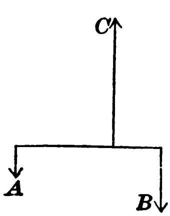

FIG. 33.

We know that—

1*st*. *All the forces cannot point in the same direction.* Let us suppose that C is opposite in direction to A and B.

2*d*. *The force C must be equal to the sum of the two forces A and B.*

3*d*. *The line along which C is applied must lie between the lines along which A and B are applied.*

4th. A × shortest distance from line of A to line of C = B × shortest distance from line of B to line of C. (See Fig. 33.)

These rules apply as well to horizontal forces as to vertical forces. Try three spring-balances laid parallel to each other on a table and pulling at some light horizontal bar—a lead-pencil, for instance. (Or try the "checker-board" with larger spring-balances, if the apparatus of the Second Part is available.)

58. Pulleys.—We have already learned to consider a disk pivoted at the centre as a kind of lever. When such a lever is worked by means of a cord or band lying upon its circumference, it is called a *pulley*. We shall now see that the pulley form of lever has some great advantages.

EXPERIMENTS.

(1) Take the pulley shown in Fig. 34 (No. XV), and let us first use the largest circle only.

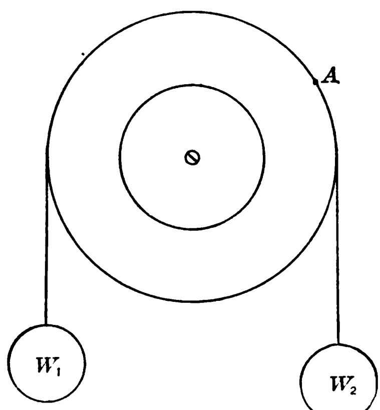

FIG. 34.

If we fasten two equal weights, W_1 and W_2, to the ends of a string and pass the string across the top of the pulley, we shall of course find that they balance each other.

But suppose we used two strings, one for W_1 and the other for W_2, fastening each string to a pin or tack at point A, but letting each string rest in the groove of the pulley, so that the final position of the two strings will be represented by Fig. 34. Will two equal weights balance each other under these conditions? The question is quickly answered by trial, and by turning the pulley a little one way or the other we can try the experiment with A in a variety of positions.

(2) Next try the effect of a horizontal pull, P, applied by a spring-balance at the top of the pulley to balance a weight W, as in Fig. 35. (Remember that in this position the *reading* of the *ordinary* 8-oz. balance is about $\frac{1}{4}$ oz. less than the real force exerted by it, be-

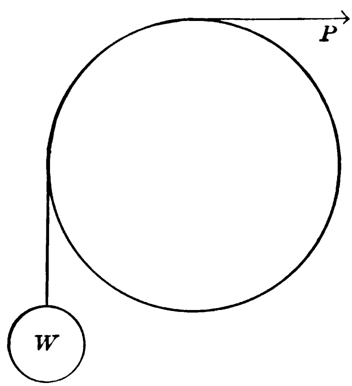

FIG. 35.

cause the spring of the balance does not now support the weight of the hook and bar, which is about $\frac{1}{4}$ oz.). Find by experiment whether the force P must be greater or less than, or equal to, the direct pull of the weight W.

(3) Balance a weight on one circle of the pulley by a weight on another circle, and find the simple relation which holds between the balancing weights and the radii of the circles.

59. Advantages of a Pulley.—We see that the advantage of a pulley, as compared with a simple bar-lever, is that the pulley enables us to vary the direction of our power at will

and to lift a weight a much greater distance than we could with a bar-lever no longer than the diameter of the pulley. In fact, the distance through which we can lift the weight by means of the pulley depends merely upon the length of the string that supports the weight.

60. Windlass, Capstan, etc.—The *windlass* (see Fig. 36) is a familiar apparatus consisting of an elongated pulley, *d*, called the drum or cylinder, turned by a power applied at the handle, *h*, and acting through the lever, or crank, *c*.

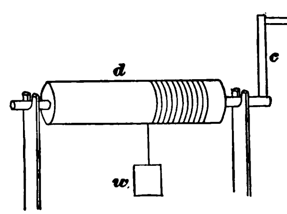

FIG. 36.

The crank and handle are sometimes called a *winch*. The same name is sometimes applied to the whole windlass.

If the power is applied at right angles with both *h* and *c*, and if the cord sustaining the weight is small, we have, very nearly,

$$P \times \textit{length of } c = W \times \textit{radius of } d.$$

A *capstan* is the same in principle as a windlass, but has a vertical drum, so that the lever travels in a horizontal circle.

On ship-board capstans are frequently worked by means of a number of men walking about the drum and pushing against the levers, or *capstan-bars*, of which there may be several applied to one cylinder.

61. Movable Pulleys.—In the pulleys thus far studied by us the pivot has been fixed in position; but pulleys with movable pivots are frequently used.

EXPERIMENTS.

(1) Take the small metal pulley (No. XVIII) and arrange it according to the indications of Fig. 37, *P* being the pulley, *M* a weight

suspended from an axis through the centre of the pulley, *B* a spring balance, and *h* a hook to which one end of the string passing beneath the pulley is attached.

To what class of levers does the pulley in this position belong? What, then, should be the relation between the weight, which is *M plus the weight of the pulley itself*, and the pull exerted by the spring-balance? Find by experiment whether the conclusion reached is correct.*

(2) Let us now try an arrangement, like that shown in Fig. 38, in which we have one pulley, *A*, hooked to a bar overhead, and a double pulley (No. XIX), *B*, which moves up and down with the load *M*.

Let us consider what should be the relation between the pull *P* and the weight *W*, which is *M plus the weight of B*, in this case.

In the case tried in Experiment 1 we had two strings holding up the pulley *P*. We have now four strings holding up the pulley *B*. After thinking upon the matter for a little time, trying to study out what is the relation between *P* and *W* with this arrangement, let us try the experiment as we have already tried it in the simpler case, noting the force shown by the spring-balance when *M* is moving steadily up, and again when it is moving steadily down, and taking the mean between these two forces as the one that would be required to balance the weight, *W*, if there were no friction.

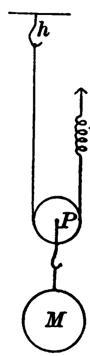

Fig. 37.

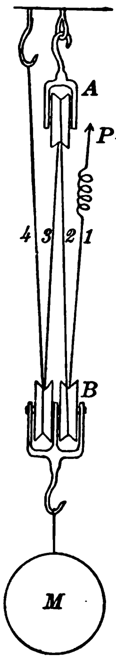

Fig. 38.

* In making this trial one must remember that friction is often large in pulleys, even when they are well oiled, as this one should be. Now when the load is being steadily raised the hand carrying the spring-balance must lift harder than it would if there were no friction, but when the load is being steadily lowered, the hand, pulling just hard enough to prevent the load from *hurrying*, is assisted by the friction. The mean between the reading of the balance going up and the reading of the balance coming down will show, very nearly, what the pull required to sustain the load would be if there were no friction.

62. Another Law for Relation of Power to Weight.—The law, "4th" in § 57, for the relation between *power* and *weight* is extremely useful, and, properly applied, is sufficient for very complicated cases of the lever and pulley; but in some cases it is more convenient to make use of a different form or statement of this law, a form which makes use of the *relative distances moved over* by the power and the weight in any operation of the lever, or pulley, or combination of these, that may be in action.

63. Search for the Law.—If we study the various cases of lever, pulley, and combination of pulleys that have been described in the preceding pages, we shall see that whenever the *weight* is greater than the *power* the weight moves a less distance than the power does in any given *operation* of the apparatus; but whenever the weight is less than the power, friction being supposed zero, the weight moves a greater distance than the power does in the operation of the apparatus.

64. Statement of the Law.—If we study the matter more closely, we shall find the following rule or law *suggested*, though we cannot say that it is *proved* by our preceding experiments:

$$P \times D_p = W \times D_w,$$

where P stands for the power applied;
W " " " weight lifted;
D_p " " " distance the power moves;
D_w " " " " " weight moves.

APPLICATIONS OF THIS LAW.

(1) It is evident that this law can be readily applied to a case like that of Fig. 38. We can see at once that if the pulley B were lifted one inch while P remained stationary there would be *four* inches of loose string under B. To make the string taut again, P would have to rise four inches. In actual use P and B rise at the same time, P moving four times as fast as B.

(2) In Fig. 39, representing the rear wheel and gear of a bicycle, let
the diameter of the tire be 28 in.;
" " " " small sprocket-wheel be $2\frac{1}{2}$ in.;
" " " " large sprocket-wheel be 5 in.;
" length of pedal radius be 6 in.

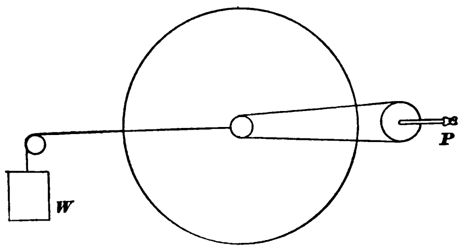

FIG. 39.

If the weight of the rider, 150 lbs., rests entirely upon the pedal shown, in its present position, how great a weight acting in opposition, as in the figure, will just neutralize the driving-power?

The circumference of the circle described by the pedal is (see Ex. B) $2\pi \times 6$ inches, or 12π inches.

One revolution of the pedal-crank makes the tire revolve 2 times, which drives the bicycle forward

$$2 \times 2\pi \times 14 \text{ inches} = 56\pi \text{ inches.}$$

The weight W will be lifted as fast as the bicycle moves forward.

If D_p stands for the distance the power P moves downward from its present position during any very short time, and if D_w stands for the distance W moves upward during the same time, we have

$$D_p : D_w :: 12\pi : 56\pi.$$

Hence the law $W \times D_w = P \times D_p$ (see § 64) gives

$$W \times 56\pi = P \times 12\pi,$$

or $$W = \tfrac{3}{14} P = \tfrac{3}{14} \times 150 = 32.1 + \text{(lbs.)}$$

QUESTIONS.

(1) A large pair of shears is used to cut a wire. One handle of the shears being held fixed, a power of 25 lbs. is applied to the other handle at a distance of 2 ft. from the pivot. The wire cut is placed 2 in. from the pivot. How great is the resistance offered by the

wire; that is, how great a force applied just over the wire would drive the blade through it?

(2) A man lifts 10 lbs. of coal on a shovel. His left hand is at the end of the handle; his right hand is 18 in. distant from his left hand; the centre of gravity of the coal is 36 in. distant from the left hand. The shovel itself weighs 6 lbs. and its centre of gravity is 21 in. from the left hand.

(*a*) What is the direction and magnitude of the force exerted by the left hand? (Consider the right hand as the fulcrum.)

Ans. 11 lbs.

(*b*) What is the direction and magnitude of the force exerted by the right hand? (Answer this question as if the left hand were replaced by a weight.)

(3) A body weighing 160 lbs. is suspended from a pole resting on the shoulders of two men, A and B, of equal height. The point of suspension is 3 ft. from A's shoulder and 5 ft. from B's. How much of the weight does each man bear?

(4) Six men are working at a capstan, each exerting a force of 20 lbs., each at a distance of 6 ft. from the centre. The diameter of the coils in which the rope is being wound on the drum is 1 ft. How great is the strain on the rope, all friction being neglected?

(5) A man finds that by moving one point of a machine, consisting of levers and pulleys, forward 10 in. he can move another point of the machine 1 in. If a force of 5 lbs. is applied at the first point, how great a resistance applied at the second point will be required to neutralize it, if there is no friction in the machinery?

(6) Can the class name any tools or machines, not already mentioned in this book, in which levers or pulleys are used?

CHAPTER V.

THREE FORCES DIRECTED THROUGH ONE POINT: THE PARALLELOGRAM OF FORCES.

65. Introductory. — In studying the lever we have usually, though not always, had parallel forces to deal with, forces acting straight up or straight down. But very often we have to do with bodies that are acted upon by forces not parallel to each other. Thus when a ladder standing upon the ground leans against a house we have at least three forces acting upon the ladder: 1st, the earth's attraction, or, as we call it often, the *weight* of the body, which acts as if the whole substance were at the centre of gravity; 2d, the push of the ground against the foot of the ladder, which push is not straight upward; 3d, the push of the wall against the top of the ladder.

Again, a flying kite is acted upon by the earth's pull, straight downward; by the force exerted by the air, which force, because of the wind, is not straight upward; by the pull of the string, which pull is not straight downward.

The way to begin the study of such cases is to study the case of three forces all acting straight from or straight toward a single point. We shall take such a case in Exercise 13, measuring the forces by means of spring-balances.

66. Errors of Spring-balances.—It is quite possible in specific-gravity work to get accurate results with an inaccurate balance; for the specific gravity of a body is found by taking the *ratio* of two quantities, both found by weighing with the same balance, and if the balance should give

the weight of each as n times its true value the ratio of the two false weights would be the same as the ratio of the true weights, whatever the value of n.

But though a balance *may* give the weight of everything as n times its true weight, n being the same for all parts of the scale, it is very unlikely that several balances will all be wrong in just the same way; and as in Exercise 13 we shall need to use three balances in combination, it is necessary for us to give more careful attention to their errors than we have given heretofore.

The following Exercise is intended to show how the errors of a spring-balance may be found and may be recorded in a form convenient for future use.

EXERCISE 12.

ERRORS OF A SPRING-BALANCE.

Apparatus: A spring-balance (No. 7). A set of weights (No. 19). Thread for suspending the weights. A measuring-stick (No. 3). (Although weights reckoned in ounces are referred to in this Exercise, it may be performed equally well with suitable weights reckoned in grams.)

Balance in the Vertical Position.—Suspend the balance by its ring from some convenient support so that the index will be not higher than the eye of the observer.

Make five careful readings with the loads indicated in the first column below, and record these readings in a second column ; thus, for example:

True Load.	Reading.	Error.*
0 oz.	– 0.2 oz.	– 0.2
2 "	2.0 "	0.0
4 "	4.2 "	+0 2
6 "	6.3 "	+ 0.3
8 "	8.1 "	+ 0.1

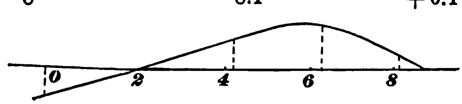

Fig. 40.

* That is, the quantity which must be *subtracted* from the *reading* in order to find the true weight.

Now draw in the notebook a straight line a little more than twice the length of the scale of the balance, and mark off on it points corresponding to the *loads* and *readings* given above.

Then represent the errors by vertical distances, measured *down* from the points indicating the *readings* when the errors are negative, and *up* when they are positive, showing these errors on a rather large scale, 0.5 cm. per 0.1 oz., for instance.

Draw a curve through the extremities of these vertical distances, as in Fig. 40. This curve will enable us to tell with sufficient accuracy for our present purposes the errors of any other readings made with the balance in the vertical position; for instance, if the balance to which Fig. 40 relates *reads* 1 oz., we may assume that the error is very nearly 0.1 oz. and that the true weight is very nearly 1.1 oz.; if the reading is 3 oz , we may assume that the true weight is about 2.9 oz., and so on, the error for each case being found by measuring from the point indicating the reading up to or down to the curve.

THE SAME BALANCE IN THE HORIZONTAL POSITION.—We will suppose at first that the same index is used for the horizontal as for the vertical readings.

Lay the balance flat on its back and tap it gently several times.* Then take its reading, which we will suppose to be — 0.5 oz. In this case it would require a force of 0.3 oz. to pull the index down to the position it occupied with no load in the vertical test. To bring the index to any reading in the horizontal use of this balance will, therefore, require a force 0.3 oz. greater than the weight which, applied to the hook in the vertical use of the balance, would bring the index to the same point.

It would in the case here supposed be sufficiently accurate for our purposes to add 0.3 oz. to any horizontal reading, and then correct this increased reading by means of the curve given in Fig. 40.

When a different index is used for horizontal readings, the principle is the same. For example, let us suppose the second index reads — 0.1 without pull in the horizontal use of the balance. We say, this reading exceeds the vertical reading without load by 0.1 oz. We must, therefore, subtract 0.1 from all horizontal readings made with this second index, and then correct these reduced readings by means of the curve in Fig. 40.

* This method assumes that the movable parts of the balance are restrained by friction only, which the tapping overcomes. Sometimes the slot in the balance-face is not long enough to allow the index to find its unrestrained position.

The curve so often referred to should be marked with the number of the balance and kept for future use.

EXERCISE 13.

THREE FORCES IN ONE PLANE AND ALL APPLIED AT ONE POINT: PARALLELOGRAM OF FORCES.

Apparatus: Three 8-oz. spring-balances, each provided with two small blocks (No. 22) to go under its sides and hold it flat on its back when it is lying upon the table. The rectangular block (No. 9). The measuring-stick (No. 3). A sheet of paper. Thread.

Take two pieces of strong thread, one about 12 inches, the other about 6 inches, long, and tie one end of the short thread to the middle of the long one. Fasten the three loose ends to the hooks of the spring-balances; then lay the latter upon the table, putting the blocks under their sides, as in Fig. 41, and let one student pull at each bal-

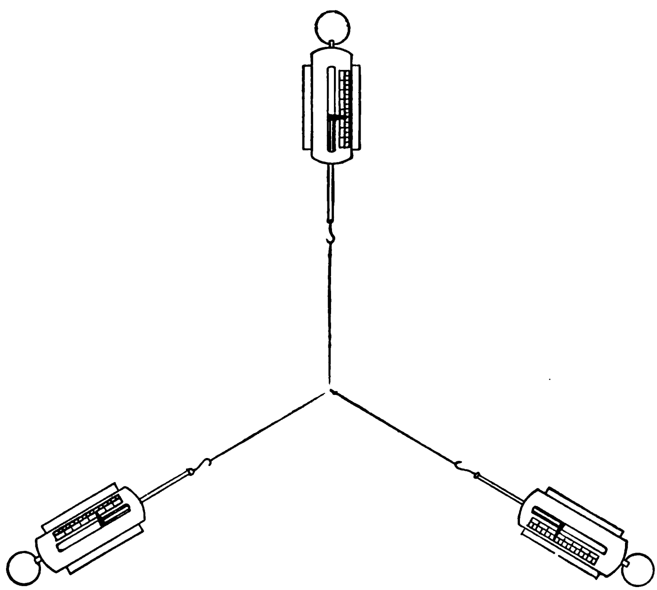

FIG. 41.

ance, taking care that the slit of each balance-face is in a straight line with the thread, until no one of these reads less than 3 oz.

It will be found that any variation in the angles which the strings make with each other will require a change in the forces. Evidently there is some connection between the directions of the strings and the forces necessary to balance each other. The object of this Exercise is to make out what this connection is.

Put under the threads a sheet of paper, and draw on this paper, just under each thread, a pencil-mark parallel to the thread, and then write down alongside each pencil-mark the force in the direction of that line, as shown by the spring-balance. The balances must be held very still while these lines are being drawn, and must be read before any change occurs in the direction of the lines.* To draw a line place one side of the block (No. 9) close alongside one branch of the thread, taking care not to push the thread out of place, and then run the point of a well-sharpened pencil along the edge of the block under the thread. Draw the other lines in the same way, doing all very carefully.

Each student in turn should make a set of lines, and record along-

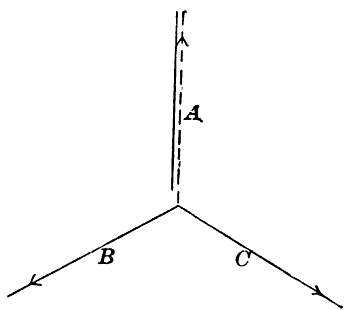

FIG. 42.

side them the proper forces. The directions of the pulls should be varied somewhat by each experimenter, in order that his lines and forces may not be exactly like those of others.

Take now the wooden ruler (No 3), and extend the three lines toward each other till they meet at one point. This they will do if

* It is well to fasten the *ring* of each balance to some object heavy enough to hold the balance in place, thus relieving the experimenters, who might grow tired and unsteady in holding the balances long enough to permit of drawing the pencil-marks properly.

they have been drawn originally just *under* the threads. If they do not all meet at one point, a new line should be drawn parallel to one of them in such a position as to pass through the crossing of the other two lines, and this new line, the dotted line in Fig. 42, is then to be used in place of the original line. The three lines as now drawn will represent accurately the *directions* of the three forces.

Now measure off from the common point along the line A a distance of 1 cm. for each ounce (or each 30 gm., if the forces are measured in grams) of the force which was exerted along that line, and put a small arrow-head (see Fig. 42) at the end of this measured distance. Erase that part of line A which lies beyond the arrow-head.

Do the same with lines B and C that has been done with A. The three arrows thus obtained, all reaching from the same point, represent the magnitude and the direction of the three forces exerted by the spring-balances.

Now with A and B of Fig. 42 as two of the sides draw a parallelogram, taking pains to make it accurate.* Then make a parallelogram with B and C as sides, then one with A and C as sides. Compare the

* One line may be drawn very nearly parallel to another by means of a device illustrated by Fig. 43. Ll is a line already drawn. The block (No. 9) is so placed that for an eye placed at E the edge mn

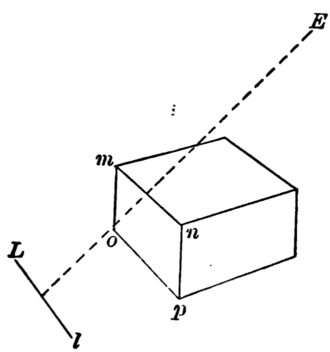

Fig. 43.

appears to be close to Ll and parallel to it. Then a pencil-mark is made along the edge op.

A better method is to set the edge op on the line Ll and then guide the block to a new position by sliding it along the straight edge of a ruler at right angles with Ll.

length and direction of the line C with the length and direction of the diagonal of the parallelogram AB; the line A with the diagonal of the parallelogram BC; the line B with the diagonal of AC.

From a study of the Exercise make a rule showing how to find the direction and magnitude of a force C which put with two forces represented by the lines A and B (Fig. 44, below) will just balance them.

Applications of the Parallelogram of Forces.

The rule found in the preceding Exercise, and which is called the *parallelogram of forces*, is peculiarly easy to apply in cases where two of the forces are at right angles with each other. A number of such cases will now be discussed.

(1) A force of 7 lbs. pulls north from a certain point, and a force of 4 lbs. pulls east from the same point. How large must a third force be to hold them in check, and what will be its general direction?

Solution.—Fig. 45 indicates the method of working the problem.

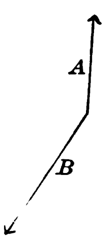

FIG. 44.

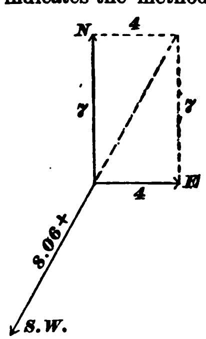

FIG. 45.

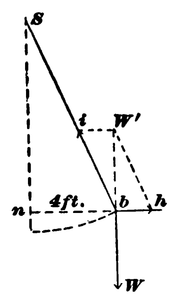

FIG. 46.

The *direction* of the third force is evidently southwest, in a line with the diagonal of the rectangle, of which the base is 4 and the height 7. The *magnitude* of the third force is evidently equal to the length of the diagonal, which a simple rule of geometry shows to be

$$\sqrt{4^2 + 7^2} = 8.06 +.$$

(2) A boy weighing 50 lbs., represented by the point b in Fig. 46, is seated in a swing 10 ft. long, represented by the line Sb. A horizon-

tal pull holds him 4 ft. to one side from the natural position of the swing.

(*a*) How great is the pull of the swing-rope ?

(*b*) How great is the horizontal pull ?

Solution.—Represent the weight of the boy by the line bW, made 1 cm. long, we will suppose. The other two forces acting on b must be such as to make a balance with this force. Draw the line bW' exactly equal and opposite to bW, and complete the parallelogram as in the figure.

A little geometry shows that the triangle biW' is similar to the triangle Sbn.

The line bi represents the pull of the swing-rope, and, as the triangles just mentioned are similar, we have

$$bi : bW' :: Sb : Sn,$$

or

$$bi = (Sb \div Sn) \times bW'$$
$$= (10 \div \sqrt{10^2 - 4^2}) \times bW' = 1.09 \times bW'$$

But $bW' = bW$, which represents a force of 50 lbs. Hence bi represents 54.5 lbs. *Ans.* to (*a*) = 54.5 lbs.

The pull upon the rope is therefore greater than the boy's weight.

The line bh, which is $= iW'$, represents the horizontal pull.

We have

$$bh : bW' :: nb : nS,$$

or

$$bh = (4 \div \sqrt{10^2 - 4^2}) \times bW' = 0.436 \times bW'.$$

The force represented by bh is therefore

0.436×50 lbs. $= 21.8$ lbs. *Ans.* to (*b*) = 21.8 lbs.

(3) A mass of 20 lbs. is suspended from the point p (Fig. 47), where a string is bent at a right angle. The ends of the string are fastened to two nails, N_1 and N_2, which are at the same height. The part N_1p is 4 ft. long, the part N_2p is 8 ft. long.

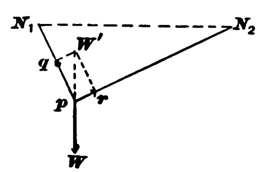

FIG. 47.

(*a*) How great is the pull upon N_1 ?

(*b*) How great is the pull upon N_2 ?

Solution.—Represent the weight by pW. Draw pW' equal and opposite to pW. Complete the parallelogram.

The triangle pqW' is equal to the triangle $W'rp$ and similar to the triangle N_2pN_1. The side $N_1N_2 = \sqrt{8^2+4^2} = 8.94$ ft. nearly.

The pull on N_1 is represented by pq, and we have

$$pq : pW' :: N_2p : N_2N_1, \text{ or } pq = (8 \div 8.94) \times pW'.$$

Therefore pq represents very nearly 0.895×20 lbs. $= 17.90$ lbs., which is the answer to question (*a*).

Similarly we find the pull on N_2 to be very nearly $(4 \div 8.94) \times 20$ lbs. $= 8.95$ lbs., which is the answer to question (*b*).

The pull along the 8-ft. part of the string is just one-half as great as that along the 4-ft. part.

The Inclined Plane.

67. Introductory. — The parallelogram of forces will enable us to understand a contrivance very often used for raising heavy weights. It is a common thing to see barrels of flour or other heavy objects loaded upon wagons by rolling them up a plank or a pair of rails, placed with one end on the ground and the other upon the wagon, so as to make the ascent gradual instead of straight up. The flat slanting surface up which the body is rolled is called an *inclined plane.*

Sometimes a body is lifted by forcing an inclined plane, the slanting face of a *wedge*, under it, as in Fig. 48.

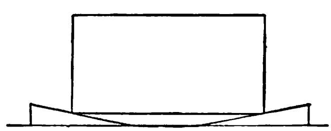

Fig. 48.

Sometimes the force used by an experimenter or a workman with the inclined plane is parallel to the inclined surface; sometimes it is parallel to the base-line of the plane, the horizontal surface of a wedge, for example.

We will consider each case in order, seeking for the con-

nection between the weight, steepness of incline, and force to be applied.

68. Force Applied Parallel to Incline.—This case is illustrated by Fig. 49, where

L reoresents the *length* of the incline;
B " " *base* " " "
H " " *height* " " "
W " " *weight* of the body on the incline, applied straight downward from the centre of gravity of the body;
W′ is the equal and opposite of *W*;
N represents the force exerted upon the body by the plane *L*, a force which is straight outward from the surface of the incline if there is no friction (see Chap. VI) between the body and the incline;
P represents the pull, parallel to the plane *L*, which with the force *N* will just balance *W*.

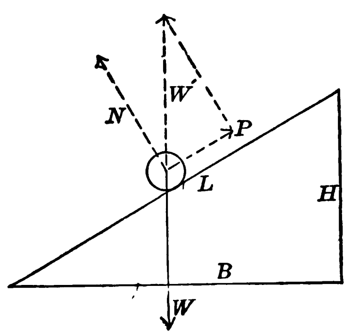

Fig. 49.

By comparing the dotted triangle with the triangle whose sides are *L*, *B*, and *H* we see that

$$P : W' \text{ (or } W) : : H : L,$$

or

$$P \times L = W \times H,$$
$$P = W \times (H \div L).$$

EXPERIMENTS.

Take apparatus No. XX and adjust it as indicated by Fig. 50, putting 7 oz. upon the pan, so that $P = 7 + 1 = 8$ oz. Then raise or

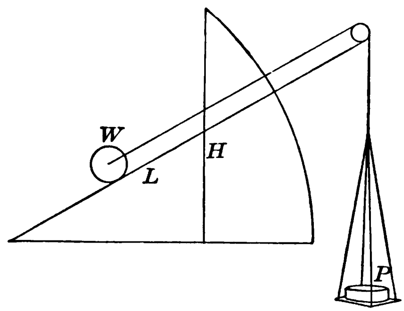

Fig. 50.

lower the incline till the weight W will barely roll up the incline when the apparatus is purposely jarred slightly. (The incline cannot be quite so steep when this takes place as it might be if there were no friction. If a knot is made in the thread near where it passes over the pulley at the top of the incline, a very slight movement up or down the incline can be detected by watching the position of this knot. A slight movement is enough.)

As soon as this adjustment is made read H, the length of the vertical scale from the top of the base-board to the under side of the incline, and record in the way indicated in the table below (upper row of numbers).

Then without changing P raise the incline somewhat more, until W will, when the apparatus is jarred, barely roll down the incline. (The incline must be somewhat steeper for this than it would have to be if there were no friction.) When the proper adjustment is made, read the new value of H and record it in the second line of the table below.

To find the H that would make P just balance W if there were no friction, take the mean between the two values now recorded. Then

find the L that would correspond to this value of H, L being the distance along the inclined scale from the hinge to the point of crossing the vertical scale.

	P	W	H	L
Going up...	8 oz.	16 oz.	....	....
" down..	8 "	16 "	....	....
To balance...	8 oz.	16 oz.	....	....

If time permits, make $P = 6$ oz., then 4 oz.; and in each case repeat the operations just described.

69. Force Applied Parallel to Base.—This case is illustrated by Fig. 51.

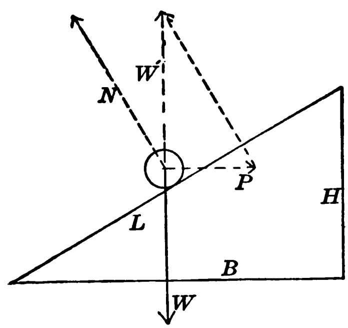

Fig. 51.

The line W' is not here, as it is in Fig. 49, the hypotheneuse of the dotted triangle; but it is evident that the dotted triangle is similar to the triangle made up of L, B, and H. P is the force applied parallel to the base, and just sufficient, with N, to balance W. We have, from a comparison of the triangles,

$$P : W' \text{ (or } W) :: H : B,$$

or

$$P \times B = W \times H,$$

$$P = W \times (H \div B).$$

EXPERIMENT.

For experiments in which the power is applied parallel to the base-line we cannot well make use of a string running over a pulley. We must apply the power by means of the spring-balance, as shown in Fig. 52, the long slot cut through the incline lengthwise allowing us to do so.

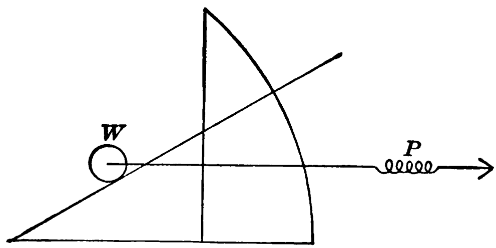

FIG. 52.

Find by trial a steepness of incline that will make P about 7 oz., and, keeping this steepness unchanged for the time, find how large P is when it is pulling W slowly and *steadily* up the incline, and how large when it is letting W run with equal slowness and steadiness down the incline. Take the mean* of these two values as the one that would be needed to balance W if there were no friction.

We record, then, for this case:

	P	W	H	B
Going up.........	....	16	....	
" down.......	....	16	....	
To balance....	....	16	....	....

where B is the length of the *base-line* from the hinge to the foot of the vertical line, along which H is measured.

If time permits, lower the incline and try various degrees of steep-

* The mean of the two values of P is not, in this case, exactly the quantity wanted, because the greater pull of P when W is going up the incline makes W press harder against the incline when going up than when going down, thus increasing friction. The mean value of P, as now found, is a little greater than the value wanted, but so little that the error is not important.

ness, so that P will be in one case about 5 oz. and in another case about 3 oz.

70. The Wedge.—The *wedge*, as commonly used (see Fig. 48), is a case of the inclined plane with the applied force parallel to the base. It differs from the case shown in Fig. 51 in one respect. In Fig. 51 the body raised has a motion parallel to the base of the plane, while the plane itself has not. A wedge commonly has a motion parallel to its own base, while the body raised or otherwise moved by it does not have a motion in this direction. The principle involved in the two cases is quite the same, and for a wedge used to lift a weight we have, leaving friction out of account as before,

$$P = W \times (H \div B),$$

where H stands for the thickness of the wedge, and B for its length.

71. The Screw.—The *screw* is an ingenious form of the inclined plane, as the following experiment will show.

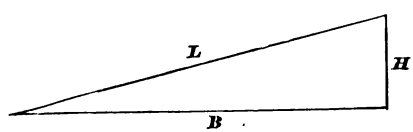

Fig. 53.

EXPERIMENT.

Cut out a long narrow triangle of paper, (see Fig. 53), and then wind it upon a lead-pencil, beginning at the end H and keeping the line B all the time at right angles with the length of the pencil. The line L will make a regular spiral around the pencil, corresponding to the *thread* of a screw.

72. Pitch of a Screw.—The distance from one turn of the thread to the next turn, measured parallel to the length of the screw, is called the *pitch* of the screw.

It is evident that in the ordinary use of a screw one revolution moves it foward or backward the length of its pitch.

73. Use of the Screw in Lifting.—A very large iron screw, called a jack-screw, is frequently used for lifting very heavy bodies. The power is applied to such a screw by means of a long handle or lever, which projects from the head of the screw at right angles with its axis, its central lengthwise line. Leaving friction out of account, we can find the relation between power applied and weight of the body lifted thus:

Let $P =$ the power applied to the handle at right angles with the handle and with the axis of the screw;

$A =$ the distance from the point of application of P to the axis of the screw;

$r =$ the radius of the screw itself;

$p =$ the pitch of the screw;

$W =$ the weight of the body lifted.

The force P produces at the thread of the screw a force $P' = P \times (A \div r)$. (See Exercise 10.)

This force at the thread is like the power used to drive a wedge. The circumference of the screw at the thread, which $= 2\pi r$, corresponds to the length of the base of the screw, while the pitch corresponds to the thickness of the wedge. We have, then

$$P' \times 2\pi r = W \times p,$$

$$P' = W \times (p \div 2\pi r),$$

or

$$P \times (A \div r) = W \times (p \div 2\pi r);$$

whence

$$P \times 2\pi A = W \times p.$$

Observe that we have here, as we have had so often

before, the rule, *Power* $\times$ *distance the power moves* $=$ *weight* $\times$ *distance the weight is lifted.*

DEFINITIONS.

74. Equilibrant.—A single force that will just balance, or make equilibrium with, two or more others is called their *equilibrant*.

In Fig. 46 bW is the equilibrant of bi and bh;
bi " " " " bW " bh;
bh " " " " bW " bi.

75. Resultant.—A single force that can exactly replace two or more others, so as to produce the same effect upon the body acted on, is called their *resultant*.

In Fig. 46 bW' is the resultant of bi and bh.

The resultant and equilibrant in any given case are equal in magnitude, but opposite in direction, so that the two would exactly balance each other.

QUESTIONS.

(1) What is the resultant of pq and pr in Fig. 47? What is their equilibrant?

(2) Draw three lines leading from one point, giving to them such magnitudes and directions that they will represent three forces in equilibrium with each other.

(3) Replace two of the lines in the preceding problem by two others that will also represent equilibrium with the third line.

(4) A telegraph-wire pulls north from a post with a force of 12 lbs.; another pulls west from the same post with a force of 16 lbs.

(*a*) How great is the resultant pull on the post?

(*b*) If a third wire is put in to neutralize the pull of the other two, should it pull more nearly south than east, or more nearly east than south?

(5) Two sticks of equal length, OA and OB in Fig. 54, each resting one end upon the ground, meet at a right angle in a frictionless joint O. From this joint is suspended a mass of 5 lbs., the weight of which is represented by the line OW.

(*a*) How great is the total force exerted by each stick against the ground, the weight of the stick being left out of account? *Ans.* 3.54 – lbs.

(*b*) How great is the vertical push exerted by each stick against the ground? *Ans.* 2.5 lbs.

(*c*) How great is the horizontal push exerted by each stick against the ground?

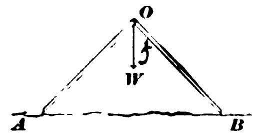

FIG. 54.

Ans. 2.5 lbs.

(We see from this problem that the total vertical force exerted at *A* and *B* is just equal to the weight of the suspended mass, the weight of the sticks not being considered. If we think of *AOB* as one end of the roof of a house, the answer to (*c*) shows the tendency of the roof to push the walls apart. This tendency is met in actual roofs by beams or rods connecting *A* and *B*.)

(6) A wedge 1 ft. long on its base and 2 in. thick is used to lift a weight of 300 lbs. in a case where friction may be left out of account. How great is the force, parallel to the base, required to drive the wedge?

(Friction not being considered, the force required to keep the wedge moving after it is started is no greater than the force required to hold it in place so as to make equilibrium, as in the discussion of § 70.)

(7) A safe weighing 2000 lbs. is resting on an inclined plane 12 ft. long, one end of which is 2 ft. higher than the other. How great is the force, parallel to the incline, required to keep it from sliding down?

(8) A jack-screw having a pitch of $\frac{1}{4}$ in. and a handle 2 ft. 1 in. long is used to lift a mass of 5000 lbs. How great must be the power applied to the end of the handle?

CHAPTER VI.

FRICTION.

76. Introductory.—When we push a heavy block along on the top of a table we feel a certain resistance. We know from experience that by making the surface of the table and the surface of the block very smooth we can lessen the resistance. *This resistance, the amount of which depends upon the condition of the rubbing surfaces, is called Friction.*

Friction always opposes motion, whatever may be the direction of the motion, that is, it merely tends to stop the motion. It never helps to push the block back to the position where it started.

We shall in Exercise 14 measure in a number of cases the force required to keep a block moving steadily along on a sheet of paper laid upon a level board, and shall study these cases with the purpose of finding out some useful facts, or laws, concerning friction between solid bodies.

EXERCISE 14.

FRICTION BETWEEN SOLID BODIES.

Apparatus: A spring-balance (No. 7). A rectangular block (No. 9). Set of weights (No. 19). A smooth sheet of paper about 1 ft. wide and 1½ ft. long. Thread.

We shall first consider the velocity of the motion, that is, we shall ask whether the force required to keep up a slow steady motion is greater or less than that required to keep up a more rapid steady motion.

Lay the block on one of its broad sides, and attach it to the spring-balance by a thread passing around but not under the block. Load the block with weights until the force required to maintain a slow steady motion is about 3 oz. Draw the block parallel to its *grain* along the sheet of paper several times with a very slow steady motion, and then several times with an equally steady motion two or three times as fast. (As the paper is likely to grow somewhat smoother under the repeated rubbing, the experimenter should not make all his slow trials first, but should change from slow to fast and fast to slow a number of times.)

Record your conclusion as to whether the slow or the more rapid motion requires the greater force.

We shall next try to find out whether, the total weight being the same as before, it is easier or harder to draw the block on a narrow side than on a broad side.

Use the same block and the same load of weights, pulling it now, as before, parallel to its grain.

(The side upon which the block slides should in all cases be clean, and the broad and narrow sides which are compared should be, as nearly as practicable, equally smooth. The thread must not be between the rubbing surfaces in any case.)

Record your conclusion as to whether the broad side or the narrow side offers the greater resistance to the motion.

Finally, we shall ask what connection there is between the total mass drawn and the force required to draw it.

For this purpose vary the weights placed upon the block, using not less than 6 oz. for the least and as much as 16 oz. for the greatest load.

Add to the load in each case the weight of the block itself, and make the record in the following form, W being the load and b the weight of the block:

$W + b$.*	F (Force Required).
. . . .	. . . . .
. . . .	. . . . .
. . . .	. . . . .

Look for any simple relation between $(W + b)$ and F.

* It is well to begin with the lightest load, proceed in regular order to the heaviest, then go back in exactly the reverse order to the lightest, recording both trials made with each load and taking the mean of the two for final study.

The experiments just described will teach a number of useful facts about friction between two solid substances, but one must be careful not to apply the conclusions here arrived at to extreme cases, extremely slow or very fast motion, for instance; or to cases where the pressure is great enough and the edge of the sliding body narrow enough to cause an actual cutting of the body into the surface over which it should slide.

77. "Laws of Friction."—The so-called laws of friction between solids are:

1*st. Friction is independent of the velocity of one surface across the other, other things being equal.*

2*d. Friction is independent of the area of the rubbing surfaces, other things being equal.*

3*d. Friction is proportional to the total pressure of one surface against the other, other things being equal.*

The experimenter in Exercise 14 need not be surprised or disappointed if his observations do not agree exactly with these statements. In fact, the "laws" are not strictly true; but they are near enough to the truth to be of very great use.

78. Coefficient of Friction.—*If the pressure between two surfaces, at right angles with each of them, is called P, and if the friction between the two surfaces is called F, the ratio $F \div P$ is called the coefficient of friction.*

In Exercise 14 the ratio $F \div (W + b)$ is the coefficient of friction.

There is a method of finding this coefficient without measuring either P or F. It makes use of an inclined plane and the parallelogram of forces.

In Fig. 55 O is supposed to be a body resting upon the incline AB, which is just steep enough to keep O moving with uniform velocity, in spite of friction, if it is once started down the incline.

The line OW represents the weight of the body. This

is equivalent to a force OP at right angles with the incline and a force OM down the incline. It is the force OP that

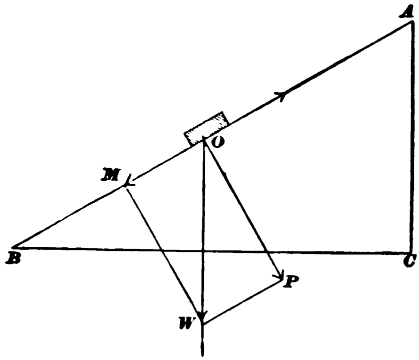

FIG. 55.

causes the friction. It is the force OM that maintains motion in spite of the friction.

If the body moves with *uniform* velocity down the incline, as we have supposed, the force OM must be exactly equal and opposite to the resistance of friction. For if OM were greater than the friction, the body would move faster and faster down the incline; while if OM were less than the friction, the body would move more and more slowly down the incline.* While the body is moving downward friction is represented by the arrow pointing from O toward A, equal and opposite to OM.

Therefore, in accordance with the definition given at the beginning of this article, we have

$$\textit{coefficient of friction} = OM \div OP.$$

* It requires force to set any body in motion and it requires force to stop any body that is in motion. If a body is moving along in a straight line with uniform velocity we know that the various forces acting on it balance each other. This matter is discussed further in the Second Part.

A comparison of the triangle OWM, in which $WM = OP$, with the triangle ABC shows that they are similar, and hence

$$OM : OP :: AC : BC.$$

That is, the

$$\textit{coefficient of friction} = AC \div BC.$$

EXERCISE 15.

COEFFICIENT OF FRICTION.

Apparatus: The same block that was used in Exercise 14. A flat board (No. 20) about 15 cm. wide and 50 cm. long for the block to slide on. A sheet of paper to cover one side of this board. Some means of raising one end of the board and adjusting it so that the block will just slide down it; another block similar to the one which slides, or any similar object, will do for this purpose. A 30-cm. measuring-stick.

Place one end of the board on the table and the other on the support. Vary the steepness of the board by varying the position of the support, until such an inclination is found that the block, once started slowly, will barely continue in motion down the board.

Then lay off on the table beneath the board a distance of 30 cm., measured from the edge where the board rests upon the table, and from the end of this line measure H, the vertical distance up to the under side of the board. The coefficient of friction will be $H \div 30$.

If the same block, the same side of the block, and the same kind of paper are used in this Exercise as in Exercise 14, the value of the coefficients obtained in the two Exercises should be compared.

79. Friction in Applied Mechanics.—Friction is one of the most important conditions in the construction and operation of very many mechanical appliances. It enters largely into the list of resistances to be overcome, as in the rolling friction of the car-wheels upon the track or of wagon-wheels upon common roads. Every axle revolves in its bearings with a measurable amount of friction, which can be diminished but not overcome by oiling the surfaces

in contact. On the other hand, many machines and mechanical appliances would be valueless without friction. Upon this the efficiency of belting, of brakes, of nails and screws of every description, is dependent. The driving-wheels of engines or of electric street-cars, the feet of men or of horses, would be unable to produce or maintain locomotion without the aid of friction. If its operation were suspended, every river would become a cataract, soon running itself out.

Rolling Friction.

80. Introductory.—The friction encountered by a moving body is usually much less when it is on wheels or rollers than when it slides, though it is true that on snow *runners* are better than wheels. The wheels of ordinary carriages do not get rid of sliding friction altogether, for the surfaces of the axle and the hub slide over each other. The "ball bearings" of bicycles do away with this sliding friction almost completely.

81. Coefficient.—The coefficient of rolling friction of iron wheels on iron rails may be as small as .002,* so that a pull of 4 lbs. may keep in motion a carriage weighing 2000 lbs.

The coefficient of sliding friction of smooth dry iron upon iron is perhaps .15 or .20.

82. Slipping of Wheels.—When people were first considering the use of steam for dragging railroad trains, they thought it would be necessary to provide the driving-wheels of the locomotive with cogs fitted to a cogged rail along the track. This device was found to be unnecessary for ordinary work, but it is used on very steep inclines running up the sides of mountains.

* Rankine, *Civil Engineering.*

Even upon ordinary railroads, when the rails are wet and there is a heavy train to be set in motion, the driving-wheels sometimes slip and revolve, while the train refuses to start. The frequency of the puffs from a locomotive depends upon the speed of revolution of the driving-wheels, and when an engine that has been puffing very slowly in starting a train suddenly gives three or four puffs in very quick succession, we may conclude that the driving-wheels are slipping on the rails. Engines are provided with sand-boxes, from which sand can be sprinkled upon the rails in front of the driving-wheels when slipping occurs.

Friction between Solids and Fluids.

83. Unlike Friction between Solids.—The laws of friction between solids and fluids are very different from those which hold between solids. Friction between solids and fluids changes comparatively little with change of pressure, but it changes a good deal with change of velocity. The resistance of the air is an important obstacle to rapid motion, as in the case of a railroad train, and the frictional resistance of the water to the hull and propeller of a steamer demands most of the steam-power required to propel the vessel.

84. Friction in Tubes.—The friction of liquids or gases flowing rapidly through long tubes is very considerable, as the following experiments will show.

EXPERIMENTS.

(1) Take a rubber tube 2 or 3 m. long and about 0.6 cm. in diameter of bore. Cut off a piece about 20 cm. long. Fill a large glass jar with water.

Using the short piece as a siphon, keeping the lower end about 10 cm. beneath the surface of the water in the jar, find the number of seconds required to fill a small tumbler with the water delivered.

Try the same experiment with the long tube, keeping its outlet also 10 cm. below the surface of the water in the jar.

Compare the rates of delivery in these two cases.

(2) Take a rubber tube 2 or 3 m. long and about 0.15 cm. in diameter of bore. Cut off a piece 10 cm. long. Light a candle.

Put out the candle-flame by blowing through the short tube. See how far from the outlet of the tube the flame must be placed in order to survive the blowing.

Repeat the trial, using now the long tube.

Compare the distances in the two cases.

Something more concerning friction of water in tubes is given in the Second Part.

QUESTIONS.

(1) A body weighing 20 lbs. rests upon a horizontal surface upon which its coefficient of friction is 0.2. How great is the force required to keep the body moving along the surface?

(2) It requires a force of 20 lbs. to keep a certain body moving along a horizontal plane, the coefficient of friction being 0.3. What is the weight of the body?

(3) A sledge weighing 10 lbs. can be drawn along a certain level surface by a force of 0.25 lb. How great may we expect the force to be which will just maintain motion when a load of 50 lbs. is placed on the sledge?

(4) A sledge weighing 50 lbs., having runners 1 in. wide, is dragged along a floor by a force of 15 lbs. How great a force would be required if the runners were twice as wide?

(5) According to Rankine's *Civil Engineering*, the coefficient of sliding friction of loose earth on earth may be as much as 1, although it is generally less. Suppose a bank of earth, with 1 for the coefficient of friction, to be made of such steepness that the outer surface, if started, will continue to slide downward.

(*a*) If a pole reaches 10 ft. straight downward into such a bank, how far along a horizontal line is the lower end of the pole from the surface?

(*b*) How great is the angle which the surface of such a bank makes with a horizontal plane?

CHAPTER VII.

THE PENDULUM.

85. Use in Clocks. — Before leaving the subject of Mechanics and going to that of Light it is well to learn something about pendulums, which are used to control the motion of clocks.

If you were to examine the works of an old-fashioned clock, you would find the power which drives it in a heavy weight working upon a kind of pulley by means of a long cord, but the device which governs the speed of the works and allows the motion to be neither too fast nor too slow is the pendulum. As a crowd of men at a turnstile, however they may try to force their way, can pass no faster than the swinging turnstile permits, so the clock-weight, which if the control were removed would run down at once with a furious buzzing of the wheels, is allowed by the pendulum to descend only very slowly, a very little distance at every swing of the pendulum, and not at all when the pendulum does not move.

The rate at which the clock-wheels can move, then, depends upon the length of time required for each swing of the pendulum. We will try a few simple experiments to find out something about the laws of pendulum motion.

EXPERIMENTS.

Description of Apparatus.—A convenient method of suspending a simple pendulum is shown in Fig. 57, where *B* is one end of a wooden bar, which is bevelled off on the side from which the pendulum

hangs. *C* is a cork fastened to the top of the bar and having in it a slit made by a sharp knife, through which slit the silk thread, *S*,

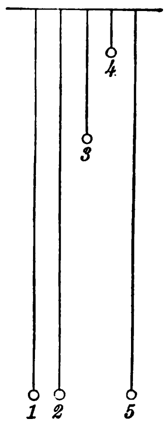

FIG. 56.

FIG. 57.

passes. If this part of the thread is waxed, the fastening thus obtained holds the pendulum securely, although it is very easy to increase or decrease the length of the pendulum at will. The length of the pendulum is to be measured from the under side of the bar to the centre of the ball. It is intended that the length of No. 2 and No. 5 in Fig. 56 shall be the same as that of No. 1, that the length of No. 3 shall be one-fourth that of No. 1, and the length of No. 4 one-ninth that of No. 1. It is therefore convenient to make the length of No. 1 just 36 inches, which will require 9 inches for the length of No. 3, and 4 inches for that of No. 4. The suspended body is a bullet in the case of each pendulum except No. 5, where it is some lighter object—a marble, for example.

The whole apparatus is called No. XXI.

(1) *How does the time required for a single swing depend upon the length, or width, of the swing?*

Set No. 1 and No. 2 swinging at the same instant and with the same width, or length, of swing, and watch them both for a little while until it is plain that under these circumstances they keep to-

gether, No. 1 taking just as long a time for one swing, or for any number of swings, as No. 2 does.

Then draw the ball of No. 1 about one inch aside from its position of rest, and the ball of No. 2 about fifteen inches aside from its position of rest, and release both balls at the same instant. Watch the two for some little time, a quarter of a minute or longer, and see whether at the end of that time they begin each swing together, as they did at first. If they do not, observe which one has gained upon the other, and, after one or two repetitions of the experiment, write down an answer to the question which the experiment was intended to meet. This answer should state which swing, the long or the short, if either, takes the longer time, and whether the difference in time is large or small compared with the time of either swing.

(2) *How does the time required for a single swing depend upon the length of the pendulum from the support down to the centre of the ball?*

Let one person, holding a watch in his hand, draw ball No. 2 several inches aside from its position of rest and, releasing it at a convenient moment, give a signal to the class, and let the class count the number of single swings till, at the end of 20 seconds from the start, a signal is given to stop counting.

In a similar manner the number of swings of No. 3 in 20 seconds and the number of swings of No. 4 in an equal time are found, and the observations for the three pendulums are recorded in a table, as follows:

Pendulum.	Whole Time.	Number of Swings.	Time of One Swing.	Length of Pend.	Square Root of Length.
No. 2	20 sec.	....	....	36	6
" 3	"	....	....	9	3
" 4	"	....	....	4	2

The numbers to fill the fourth column must be found from those in the second and third columns. A comparison of the fourth column with the sixth column will probably show that there is a close relation between the time of swing and the length of a pendulum.*

(3) *Weight of Pendulum-ball.*—Finally, a comparison of No. 1 and No. 5, set in motion at the same time and with the same width of

* It is interesting and even amusing to watch pendulums 1 and 3 or 3 and 4 swinging at the same time, both being started at the end of a swing at the same instant.

swing, will show whether the time of swing depends much upon the nature of the suspended body.

It will doubtless be noticed that the *width* of swing of the lighter body diminishes more rapidly than that of the heavier one. This gradual loss of motion is due to the resistance of the air. The resistance is about the same for both bodies if they have the same size, shape, and velocity, but a light body is more quickly stopped by a given resistance than a heavier body. This is the reason why one cannot throw an acorn or a piece of cork so far as one can a stone of the same size.

86. Springs in Place of Pendulums.—It has been said above that pendulums are used to control clocks, but many clocks and all watches are controlled by means of vibrating *springs;* for these, like pendulums, are very regular in their swings and so are good time-keepers. The controlling springs (see the "balance" of a watch, Second Part) must not be confused with the much larger *driving*-springs, or "*main*-springs," which are used in watches and in most clocks of the present day.

More will be said about pendulums in the Second Part.

QUESTIONS.

1. If one pendulum is 9 inches long and another is 64 inches long, how will the time of vibration of the first compare with that of the second?

2. If pendulum A, 39 in. long, vibrates once in a second and pendulum B vibrates once in 5 seconds, what is the length of B?

CHAPTER VIII.

NATURE OF LIGHT: VISIBILITY OF OBJECTS.

87. Light is Something that Travels.—We say that a lamp *gives*, or *gives out*, light. This is true. Light is something that comes to our eyes from any object and enables us to *see* the object.

A substance through which light can travel is called a *medium* for light. We have ways of measuring the time required by light to travel a given distance in air and in many other media.

88. Measurement of the Velocity of Light.—One of the simplest methods for measuring the velocity of light is that devised by the French physicist Fizeau.

It consists essentially of a source of light, from which a bright beam may be obtained, a toothed wheel which may be made to revolve in a plane at right angles to the course of the beam of light, and a plane mirror. Apparatus is provided by means of which the rate at which the wheel revolves can be exactly measured.

The beam of light passes through the space between two adjacent teeth of the wheel, travels a distance of several kilometers, is then reflected by the mirror, and returned over the same path by which it passed out. If the wheel is at rest, the beam as it returns will repass the aperture between the teeth through which it passed out. But it is easy to see that if the wheel could be revolved fast enough a tooth might be brought into the path of the returning

rays in time to intercept them. Still more rapid revolutions would bring a new gap between teeth into the path of the returning rays, and so on. In fact alternate eclipses and appearances of the returning rays are produced when the wheel is revolved at a high and continually increasing velocity. From the rate of motion of the wheel and the distance traversed by the beam it is not difficult to calculate the velocity of light.

As a result of measurements made by somewhat different means from those just described, the velocity of light has been ascertained to be about 300,000 kilometers, or 186,000 miles, per second in a vacuum. The velocity in air is a little less.

89. Light is of Various Kinds.—Light as it comes from the sun, or from most lamps, is of many different kinds, all blended together so that the eye does not distinguish one kind from another; but when this mixture of light falls upon certain objects, pieces of glass called *prisms*, for instance, the mixture is broken up and we see the different *colors*.

EXPERIMENT.

Hold a glass prism (No. XXXI) in the direct sunlight in such a position that light after passing through the prism will fall upon a white surface not in the direct sunlight

This breaking up of light is considered further in § 134.

90. Light a Wave-motion.—Before the nineteenth century many people believed light to consist of particles of matter, actually shot out in some way from the luminous body. These supposed particles were called corpuscles (that is, *little bodies*), and this theory as to the nature of light was called the *corpuscular theory*.

We now believe that light is not a substance, but a kind of wave-motion, a shiver, which is sent along through bodies with great velocity and to very great distances,

although the particles of the body, or *medium*, transmitting this wave-motion travel very small distances on either side of their positions of rest. More will be said about this in the Second Part of this book.

91. Color and Wave-length.—The different kinds of light, which produce in us the sensations of different *colors*, are distinguished from each other by differences of wave-length. Waves which produce the sensation of red, and which we often call red waves, are longer than the so-called blue waves, which produce the sensation of blue. One tint of red has a wave-length of one thirty-thousandth part of an inch. One tint of blue has a wave-length of one fifty-five-thousandth of an inch.

92. Light Travels in Straight Lines.*—When direct sunlight enters a darkened room through a small hole, one can usually trace its course and boundary in the room by means of the air-borne dust particles which are lighted up by it. It is easy to see that the boundary, the side, of the *beam* of light is straight. This is one of the familiar facts which show that light travels in straight lines. Practical applications of this property of light are found in the practice of *sighting* rifles, cannon, and other firearms; in the method of glancing along the edge of a board, which the carpenter adopts to see whether it is straight; and in the various surveying operations, in which points are located by sighting with the unassisted eye, or by means of fine slits in metal plates, or by the aid of small telescopes.

93. Light "Pencils and Rays."—If a beam of light is

* This statement holds good only in cases in which the light travels in a medium or substance of uniform composition throughout. Even under such circumstances there are certain exceptions to the general rule of rectilinear propagation. These occur where light passes close by the edges of objects, but the effects produced, although very interesting and beautiful, are not sufficiently prominent to make their study in this book necessary or desirable.

slender, it is a *pencil* of light. If the pencil is very slender indeed, it is called a *ray* of light, and is represented in drawings by a single line.

94. Camera Obscura.—This name means *dark chamber.*

EXPERIMENT.

Push the small tube of No. XXIV, closed end foremost, into the larger, and then, pointing the apparatus toward a window, look into the smaller tube and move it back and forth in the other till the best image of the window or of objects outside is obtained.

It is evident at once that the image is upside down, that is, that the bottom of the image represents the top of the object. This is due to the fact that the light-rays, coming from the object and traversing the very small aperture in the end of the tube, *cross each other in their passage,* as in Fig. 58, where the object is represented by the arrow *AB.*

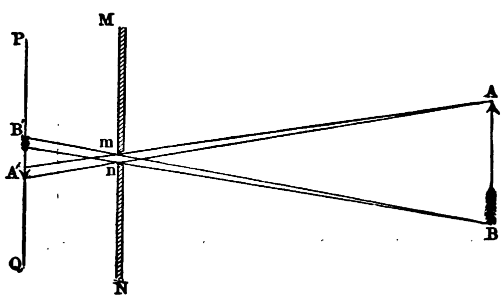

FIG. 58.

For instance, the cone of rays AA' from the tip of the arrow and the cone of rays BB' from the other end, cross at *mn*, and appear in the image at the spots A' and B' respectively.

If the aperture *mn* were gradually made larger, the spots A' and B', illuminated from A and B respectively, would grow larger and larger. The same would be true of the spots illuminated from other points of the arrow; and at last the growing spots would so overlap each other that the image would be lost in a mere blur of light on the screen.

95. Shadows. — From the fact that light travels in straight lines, it is easy to see that it will be cut off from a portion of space behind any illuminated opaque object, just as waves of water are cut off by a breakwater, leaving a region of calm water behind it. The simplest case is that in which the light-giving object is as small as possible.

EXPERIMENT.

Light a bat-wing gas-jet or a kerosene lamp with a broad, thin flame, and cast the shadow of a lead-pencil, held vertical, on a sheet of white paper, having first the edge and then the broad side of the flame toward the pencil. Note the great difference in the sharpness of outline in the two cases.

96. Umbra.—A shadow with a perfectly sharp outline could only be obtained by using as the source of light a mere point. To illustrate what would be the result if this

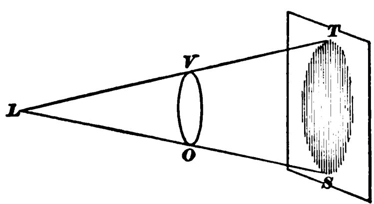

FIG. 59.

could be done, the student should examine Fig. 59.

Of the light-rays proceeding from the point L, the circular opaque object OV intercepts all which strike its surface, thus forming a shadow whose shape is in this case the frustum of a cone, $OSTV$.* The black space ST in the screen is not the whole shadow, but a *section* of the entire shadow $OSTV$. A perfect shadow like this, equally dark at all points, is called an *umbra*.

97. Penumbra.—Suppose now that the source of light is of appreciable size, a candle-flame, for example: then the

* That is, a cone with its top sliced off by a section parallel to the base.

opaque object *O* cuts off all illumination from some portions of the screen, and from other portions cuts off only a part of the light, as in Fig. 60.

That part of the screen which receives light from part of the flame *AB*, but not from all of it, will appear a partially shaded ring, *P'S'SP*, around the central area of total shadow. This ring forms what is known as the *penumbra* (from two Latin words meaning *almost* and *shadow*).

On account of the comparatively large size of most sources of light most shadows are surrounded by a wide margin of penumbra. The student will find the best examples of clear-cut shadows in those cast upon near surfaces by opaque bodies exposed to electric arc-lights, and he

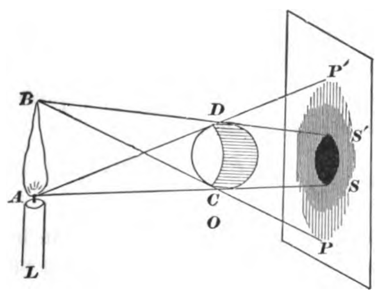

FIG 60.

may compare the dim and indistinct shadows of the leaves of shade-trees exposed to the sun, with those cast by the same objects exposed to the electric light at night, in which even the serrated margins of the leaves are sometimes clearly outlined.

98. How Light Weakens with Distance: Law of Inverse Square.—If two equally large surfaces are turned toward a very small flame, distant 1 ft. from one and 2 ft. from the other, the nearer surface will receive very nearly four times as much light from the flame as the more distant sur-

face. It is easy to prove that this is true if light travels in straight lines diverging from a point (Second Part).

If one surface is three times as far away as the other it will receive only one ninth as much light as the nearer one, and so on.

The law, which holds when the diameter of the light-giving spot is very small compared with the distance from it to the receiving surface, may be stated thus: *The amount of light received on a surface of given area from a given source of light is inversely proportional to the square of the distance from the source to the surface.*

This is called the *law of inverse square.*

It follows from this law that if a lamp L sends to a given surface at a distance D just as much light as another lamp L' sends to the same surface at a distance D', the light-giving powers of the two lamps, which powers we will call P and P', must be such that

$$P : P' :: D^2 : D'^2.$$

Illustration.

A candle-flame 30 cm. from a white card and an incandescent electric lamp 120 cm. from the same card light it up equally. What is the relative power of the two sources?

$$P_l \text{ (for the lamp)} : P_c \text{ (for the candle)} :: 120^2 : 30^2.$$

Hence $$P = P_c \times 16.$$

99. Photometry; Rumford's Photometer.—It is a matter of great practical importance to compare the illuminating power of different lamps. This operation is called *photometry*, or *light-measurement*. It cannot be done by merely observing the lamps directly; for the eye is unable in this way to distinguish slight differences of power, and if the lights are of somewhat different colors the unaided eye gives only the vaguest indications in regard to their comparative efficiency.

One of the simplest devices for measuring the relative power of two sources of light is Rumford's photometer, which compares the shadows cast by a rod placed in front of them.

EXERCISE 16.

USE OF RUMFORD PHOTOMETER.

Apparatus: Two small kerosene lamps like No. 33. A cardboard screen and its support (No. 32 and No. 21). Any opaque rod about 1 cm. in diameter and 10 or 15 cm. tall, supported upright; e.g., No. 13 standing in a hole bored in a small block, or a Bunsen burner. A meter-rod.

The object of the experiments will be to find whether a flame sends more or less light from its broad side than from its edge, and, if so, how much. The flame should be made as large as they can well be without smoking.

The apparatus should be arranged as in Fig. 61.

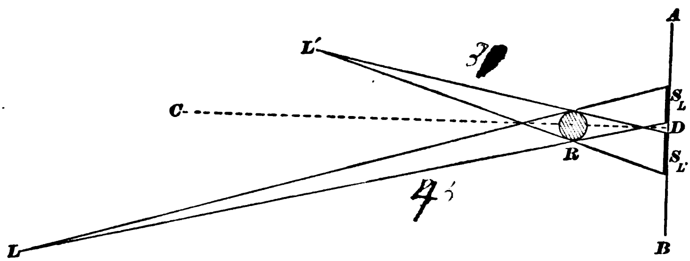

FIG. 61.

L is one of the lamps to be compared, and L' the other; R the rod, AB the screen, and S_L and $S_{L'}$ the shadows. The lamps should be so arranged that lines drawn from their centres to the centre of R will make nearly equal angles at R with the line CD, drawn at right angles to the screen through the centre of R, and on this line the observer should be placed. The shadows should be near each other, but must not overlap.

It is plain that the shadow corresponding to L is illuminated by light from L' and that the one corresponding to L' is illuminated by light from L.

Place the lamps equidistant from the rod. and, shielding the eyes

from the direct light of the flames, adjust the flames, turned edgewise to the rod, until the shadows are of equal darkness.

Then turn one of the lamps about in place until its flame is flatwise to the rod, and compare the shadows again, fixing the attention upon the middle of the more blurred one. If the shadows still appear to be of equal darkness, record the fact. If they do not, move one of the lamps toward or from the rod until the shadows appear equally dark, and then record the distance of each flame from the corresponding shadow.

Try each lamp in turn flatwise, the other being edgewise. Between the trials test the flames again in their original position, to make sure that they are still equal.

If, on the whole, it appears that one aspect of the flame, broad side or edge, is more effective than the other, estimate the relative light-giving power of the two aspects from the measured distances, making use of the law of inverse square.

100. Bunsen's Photometer.—The form of photometer devised by the German chemist and physicist Bunsen yields, under suitable conditions, more accurate results than the apparatus just described, and is equally simple, but more difficult to use in an undarkened room.

EXPERIMENT.

Drop a little paraffin on a sheet of heavy, unsized, white paper,—thin drawing paper, for instance. Heat the paper by placing on it a moderately hot iron weight or a can of hot water, until the paraffin is entirely melted and soaked evenly into the paper, so as to make a roughly circular spot about 3 cm. in diameter. Cut out of the paper a circle about 12 cm. in diameter with the spot just prepared in its centre. It will be noticed that the spot is *translucent;* that is, it allows some light to pass through it, although objects cannot be clearly seen through it. If one looks from a darker portion of the room toward a brigther portion with this screen interposed, the translucent spot will appear brighter than the ring of opaque paper around it, while, under the reverse conditions of illumination, the opaque ring will appear brighter than the spot.

Mount the screen in any convenient way; for example, in a block (No. 21). In making photometric observations with this screen the

illumination from one source of light is to be allowed to fall at right angles on one side of the screen, and that from the other source is to fall at right angles on the other side. The screen is then to be moved back and forth between the two lights until a position is found in which the appearance of the screen, as tested by the contrast between the central spot and the rest of the surface, is exactly the same on both sides when viewed from the same angle. The illumination on the two sides is then equal, and the distances from the lights to the screen will afford a means of comparing the power of the lights, as already indicated in § 98.

It is hardly worth while to attempt this experiment in an undarkened room.

101. Effect on Light of the Body on which it Falls.—When light-rays meet the surface of a body they may be:

a. Regularly reflected: that is, sent off from the surface in a direction which can be calculated or foretold, if we know the direction in which they are to strike the surface, as sunlight is reflected by a mirror.

b. Irregularly reflected or scattered: that is, sent back or off from the surface in many different directions, as sunlight is sent back from the surface of white cloth or white paper.

c. Transmitted: that is, allowed to pass through as sunlight through clear window-glass.

d. Absorbed: that is, neither reflected nor transmitted, but swallowed up, as sunlight by a lamp-black surface upon which it falls.

It usually happens that more than one of these effects is produced by the same body at the same time.

102. Visibility of Objects.*—Very few of the objects we see shine by their own light, as we can tell by testing them in the dark. They merely give off the light, or some part

* For much interesting and valuable matter upon this subject see Rood's *Text-book of Color*, Appleton & Co.

of the light, which has fallen upon them from the sun, or from some other light-giving body.

Of course we see many things every day upon which neither the sun nor any lamp is directly shining. We see them by what is called "daylight." This, however, is sunlight, although it may not have come straight from the sun to the objects that we see lighted up by it. It may have gone from the sun to a mass of clouds, from the clouds to the surface of fields or streets or walls of houses, and from these surfaces into corners where the sun itself is never seen.

It is extremely fortunate for us that all external objects do not treat the light which falls upon them in exactly the same way. If they did, all things would be of one color, and we could distinguish only light and shade. We have something like this condition after a fresh fall of snow which has covered roofs and trees as well as the ground. There are always, however, parts of trees and houses not completely covered by the snow, and this fact enables us to keep our bearings fairly well. If *everything* were covered by the snow, our eyes would not be of much more help to us in broad daylight than they are in the dead of night.

103. Colors of Transparent Bodies.—Colored pieces of glass, colored liquids, and other *transparent* bodies, generally owe their color to the fact that they are not transparent to all kinds of light. The light which enters them, sunlight, for example, usually consists of many different colors blended together; and they rob this light of those colors which suit their own constitution, *transmitting* the rest. It is the transmitted, the rejected, light which we get from them that gives them their apparent color. The light which they *absorb* is turned to something else in the absorption, and is no longer light. It is usually turned into heat.

104. Colors of Opaque Bodies.—Most bodies with which we are familiar do not appear to transmit light. We cannot see through them, and we call them *opaque* bodies.

In fact, most so-called opaque bodies are not perfectly so. If they are made into very thin sheets, the sun can shine faintly through them. Even when they are in thick masses, the light penetrates a very little distance beneath the surface, where some of it is absorbed, and some, being reflected by interior particles, returns to the outside. This returning light is usually different in color from the mixture of lights that entered, certain parts having been absorbed more than others.

105. Light from Surface of Colored Bodies.—The light reflected from the real external surface of non-metallic colored bodies receiving white light is usually not colored. The following experiment shows an illustration of this fact:

EXPERIMENT.

Let a beam of direct sunlight, entering a window, fall very obliquely upon a sheet of colored glass in such a way that the reflected beam will fall upon a white surface. Observe the color of the reflected light.

Certain materials, silks, for example, may reflect white light, from the outer surface, together with considerable colored light that has penetrated this surface and has been sent back from the interior. The white light gives the *sheen*, but in the spots where this is strong the *color* is not at the same time very evident, being made to look pale by the large amount of white light mixed with it.

The following experiment will show how the *color* coming from an object may be deepened by diminishing the amount of white light reflected from the external surfaces of its numerous particles.

EXPERIMENT.

Grind a lump of sulphate of copper to a fine powder and observe how faint the blue color becomes; then wet the powder with water,

which *adds* nothing but prevents some of the external reflection, and note the decided deepening of the blue.

In velvet * the ends of the fibres, which reflect but little white light, are turned outward, and the light which penetrates the surface and then returns to the outside is deeply colored.

* See Rood, p. 79.

CHAPTER IX.

REGULAR REFLECTION OF LIGHT.

106. Reflectors.—Smooth, even surfaces, like the surface of still water, polished glass, or polished metal, reflect light regularly (§ 101).

Transparent reflectors are not convenient for ordinary use: partly because light which we do not want may come through them from behind; partly because they reflect really well only such light as falls upon them very obliquely.

EXPERIMENT.

Let M, Fig. 62, be a piece of clear window-glass, L a lamp, and E the position of the observer's eye. The rays LM and ME make a

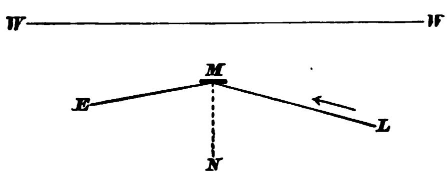

FIG. 62.

large angle with the line MN, which is the *normal* to the surface of the glass.

Observe the comparative brightness of the flame itself, and its picture or *image*, seen by reflection from M. Notice with what degree of clearness objects back of M, as, for instance, points on the wall WW, can be seen *through* M in the direction EM.

Gradually move the lamp and the eye toward the point N, until at last both lamp and eye are as nearly as possible on the line NM. While making these changes of position observe any resulting changes in the brilliancy of the image of L, and in the clearness with which objects on the line WW' are seen through the glass.

The reflecting surface which we make use of in a common mirror is not the front surface of the glass, but the metallic surface at the back. The glass is merely a convenient transparent support for the metallic layer, keeping it in shape and protecting it from being tarnished, as it soon would be if exposed to the air.

Reflection from a Plane Mirror.

107. Where the Image Is.—A *plane* mirror is a *flat* mirror. We shall study curved mirrors later.

When we place an object in front of a plane mirror and stand in a proper position we see an image, or "reflection," of the object, and we say that we see the object, or its image, *in* the mirror. If M, Fig. 63, is the mirror, O a point of the object, and P_1, P_2, P_3, and P_4 are the positions of four eyes, all may see at the same time an image of the point O in the mirror. Our first Exercise in light is intended to answer the question whether all these eyes see the *same* image, that is, whether all are looking toward the same point, and if so, where this point is—in front of the mirror, or behind it, or at its surface.

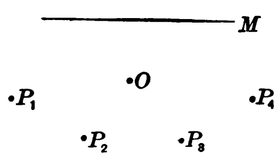

FIG. 63.

EXERCISE 17.

IMAGES IN A PLANE MIRROR.

Apparatus: A mirror (No. 23). A rectangular block (No. 9). A rubber band to hold the mirror to the block. Two straight-edged wooden rulers (Nos. 24A and 24B). A measuring-stick (No. 3). A sheet of thin white paper about 12 inches by 20 inches. A small block (No. 25). Attach the mirror to the large block by means of the rubber band in the manner shown by Fig. 64.

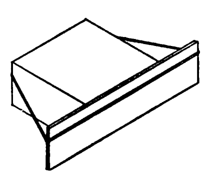

FIG. 64.

Draw a straight pencil-mark across the sheet of paper at its middle, and set the *back* surface of the mirror

directly over and parallel to this line, the middle of the mirror being very near the centre of the sheet. See Fig. 65.

Draw on the sheet of paper in front of the mirror a triangle, each side of which shall be several inches long, and no corner of which shall be less than three inches from the mirror. It is well to have one angle of the triangle not directly in front of the mirror, but somewhat to one side, like point No. 1 in the figure.

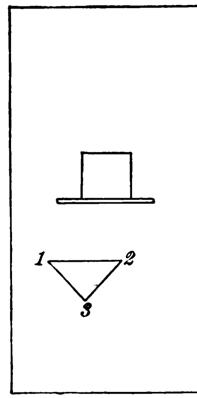

FIG. 65.

Place the small block in such a position that the vertical pencil-mark which it bears shall be directly over point No. 1 of the triangle. Then lay a straight-edged ruler (Fig. 66), upon the paper in such a position that one of its long horizontal edges, *PQ*, shall point directly toward the image of the vertical pencil-mark, as seen in the mirror.* The ruler should be so placed that the line of sight will strike near one end of the face of the mirror. Then with a well-sharpened pencil draw upon the paper a fine clear mark alongside that edge of the ruler which lies just beneath the line *PQ* (Fig. 66) along which the *sight* has been taken. Mark this line 1, because it points toward the image of the vertical pencil-mark when this mark is over point No. 1.

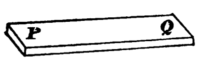

FIG. 66.

Next, without disturbing anything else, place the ruler in a new position, far removed sidewise from the position just occupied, sight as before, draw another line alongside the ruler, and mark this line also 1.

Then with the ruler in a new position, about half-way between the first two, if this is convenient, draw a third line in the same way, and mark this also 1.

All this time the small block has remained unmoved, and the pencil-mark upon it has pointed straight down at point No. 1.

* Many persons cannot do this at first unless they are especially instructed. A person who is not near-sighted should hold his eye eight or ten inches distant from *P*, and should then direct the ruler in such a way that the *point P*, the *point Q*, and the image of the vertical pencil-mark seen in the mirror shall all lie in one straight line. Do not try to look along the vertical *side* of the ruler, but hold the eye high enough to see all the time the top of the ruler.

Now place the small block so that the pencil-mark shall point straight down at point No. 2. While it is in this position draw three straight lines toward the image and mark each one of these 2.

Finally, put the pencil-mark over point No. 3, draw three straight lines toward its image, and mark each of them 3.*

When the three sets of lines have been drawn, the two blocks and the mirror are removed from the paper, and each line is then lengthened † until it crosses both the others of the same set; that is, each No. 1 line is continued toward or beyond the mirror till it crosses the two other No. 1 lines. Then the No. 2 set and the No. 3 set are treated in the same way.

After each set of lines has been extended in this way, it will be in order to answer the question whether all the lines of any one set lead to the same point or nearly so, and, if so, where is this point situated with respect to the mirror and to the point whose image it is.

If the image of each point, No. 1, No. 2, and No. 3, can be thus found, connect the image-*points* with each other by straight lines, and thus make an image-*triangle*.

Then fold the sheet of paper carefully along the pencil-mark by which the mirror was placed, and holding the folded sheet against a window, so that the light from without will shine through it, compare the size and shape of the two triangles and their relative positions with respect to the line along which the paper is folded.

* While drawing all these lines the experimenter should look frequently to see whether the back of the mirror remains in place. It may be thrown out of place by a little blow or by rubbing the paper hard to remove pencil-marks.

† If a line has to be extended far it is well to use two rulers, *A* and *B*, as shown in Fig. 67. First *A* is put into position and a line

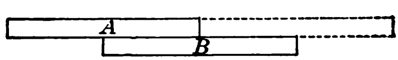

Fig. 67.

is drawn alongside it. Then, while *A* remains unmoved, *B* is carefully brought close to it, as the figure shows; then *B* is held firmly in place while *A* is pushed forward to the position indicated by the dotted lines. *B* is then removed without disturbing *A*, and again a line is drawn alongside *A*. In this way a line may be continued nearly straight for a considerable distance.

The general rule for placing the image of any point should be recorded when it is found.

The final result aimed at in this Exercise should be to enable the student to tell, without further experiment, in any new case given him (Fig. 68, for instance, in which *A B* is the line upon which the mirror stands), the position of the image of points No. 1, No. 2, No. 3, and No. 4, and so the shape and position of the image of the figure at the corners of which these points lie.

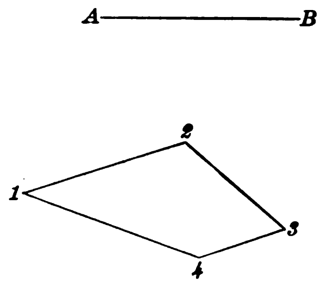

FIG. 68.

108. The Law of Reflection.—In Fig. 69, *MM* is a

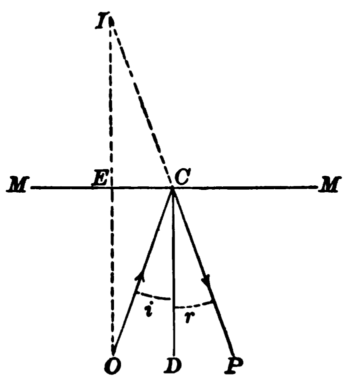

FIG. 69.

mirror surface, *CD* a *normal* to this surface, *OC* a ray *incident* at the point *C*, and *CP* the same ray after reflection.

The angle *i* is called the *angle of incidence.*

The angle *r* is called the *angle of reflection.*

The "law of reflection" is, that *the angle of reflection is equal to the angle of incidence.*

This law is easily proved on the basis of what we have

learned in the preceding exercise. The line of proof is this: *The image of O is at I. The angles at E are right angles; EI = EO; EC is common to the two triangles; hence the triangle CEI is similar to the triangle CEO. Then angle i = angle EOC = angle EIC = angle r.*

109. Real and Unreal Images.—If the rays of light proceeding from a point are by any means really brought together again at a different point, as in Fig. 79, then the second point is called a *real* image of the first. A real image has an actual existence in space, and will show as a *picture* upon a properly placed white screen.

If the rays of light proceeding from a point are by any means made falsely to appear to diverge from a different point, as in Fig. 70, then the second point is called an

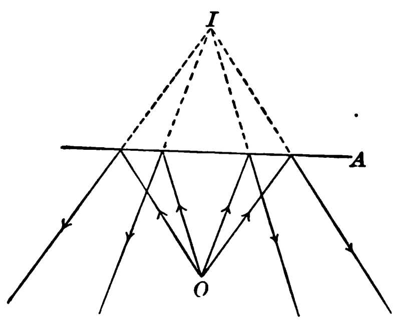

FIG. 70.

unreal, or *virtual*, image of the first. A virtual image has no real existence in space, and would not show upon a screen placed where it *appears* to be.

Evidently the image formed by a plane mirror is an unreal image.

110. Images of Images.—If any of the rays from *O* (Fig. 71) after reflection from the mirror *A* fall upon a second plane mirror *B*, they will be treated by this second mirror

just as if they really came from I_1; that is, we shall, looking into the mirror B in the right direction, see an *image* of the *image* I_1, and this second image, I_2, will appear just

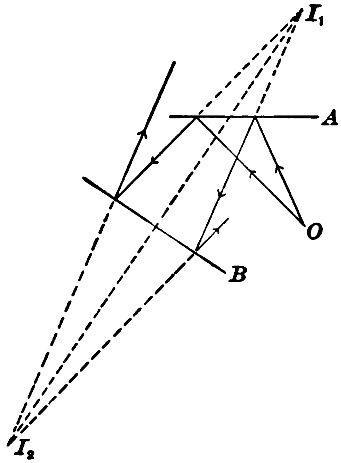

FIG. 71.

as if it were the image of an actual object, sending rays from I_1.

The rays reflected first from A and next from B might then fall upon a third mirror, and give an image of the image I_2, and so on; but at each reflection there is some loss of light, and an image formed after many reflections might be dim.

111. Positions of the Various Images.—Let A and B in Fig. 72 represent the positions of two plane mirrors meeting at right angles with each other at the point C. Let O be a small object placed between the mirror faces.

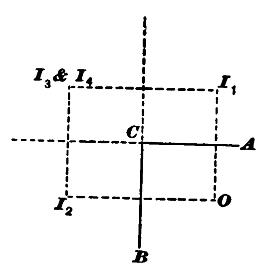

FIG. 72.

We shall have one image, I_1, formed by mirror A with-

out any help from mirror B, and another, I_2, formed by B without help from A. There is also I_3, the image of I_1, seen in B; and there is I_4, the image of I_2, seen in A. I_3 and I_4 fall at the same spot.

We cannot with this arrangement of the mirrors get images of I_3 and I_4; because rays leaving mirror A as if diverging from I_4 would not strike the face of B, and rays leaving mirror B as if diverging from I_3 would not strike the face of A.

Observe that O and its images fill the corners of a rectangle. If O were midway between the mirrors, the rectangle would be a square, with C at its centre.

If the angle between the mirrors were made a bit less than 90°, I_3 and I_4 would fall apart. If the angle were made 60°, *one sixth* of a circle, O lying half-way between them, O and its images would fill the corners of a regular hexagon having C at the centre.

If the angle were 30°, *one twelfth* of a circle, O and its images would fill the corners of a twelve-sided figure.

EXPERIMENT.

Place the hinged mirrors of No. XXV upon the board, with the *reflecting* surfaces making an angle of 90°, the point 1, Fig. 73, being midway between them. Place a lighted candle, of such length that its flame will not be above the upper edge of the mirrors, exactly over the spot 1.

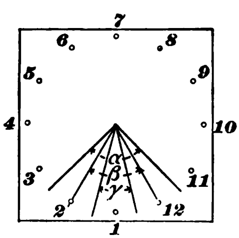

FIG. 73.

Note the positions of the images of the candle seen in the mirrors. Put pegs into the holes behind the mirrors in such positions that to an observer placed in front of the mirrors, so as to see the images in the mirrors and the pegs over the mirrors, the pegs will appear to coincide with the images.

Then make the angle between the mirrors 60° and place pegs to coincide with the images.

Finally, try an angle of 30°.

112. The Kaleidoscope.—The preceding passages give an explanation of the kaleidoscope, No. XXVI, with which most beautiful effects of endless variety can be obtained. The kaleidoscope uses bits of colored glass instead of a candle flame, and sometimes has *three* mirrors put together at angles of 60°.

QUESTIONS AND PROBLEMS.

(1) In a lighted room at night the glass of a window will serve as a mirror. In daylight unsilvered glass with a black cloth behind it may be used in the same way. Can you explain this?

(2) Soon after the moons of Mars were discovered in 1877 some one announced in a newspaper that one of these moons could be seen near Mars by looking at the reflection of that planet in a common mirror. It is true that a faint bright speck appeared near the image of Mars as thus seen, which did not appear when the planet was looked at directly, but the true moons could be seen only by the aid of powerful telescopes. Can you, after trying the experiment with any bright star, explain the appearance seen in the mirror?

(3) Write some short word as it would appear in a mirror if the printed page containing it were reflected in the mirror.

(4) A person standing in the middle of a room 20 ft. wide looks with one eye into a mirror 2 ft. square set in the wall of one side of the room. How many square feet of the wall behind himself could he see reflected in the mirror if his own image did not obstruct the view? (*Suggestion:* Draw a diagram representing the position of the observer, the mirror, the reflected wall and its image, all on a horizontal plane.)

(5) A candle-flame is placed half-way between two plane mirrors which meet at an angle of 40°. How many images appear, and how are they arranged?

Reflection from Curved Mirrors.

113. Spherical and Cylindrical.—Most curved mirrors are parts of spherical surfaces. We shall, however, study mirrors which are parts of cylinders. They are more convenient for our use than spherical mirrors, and they are less expensive.

We shall use both the *convex*, or bulging, and the *concave*, or hollowed, face of the mirror.

114. Centre of Curvature, etc.—*MM*, Fig. 74, represents a cut through a cylindrical mirror at right angles with the straight lines of its surface. This cut is of course a part of a circle.

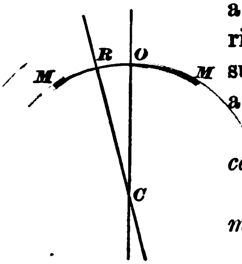

FIG. 74.

C, the centre of the circle, is called the *centre of curvature* of the mirror.

The point *O* is called the *centre of the mirror*.

The line *CO*, extended to any distance in either direction, is called the *principal axis* of the mirror.

Any straight line extending, like *CR*, through *C* and across the line *MM*, but not through the point *O*, is called a *secondary* axis of the mirror.

EXERCISE 18.

IMAGES FORMED BY A CONVEX CYLINDRICAL MIRROR.

Apparatus: The mirror (No. 27). A measuring-stick (No. 3). Small block (No. 25). Rulers (No. 24 A and B). Sheet of white paper. The plane mirror (No. 23) and its supporting block (No. 9).

Hold the mirror with its straight edges vertical, and look at the image of your own face in the convex surface. You will see that the image is distorted, appearing too narrow for its length. Hold the mirror with its straight edges horizontal, and the image will be distorted in the opposite way, appearing too wide for its length. The object of the following experiments is to give a better understanding of these curious effects.

Set the mirror on the table and bring one end of the plane mirror close to the surface of the curved mirror, as in Fig. 75 Then place the small block in front of both mirrors, as in Fig. 75, in such a position that you can see the block reflected in both mirrors at the same time.

Do the two images thus seen appear of the same height?

Do they appear of the same width?

Fill out, if you can, the following statement: *Lines of the object which are parallel to the straight lines of the cylindrical mirror appear*................*in the cylindrical mirror*.............*plane mirror.*

Lines of the object which are at right angles with the straight lines of the cylindrical mirror appear.....................*in the cylindrical mirror*....................*in the plane mirror.*

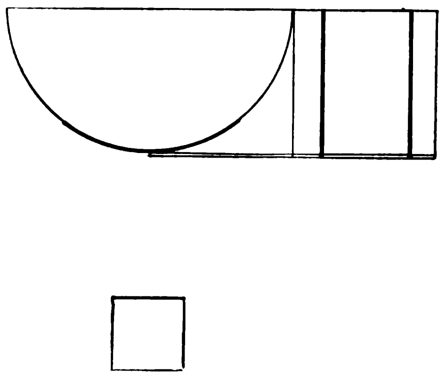

FIG. 75.

Remove the plane mirror. Holding the base of the curved mirror firmly in place, make a fine, clear, pencil-mark on the paper along the outer edge of the mirror. Then mark on the paper the point C, which is the *centre of curvature* of the mirror

About 5 cm from the front of the mirror draw an arrow 6 cm. long, marking the ends and the middle as in Fig. 76. Then place the small block so that the vertical pencil-mark which it carries will point straight down at point No. 1.

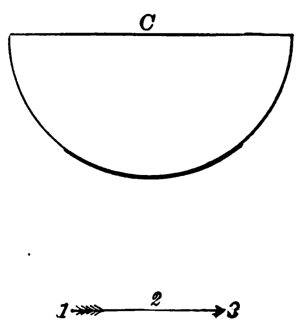

FIG. 76.

With the straight-edged ruler draw two lines, well apart, toward the image of this vertical line as seen in the mirror, avoiding parts of the mirror, if there are such, that do not give a good image of the line. Mark each of these lines 1. Then draw two lines for point No. 2 and two for point No. 3, in the same way.

Then clear the paper and prolong each pair of lines till it comes to a crossing-point. The three points thus found will locate the images of object-points No. 1, No. 2, and No. 3, respectively, and a line connecting these three image-points will give an idea of the shape of the image-arrow, whether it is straight or not, and whether its curvature, if it has any, is in the same general direction as the curvature of the mirror or in the opposite direction.

Draw a straight line from each marked object-point to the corresponding image-point, and prolong these three lines until they cross each other. Note where the crossing occurs.

Is the image longer or shorter than the object? Is it nearer to, or farther from, the mirror than the object is?

(It must be understood that the pupil is asked these questions only in regard to the particular case that he has tried. He cannot tell without further experiments or further instruction whether the answers he gives in this case would be true for all cases of objects reflected in mirrors such as he is using, for he does not know that the distance of the object from the mirror may not decide all these questions. The fact is, however, that, if he has found correct answers to the questions asked for his one case, the same answers will be true for the same questions in all cases with *convex* cylindrical mirrors. The effects seen with *concave* mirrors are much more complicated.)

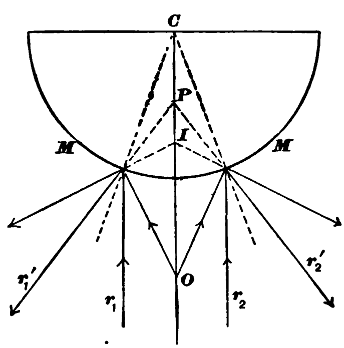

FIG. 77.

115. "Law of Reflection" Still Holds.—With curved mirrors, as with plane mirrors, the law (§ 108) *angle of in-*

cidence = angle of reflection holds. With the help of this law we can see why the image of a point is nearer the mirror, when this is convex, than the point itself is.

Let O (Fig. 77) be the object-point in front of the convex mirror MM, the centre of curvature being at C. A line drawn from C to any point of the mirror is at right angles with the mirror at the point of crossing. Two rays going from O to the mirror-front appear after reflection to come from I, which is nearer the mirror than O is.

116. Principal Focus and Focal Length.—If rays come from some very distant point on the principal axis (§ 114), they are practically parallel to each other when they reach the mirror. Two such rays are represented by r_1 and r_2 (Fig. 77). Applying the law of reflection to them, we find that after reflection they appear, as r_1' and r_2', to diverge from a point P, which is very nearly midway between the reflecting surface and the centre of curvature.

The point P is called the *principal focus* of the convex mirror. It may be defined as *the point which marks the image of an object-point situated a long distance away from the mirror on the principal axis*, or as *the point from which rays coming to the mirror parallel to the principal axis appear to diverge after reflection.*

The distance, measured along the principal axis, from the principal focus to the reflecting face is called the focal length of the mirror.

The principal focus and the focal length play a very important part in the science of curved mirrors and lenses (§ 136). More will be said of this later. See § 124.

117. Concave Mirrors.—If the *concave* side of the mirror were used, it is easy to see from Fig. 78 that rays from a point O near the mirror-front would after reflection appear to come from a point I, which is *farther* from the mirror

than O is. It is evident that the rays from O are more nearly parallel to each other after reflection than before.

Rays from a point O', somewhat farther from the mirror than O, appear after reflection to come from a still more distant point, I', and these rays are nearly parallel after reflection. It is easy to see that if the object-point were put somewhat farther still from the mirror, the rays proceeding from it might, after reflection, be parallel to each

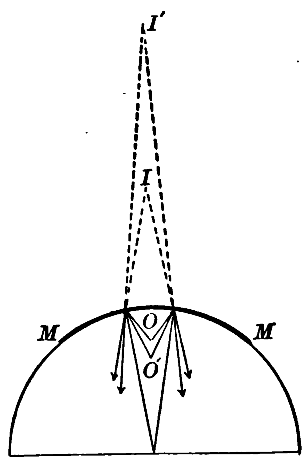

FIG. 78.

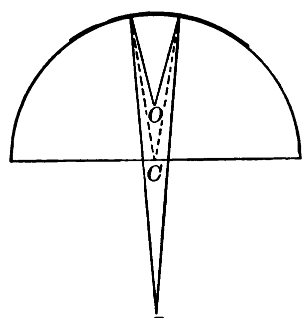

FIG. 79.

other. They would appear to come from a point as far as possible behind the mirror.

If the object-point is placed still farther away from the mirror, as at O in Fig. 79, the rays may after reflection be actually *converging*, and cross at a point I in *front* of the mirror. This image I is a *real* image (§ 109), and if O is bright enough, the image I may be seen, like a picture, on a piece of white paper or cloth placed in the right position.

We see that the centre of curvature C lies between the object-point O and the image-point I in the case shown by Fig. 79. This is always so in the case of *real* images formed by concave mirrors, unless the object-point is at C, in which case the image-point also falls at C.

If the object-point were placed where I now is, in Fig. 79, the image-point would fall where O now is.

EXERCISE 19.

IMAGES FORMED BY A CONCAVE CYLINDRICAL MIRROR.

Apparatus: The same as in the preceding Exercise, and in addition a common pin.

Preliminary.

Remove the mirror from the base-board; place the latter upon the paper and mark on the paper the point C and the curved outline of the board.

Make the distance CA 4.2 cm., and draw the arrow A 4 cm. long. Draw radii from C through the ends of A.

Make the distance CB 3.5 cm., and draw B from radius to radius.

Make the distance CD 1.5 cm., and draw D from radius to radius.

(All this should be done before the regular Exercise begins.)

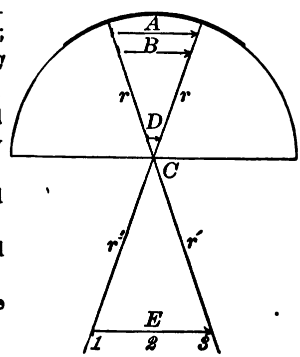

FIG. 80.

Place the mirror in position as in Fig. 80, and, keeping the eye about 20 cm. from it, look at the images of A, B, and D.

Do the images of A and B point in the same general direction, from left to right in the figure, as the arrows themselves?

Is the same answer true of D and its image?

Are the images of A and B longer or shorter than the arrows themselves?

At the centre of A stand the pin upright, and laying the two rulers on the paper, point one edge of each toward the image of the pin, contriving to have these edges make a considerable angle with each other. In this way the position of the image is located. Is it behind the mirror or in front? Is it, then, a real image or an unreal one?

By the same method locate the image of the pin when erected at the centre of B and when at the centre of D, asking and answering in each case the same questions that were asked when the pin was at the centre of A.

If time permits continue the Exercise as follows:

Extend the two radii r r by the lines r' r' drawn on the paper, as in Fig. 80. Draw the arrow E, 6 or 8 cm. distant from C, marking points 1, 2, and 3, upon it. Locate the image of each of these points by the method used in the preceding Exercise with the convex side of the mirror, drawing upon the paper the lines of sight and the image of the arrow.

118. Principal Focus of Concave Mirror.—*The principal focus of a concave mirror is the point to which rays, coming to the mirror parallel to the principal axis, converge after reflection.* In other words, it is *the point which marks the image (real) of a very distant point on the principal axis.*

As in the case of a convex mirror, the principal focus lies very nearly midway between the reflecting surface and the centre of curvature.

119. Rule for Placing Images.—Fig. 81 illustrates an

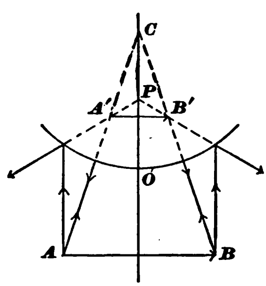

FIG. 81.

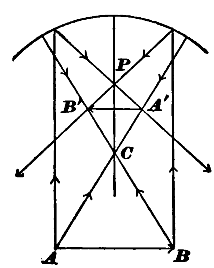

FIG. 82.

easy rule for finding the position of an image in a convex mirror.

Let AB be the object. Draw one ray from A straight toward the centre of curvature. This ray will return on

itself after reflection, coming as if from *C*. Draw another ray from *A* parallel to the principal axis. This will after reflection appear to come from *P*, the principal focus (§ 116). Both reflected rays appear to come from *A'*, which is therefore the image of *A*.

The image of *B* is found in the same way.

If the object is a straight line, as in this figure it is customary to represent the image by drawing a *straight* line from *A'* to *B'*. This is inaccurate, as Exercise 18 should show.

Fig. 82 shows the same method applied to a concave mirror.

120. Distorted Images.—In Exercises 18 and 19, and in all the figures that have been given representing cylindrical mirrors, we have been dealing with rays which are, both before and after reflection, parallel to the plane on which the mirror rests. If we make use of other rays, as we do when looking obliquely down at the mirror face, we see things sadly twisted, the effects thus obtained being too difficult for our profitable study.

121. Relation of Cylindrical to Spherical Mirrors.—If we were to use a spherical mirror, placed with its principal axis horizontal, and employ only horizontal rays striking the mirror on a narrow horizontal strip through its middle, we should get effects quite like those we have already studied. All the figures from 74 to 82 would apply as well to a spherical mirror so used as to a cylindrical mirror. Indeed, these figures are like those commonly given to show the effects obtained with spherical mirrors.

For general use spherical mirrors are better than cylindrical mirrors, because they can be used from more points of view without giving badly distorted images.

EXPERIMENTS.

With a concave spherical mirror 5 or 6 inches wide (No. XXVII) interesting lecture-table experiments may be made in a slightly darkened room, the image of a candle-flame or, better, gas-flame being thrown upon a screen so as to be visible to all in the room. The screen should be of tracing-cloth or oiled paper, so that the image upon it may be seen from both sides. An opaque screen should hide the flame itself from the eyes of the class. Fig. 83 suggests a good arrangement, *MM* be ng the mirror, *C* its centre of curvature, *L* the flame, *S* the opaque screen, and *S'* the tracing-cloth screen.

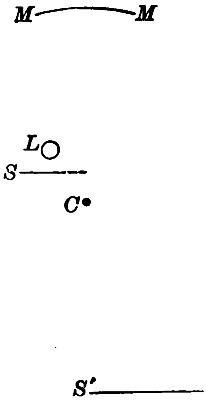

FIG. 83.

The positions of *L* and *S* may be greatly varied and may be interchanged, but the least distance of either from mirror the should be rather more than one half the radius of curvature of the mirror, if *real* images are desired.

122. Principal Use of Spherical Mirrors.—Although spherical mirrors are sometimes used to form images, as in certain telescopes, probably their most important use is to concentrate light upon some object that cannot otherwise be well seen.

Thus, the small objects which are to be looked at with a microscope need to be brightly illuminated, and a concave mirror is commonly used to throw light upon them.

123. The Ophthalmoscope.—Often a physician wishes to see what is wrong in the depths of a patient's eye. To do this the interior of the eye must be especially lighted up. If this is done by holding a flame in front of it, the flame dazzles the eye of the observer and therefore is of little use. The difficulty is overcome by means of the

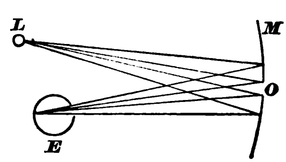

FIG. 84.

ophthalmoscope, Fig. 84, where M is a curved mirror with a hole in the centre; L is some source of light, placed so that the rays proceeding from it to the mirror pass by reflection into the eye of the patient, represented by E; and O marks the position of the observer's eye.

This simple application of the concave mirror was made by the great physicist Helmholtz, and it has probably won for him more popular fame and gratitude than all his other work. The most remarkable thing about many inventions is the fact that they were not made earlier.

124. Formulas Relating to Curved Mirrors.—In the following formulas, which are here given without proof,

D_o = the distance of object-point from mirror,
D_i = the distance of image of object-point from mirror,
F = focal length of the mirror.

For a convex mirror we have

$$-\frac{1}{D_o}+\frac{1}{D_i}=\frac{1}{F}.$$

For a concave mirror we have

$$\frac{1}{D_o}+\frac{1}{D_i}=\frac{1}{F}$$

when the object-point is farther from the mirror than the principal focus is, and

$$\frac{1}{D_o}-\frac{1}{D_i}=\frac{1}{F}$$

when the object-point is between the principal focus and the mirror.

It is doubtful whether work done with the cylindrical mirrors will be accurate enough to give results agreeing with these formulas. Similar formulas are used with respect to lenses.

QUESTIONS.

(1) A small object is placed close to a convex mirror.

(*a*) Is the image real or virtual?

(*b*) If the object is moved farther and farther away from the mirror, will the image at any time become real?

(2) If one looks at the image of his own face in a convex mirror, will the nose appear too prominent and the forehead and chin retreating, or will the opposite be true?

(3) If a small object is placed close to a concave mirror—

(*a*) Is the image real or virtual?

(*b*) If the object is moved farther and farther away from the mirror, will it reach such a position that its image will be real? If so, what is that position?

(4) (*a*) Have you in using any single mirror, plane or curved, seen a virtual image that was inverted, as compared with the object?

(*b*) Have you seen any real image, formed by a single mirror, that was right side up, as compared with the object?

(5) Do you see anything wrong with the physics of the following statement, copied from a prominent newspaper?—

There are times when the public sees things in a convex mirror, in which they appear broad, robust, and expanded. There are times when the public sees things in a concave mirror, in which they appear cramped, narrow, and contracted.

CHAPTER X.

REFRACTION OF LIGHT.

125. Introductory.—In the experiment made with a prism the class may have noticed that the light did not go in the same direction after leaving the prism as before entering it. Some members of the class in looking into pools or vessels of water may have noticed that objects beneath the surface are not exactly where they seem to be.

EXPERIMENTS.

(1) Place a straight stick in an oblique position, partly in and partly out of water. Notice the apparent bending or disconnection of the stick at the surface of the water.

(2) Place on a table a pan (No. XXVIII), 15 cm. or more in diameter and with nearly vertical sides 4 or 5 cm. high. Place a small coin on the bottom of the pan, and adjust the head in such a position that the side of the pan will just hide the more distant portion of the coin from the eye at *E* (Fig. 85). Maintain the head in this position by resting it against any convenient support; keep one eye closed, and look with the other into the pan, just beyond the farther edge of the coin, while another person slowly pours in water. Have the pouring stopped as soon as the whole of the coin becomes visible.

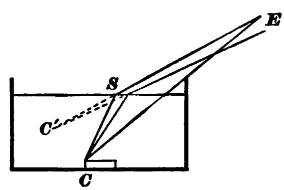

FIG. 85.

(3) Repeat experiment 2 with the eye held vertically above one edge of the coin, with a slender stick or a stout wire laid across the top of the pan, nearly in the line of vision, to serve as a point of departure from which to measure the apparent displacement of the coin, if any should be observed.

126. Interpretation of the Preceding Experiments.— Objects always appear to the eye to be in the direction *from which the rays are travelling at the moment of entering the eye.* Evidently, then, since the coin appeared to rise when the water was poured into the jar, in Experiment 2, the light-rays which proceeded from the coin must have been bent aside in some way by the water.

In Fig. 85 the straight line *CE*, which passes from the left-hand edge of the coin *C* to the pupil of the eye at *E*, represents the course of a light-ray from that point before the water was poured into the pan. Any ray that passed farther to the right than *CE* would be intercepted by the side of the pan; any ray that passed farther to the left, or more nearly vertical than *CE*, would miss the eye: hence it is evident that, so long as the pan is filled with air only, and the eye kept in the position shown, the coin cannot be seen.

But as soon as water is poured into the pan, the rays no longer travel in straight lines from the object *C* to the eye. Each ray suffers an abrupt change of direction at the surface of the water, and from this it follows that such rays as those which take the general course *CS* in the figure are finally brought to meet the eye at *E*. As a result of the bending *C* becomes visible, and its farther edge is seen apparently at *C′*, in a position somewhat raised above the bottom of the pan.

Experiment shows that the course *CSE* might be retraced by a ray. That is, a ray leaving *E* in the direction *ES* would reach *C* by the line *SC*.

QUESTIONS.

(1) If normals were drawn to the surface of the water, at the points about *S* where the rays emerge, would the bending of each ray be towards or from the normal (in the air)?

(2) If the rays were passing from E to C, would the bending at the surface of the water be toward or from the normal (in the water)?

When we look straight down into water at any small object it appears to be in its true direction from the eye, but nearer than it really is. Fig. 86 indicates why this is so. O represents the object, E the eye, much magnified, and O' the apparent position of the object.

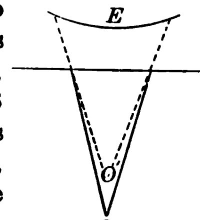

FIG. 86.

127. Angles of Incidence and Refraction.—The change of direction which a ray of light undergoes when it passes obliquely from one medium into another is called *refraction.*

The amount of the bending, or refraction, which a ray of light suffers at any surface depends partly upon the two substances which meet at this surface, and partly upon the angle, i (Fig. 87), which the ray makes with a line NN, which is at right angles with the surface at the point C where the ray strikes the surface.

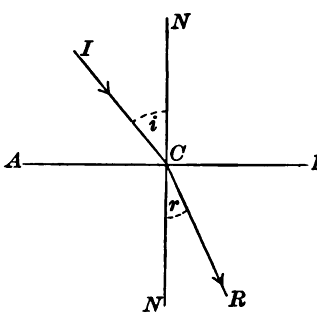

FIG. 87.

If the space above the line AB represents the air-space, and that below this line the water, or glass, or whatever substance it may be that lies there, solid or liquid, the course of the ray is changed at the surface in such a way that the angle r which it makes with NN inside the solid or liquid is smaller than the angle i.

The angle i in Fig. 87 is called the *angle of incidence.* The angle r is called the *angle of refraction.*

If the ray were represented as coming in the opposite direction, that is, first along R and then along I, r would be the angle of incidence and i would be the angle of refraction. The ray would be bent just as much at the surface as it is when going first along I and then along R.

128. Index of Refraction.—When the direction of I is changed the direction of R is changed. The way in which the change of one depends upon the change of the other is easily shown by means of Fig. 88. I, I', and I'' show

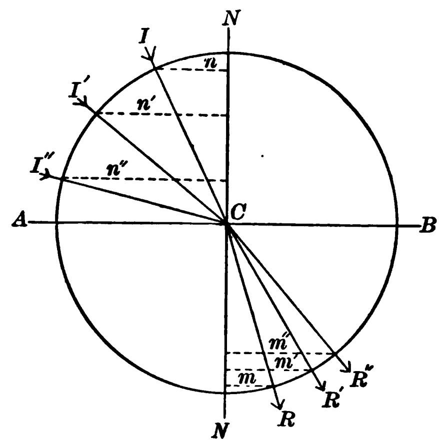

FIG. 88.

three rays all of which come to the point C and then separate, the first going along R, the second along R', the third along R''. The circle whose centre is at C is drawn with any convenient length of radius. The dotted lines, n, n', n'', and m, m', m'', are drawn from the points where the rays cut the circumference to the line NN, at right angles.

If this figure has been drawn so as to acord with the results of experiments on light-rays, we shall have

$$\frac{n}{m}=\frac{n'}{m'}=\frac{n''}{m''},$$

and *any one of these equal ratios is called the Index of Refraction of the second medium, below AB, with reference to the first medium, above AB.*

If now we can measure $\frac{n}{m}$ in any given case, we shall have a quantity which is very useful in physics, for by means of it we can *calculate* at once the value of a new m to go with any new n; that is, we can, if we know the index of refraction and the angle which any ray makes with NN in one medium, find without further experiment the angle which the same ray makes with NN in the second medium. Exercise 20 shows how to find the ratio $\frac{n}{m}$ for the case of air and glass.

EXERCISE 20.

*INDEX OF REFRACTION OF GLASS.**

Apparatus: A piece of plate glass (No. 28). Articles 8, 24A and 24B. A sheet of paper and three pins.

Place the glass, *G* (Fig. 89), on the paper *P*. Stick one pin upright at the point 1 close to one of the polished edges of the glass; stick the other pin at 2 close to the other polished edge.

Look with one eye from the position *S through* the whole width of the glass at pin No. 1. Move the eye toward 3, looking all the time through the glass at the pin. It will presently be noticed that the pin seen *through* the glass is not in the same direction from the eye as the same pin seen *over* the glass. That which is

* I owe the plan of this admirable Exercise to Mr. F. M. Gilley of the Chelsea High School. It is described in Gilley's *Principles of Physics*, Allyn & Bacon, Boston.—E. H. H.

seen through the glass is an *image* of the real pin, and it is upon this image that the attention should be fixed.

Continue moving the eye in the general direction of 3, keeping it, however, about 30 cm. from the glass, until the image of pin No. 1 is just hidden behind pin No. 2. Then place a pin at 3, in the same straight line with the eye, pin No. 2, and the image of No. 1.

Draw a fine pencil-line upon the paper close to the glass edge touched by pin No. 2. Then remove the glass.

The line now drawn marks the position of the refracting surface. The line 1–2, Fig. 89, shows the direction, within the glass, of a

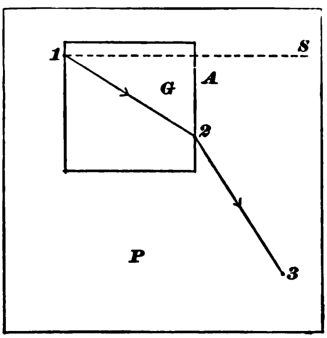

FIG. 89.

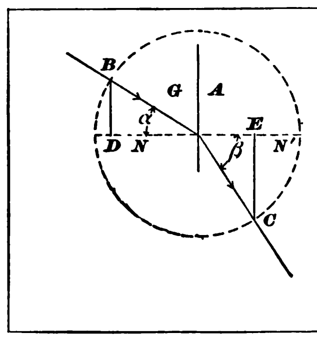

FIG. 90.

certain ray from 1. The line 2–3 shows the course of the same ray after it leaves the glass at 2. The line NN', Fig. 90, is drawn normal to the refracting surface at the point of emergence. From this point equal distances are laid off, to B and to C. Lines are drawn from B and from C to the line NN' at right angles.

$CE \div BD$ = *the index of refraction from air to glass.*

EXERCISE 21.

INDEX OF REFRACTION OF WATER.

Apparatus: Articles 3, 14, 15, 24A, 24B, 29, 30, and a sheet of paper about 6 inches square.

Put the partition N in place, as shown in Fig. 91, and pour water into the jar until its surface comes within 1 or 2 mm. of the middle tooth of the partition. Then by means of the plunger (No. 14), attached to the side of the jar by means of its clasp, raise the level of

the water till the apparent distance between the middle tooth of the partition and its reflection in the water surface is less than 1 mm. (To see this reflection well, one should look through the wall of the empty part of the jar.)

Then the brass index b is attached to the jar, as shown in Fig. 91, and is raised or lowered, *with the tip p touching the glass*, until an eye on the line Cg, 20 or 30 cm. from the jar, can barely see p, the very tip of b, apparently in a straight line with Cg. This setting should be made with care, and *after* it is made the experimenter must look to see whether the tooth at C is clear of the water. If its lowest edge *touches* the water the setting is useless, and all of the adjustments must be made anew before a *reading* is made.

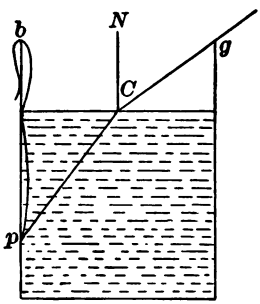

FIG. 91.

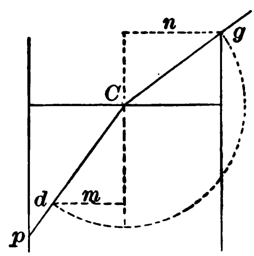

FIG. 92.

When all the adjustments have been successfully made, measure carefully the distance from the top of the jar down to the tip, p, of the index, the measuring-stick being kept outside the jar.

Measure now the inside diameter of the jar.

Measure also, unless it is already known, the distance * of C below the top of the jar.

* It is well to have this distance, which is somewhat troublesome to measure accurately, given by the teacher. Partitions of different depths might be used in order to vary the angles of incidence and refraction.

If the jar used in this Exercise is not pretty level at the top, or if the partition is not just at the middle of the jar, it is well, after making one setting of the index and one measurement of its position, to turn the jar about, transferring the index to the other side, and make a new setting and a new measurement. The *mean* of the two measurements thus made should be nearly free from any error caused by irregularity of the jar or of the partition's position.

Now make a drawing, of full natural size, of the sides of the jar (inner lines), the water surface and the partition, as in Fig. 92, continuing the partition line, by means of dots, well down into the jar. Put p in its proper place, and then draw the lines pC and Cg.

Lay off $Cd = Cg$, and then draw the lines n and m.

The index of refraction from air to water is $\frac{n}{m}$.

129. Index Different for Different Colors.—In Exercise 20 the observer may have noticed a tint of blue or of red at the edge of the image of the pin. The fact is that light of various colors comes from the pin, and that the rays are not all refracted alike, the blue being refracted more than the red. The index of refraction is therefore different for light of different colors, but for our present purpose we need not dwell upon that fact. We get a sort of *average* index by the method of Exercises 20 and 21.

130. Relation between Index of Refraction and Velocity of Light.—The velocity of light in any transparent substance depends on the nature of the substance. It is greatest in a so-called vacuum. It is least in the most highly refractive substances, and, indeed, the index of refraction for any given substance depends upon the rate at which light travels through it.

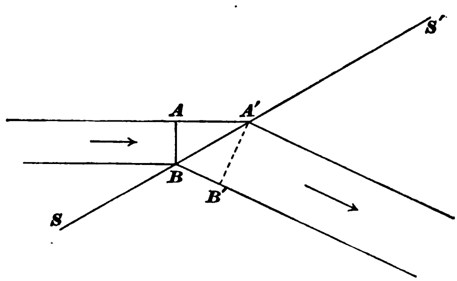

Fig. 93.

This is sometimes illustrated by an analogy suggested by

the march of troops over ground of various kinds. Suppose a column of troops to be marching over smooth ground, represented by the space to the left of the line SS' in Fig. 93. The front of the column being at AB, let the line SS' represent the border of a marsh or other difficult ground. Upon entering, the right of the column, B, first encounters the marsh, and the soldiers at B will fall behind those of the rest of the front. In consequence of this the column will, one part after another, wheel to the right until, when the whole front has entered the marsh, it will have the new direction shown by the line $A'B'$. Substitute for the column of troops a beam of light, and for the marsh a highly refractive transparent substance, and one may get some notion as to how refraction depends upon the retarding effect of refractive substances upon light-rays.

131. Total Internal Reflection: Critical Angle. — In Fig. 94 we have air above the horizontal line and water,

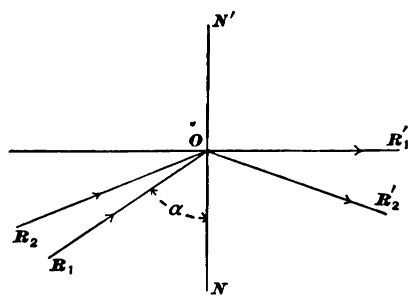

FIG. 94.

glass, or some such transparent medium below the line. A ray of light R_1 may come from beneath to the surface at such an angle with the normal that it will after refraction at O be parallel to the refracting surface. The ray R_2 coming up to O at a larger angle with the normal will not pass out to the air, nor will it skim along the surface. It will

be reflected at the point O, the surface acting as a perfect mirror, and will follow the course R'_2, the angle of reflection being equal to the angle of incidence.

The angle α, which must not be exceeded if the ray is to pass out into the air, is called the Critical Angle.

The reflection which takes place when this angle is exceeded is so good that it bears the especial name *total reflection.*

EXPERIMENTS WITH TOTAL REFLECTION.

(1) With the eye at E, Fig. 95, look at right angles into a glass prism shaped like ABC, at the same time holding an object at O. Note the position of the image O' and its remarkable distinctness.

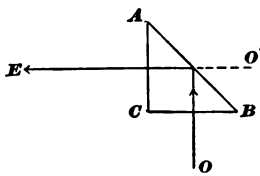

FIG. 95.

(2) In Fig. 96 SS' is a disk or square of thin wood about 10 cm. wide, LO is a piece of knitting-needle about 8 cm. long. The wood floats in water which fills a vessel to the brim AB

Push the needle down until its upper end is nearly level with the upper surface of the board, and look down *obliquely* through the water, close past the margin of the board, at the lower extremity of

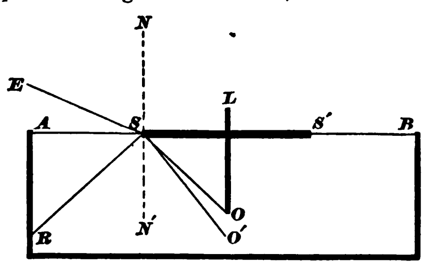

FIG. 96.

the needle. Now draw the needle up, little by little, through the floating board until the point is reached at which the needle just vanishes from view, the line of sight being made at last as nearly horizontal as possible. Lift the board from the water, and note how much of the needle still projects below the board.

When the point is at O', the light-ray going from it to S passes out into the air. When the point is at O, a light-ray OS suffers total reflection along SR. The angle OSN', or its equal SOL, is nearly equal to the critical angle. Of course no great accuracy can be expected here.

Effect of Transparent Plates and Prisms.

132. Transparent Plates.—A plate of glass, or other transparent material, with plane parallel sides, as in Fig. 97, refracts light which enters it obliquely, but refracts it equally and in the opposite direction when it comes out at the opposite side of the glass, so that the entering and emerging rays are parallel to each other, although, as Fig. 97 shows, they do not lie in one straight line.

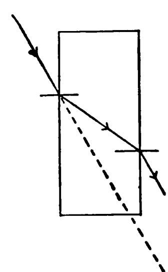

FIG. 97.

Evidently a thick plate of glass will, other things being equal, set the emergent ray farther to one side, from the line of the original ray, than a thin plate will.

133. Prisms.—A *prism*, in the study of light, is usually a piece of glass, or other transparent material, bounded by three rectangular and two triangular faces. *DEF* in Fig. 98 represents one end of such a prism.

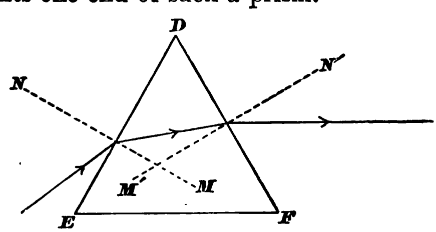

FIG. 98.

It is evident that light entering the face DE from air will be *refracted* toward the normal NM. Going through the prism to the face DF it passes out into the air, being re-

fracted again, this time *from* the normal $M'N'$, so that the two refractions have bent the ray far from its original direction.

The total bending or *deviation* suffered by a ray in passing completely through a prism depends on a number of things.

1st. *On the angle which the two faces passed through make with each other.* This angle is called the *refracting angle;* see D in Fig. 98.

The greater this angle is, other things being equal, the greater the total deflection will be. We have seen in § 132 that if the two faces are *parallel* the total deviation is zero.

2d. *On the color of the ray.*

This fact has already been noticed. Red light is deviated less than blue light.

3d. *On the angle which the ray makes with the first surface.*

The total deviation is least when the ray strikes in such a way as to follow, within the prism, a course parallel to IE, Fig. 99, which makes the distance AI equal the distance AE, and makes the refraction equally great at both surfaces.

EXPERIMENT.

Repeat the experiment of § 89, varying the angle at which the sunlight strikes the first face, in order to show that there is one in-

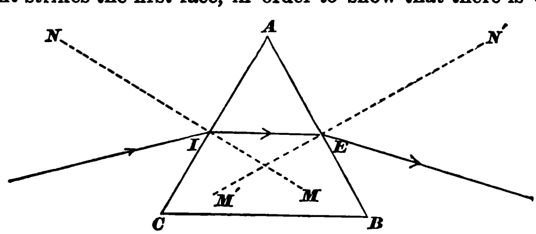

Fig. 99.

clination which gives a less total deflection of the light than any other position.

4th. *On the material of the prism.*

All kinds of glass do not refract equally.

134. Dispersion: The Spectrum.—The separation of rays of different colors by a prism is called *dispersion.*

The spot or band of colored light produced by the dispersion of a sunbeam is called the *solar spectrum.*

It is customary to divide the spectrum into seven regions, called *red*, *orange*, *yellow*, *green*, *blue*, *indigo*, *violet*, and to call the general colors of these the *primary colors*, to distinguish them from those formed by compounding two or more of them. This division of the spectrum is a mere matter of convenience. We might name a hundred colors of the spectrum if we chose to do so.

So long as we keep to any one refracting material the dispersion is, in general, greater when the average deviation of all the rays is greater. Thus with a given prism the dispersion is least when all the rays go through the prism as the ray *IE* goes in Fig. 99.

When prisms of different material are used, two kinds of glass for example, one may disperse the rays more than the other, while producing no greater *average* deviation of all the rays; or one may disperse the rays about as much as the other while deviating them, as a whole, much less.

135. "Achromatic" Prisms.—Two prisms of nearly equal dispersive power but of unequal deviating power may be combined, as in Fig. 100, making a compound prism which produces considerable deviation with very little final dispersion. Such a combination is called *achromatic*, that is, *colorless.*

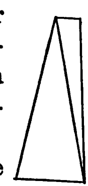

FIG. 100.

Achromatic combinations of *lenses* (§ 149) are used in many optical instruments.

Lenses.

136. Shapes of Lenses.—A *lens* is, usually, a piece of glass whose two faces are parts of spherical surfaces.

Sometimes there is a cylindrical surface between the two spherical faces.

Fig. 101 shows various lenses as they would look if cut through the middle.

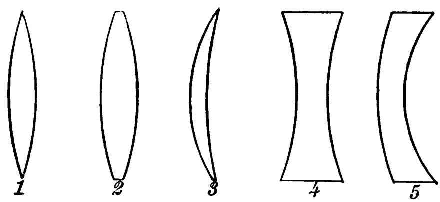

FIG. 101.

Lenses are classed as *convex*, or *converging*, and *concave*, or *diverging*. Convex lenses are all thicker in the middle than at the margin, and cause parallel light-rays to converge, as in Fig. 102. Concave lenses are thinner in the

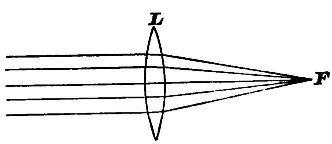

FIG. 102.

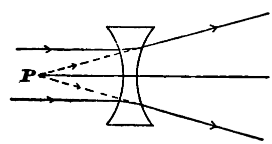

FIG. 103.

middle than at the margin, and cause parallel light-rays to diverge, as in Fig. 103.

Some of the lenses used in the most accurate optical instruments have convex or concave surfaces, which are not strictly parts of spherical surfaces. Such lenses possess certain advantages over spherical-surface lenses (see § 147).

137. Definitions Relating to Lenses.—The lenses we shall use will be much like No. 1 in Fig. 101. The two sides are supposed to be just alike.

To understand such a lens better we will make use of Fig. 104.

C is the centre of the spherical surface of which *ASB* is a part. It is called the *centre of curvature* of the face *ASB*. *C'* is the centre of curvature of the face *ARB*.

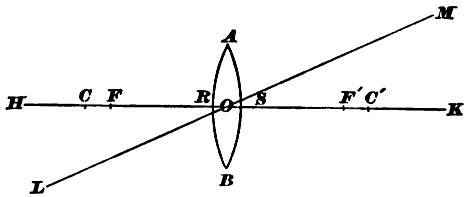

FIG. 104.

The straight line *HCOC'K*, continued to any distance in each direction, is called the *principal axis* of the lens.

Any straight line going, like *LM*, obliquely through the centre of the lens is called a *secondary* axis of the lens.

If the two faces of a lens are exactly alike, as we suppose them to be here, any ray of light going through the centre of the lens, the point *O*, will have the same direction after leaving the lens as before entering it, because the two little spots of surface at which it enters and leaves the lens are parallel to each other, so that the ray is affected just as if it were going through a plate with parallel faces.* *O* is called the *optical centre* of the lens.

Rays entering a convex lens *parallel to its principal axis*, as in Fig. 102, are refracted in such a way that after leaving the lens they will cross this axis. They do not all cross at one point, but if the faces are near together, and are *very small* parts of spherical surfaces, as in our lenses, such rays will cross at or near a certain point, *F*, on the principal

* The direction of the ray *within* the lens is, of course, not quite the same as its direction before entering. This fact is not shown in Fig. 104.

axis, and this point is called the *principal focus* of the lens. There are two principal foci, one on each side of the lens. See points F and F' in Fig. 104.

The distance from the principal focus to the nearer face * of the lens is called the *focal length* of the lens.

Focal length is a quantity of very great importance in dealing with lenses, and the next Exercise will show how to find it by experiment. For this purpose we need to have the light come to the lens in rays nearly parallel to each other and to the principal axis. This we can do by taking the light from any small spot of any distant but distinct object; for instance, a chimney or a church-spire outlined against the sky.

EXERCISE 22.

FOCAL LENGTH OF A CONVERGING LENS.

Apparatus: The lens (No. 31) mounted on a block. A meter-rod (No. 2). A small block (No. 21) bearing a white cardboard screen (No. 32). A common pin.

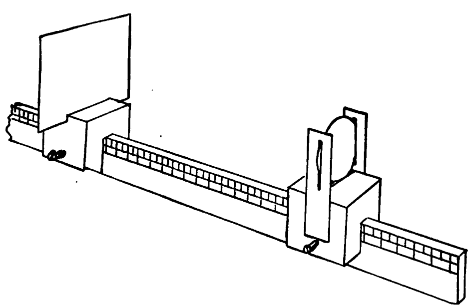

Fig. 105.

First Method.—Place the lens and the screen upon the rod, as in Fig. 105, and point the rod at some distant object, seen against the sky, in such a way that the light from this object will pass from the lens and then fall upon the screen. Move the screen back and forth

* See Appendix I.

until that part of the image * which lies on or near the principal axis of the lens is made as distinct as possible. Then by means of the graduations of the meter-rod, or by an independent measuring-stick if this is preferred, note the distance from this part of the image to the nearer face of the lens. This is the *focal length.*

SECOND METHOD.—Remove the screen from its block and put the pin upright in its place. Let the pin, thus mounted, be placed on the meter-rod, about as far from the end of the rod as the pupil usually holds a book from his eyes when reading. Place the lens somewhat farther from the same end of the rod.

Place the eye at this end of the rod and, looking sharply at the *pin*, direct the rod and adjust the lens in such a way that the light from some distant object will pass through the lens and form an image *in the air* close to the pin. To decide whether the image is nearer the eye than the pin is, move the eye to and fro, to the right and the left, watching the pin and the image.† If the pin is more distant than the image, it will, when the eye is moved toward the right, appear to move across the image toward the right. If the pin is nearer than the image, it will, when the eye is moved toward the right, appear to move across the image toward the left. The rod should not be held in the hands during this test, but should be placed on some steady support.

Continue the adjustments until the test described fails to show which of the two, the pin or the image, is nearer the eye. Then measure the distance from the pin to the lens. It should be the focal length of the lens.

Compare the values of the focal length given by the two methods. The second method is more difficult, but it is instructive, and it

* The image is formed because light coming from any one small spot of the object is brought to a small spot again by the lens. The image is made up of such small spots each in its own place. For the purposes of this Exercise the *distant* object need not be more than 30 or 40 feet from the experimenter. The images on the screen will be much more distinct if the apparatus is used in the back part of the room, well away from the windows.

† To see the reason of the test just described, close one eye and hold the two forefingers, some inches apart, in line with the other eye, so that one finger hides the other. Then move the eye to the right and left, and notice the apparent movement of the fingers with respect to each other.

can be used in cases where the image is too faint to show clearly upon the screen.

138. Discussion of Exercise 22.—It is common to speak of the rays coming to a lens from a distant object as *parallel* rays. This does not mean that rays coming from different parts of the object to the lens are parallel to each other. It means merely that rays coming from any one spot of the object to the lens are parallel, or very nearly parallel, to each other. In fact, if rays from the *different parts* of a luminous body could be converged to the same point, the result would not be an *image* repeating the features of the original objects. It would be a mere point, or very small patch of light.

The image seen in the *Second Method* is, like that of the *First Method*, a real image (§ 109), but it is *in the air.*

As there is an image in the air, we may well inquire why this image cannot be seen by a whole class at once without the use of a screen. It is because the light forming the image in the air goes straight on *through* this image, and can be received only by placing one's self behind the image. The light which forms an image upon a *screen* is by the threads of the screen reflected back in all directions, and therefore some part of it reaches every eye.

QUESTION.

If a bright point were placed at the principal focus of a lens, what direction would the rays going from this point to the lens have after passing through the lens?

139. Object-distance and Image-distance: Conjugate Foci.—*Two points so placed with respect to a lens that an object placed at either of them will have an image at the other are called Conjugate Foci of the lens.*

EXERCISE 23.*

RELATION OF IMAGE-DISTANCE TO OBJECT-DISTANCE: CONJUGATE FOCI OF A LENS.

Apparatus: The same lens that was used in Exercise 22. A meter-rod. Block (No. 9). Small block (No. 21), with a cardboard screen (No. 32). Small kerosene lamp with an asbestos band around the chimney (No. 33).

Arrange the apparatus according to Fig. 106. The hole in the as-

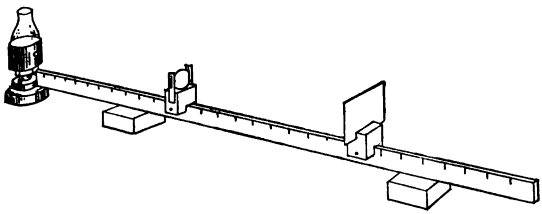

FIG. 106.

bestos band, lighted up by the flame behind, is the *object* the image of which is to be received upon the screen. One end of the meter-rod is placed vertically beneath this illuminated hole.

Place the screen at first at a distance from the object about equal to three times the focal length of the lens. Then move the lens back and forth on the rod between the object and the screen, and see whether in any position it gives upon the screen a clear image of the object. If it does, measure the distance from the lens in this position to the object, and write this distance as the first number in a record-column headed D_0 (object-distance). Measure also the distance from the lens to the *screen*, and put this distance as the first number in a record-column headed D_1 (image-distance).

If, with the present position of the screen and object, there is *no* position of the lens that will cause a distinct image of the object to fall upon the screen, move the screen one or two centimeters farther from the object, and then try again to get a good image. If still none is found, move the screen still farther away, continuing the trial till a distinct image is obtained. Then measure and record the

* To economize space upon the laboratory-tables it will probably be necessary to have pupils work in pairs in this Exercise. Each pair should know the focal length of its lens at the outset, so as to lose no time in beginning the Exercise.

D_o and D_i as already described. (Very little time need be spent upon these first successive trials.)

Then at one move place the screen about 10 cm. farther still from the object, find a position of the lens that will give a distinct image, measure and record D_o and D_i as before. Without moving the screen, see whether there is any other position of the lens that will give a distinct image; if there is, measure and record the D_o and the D_i for this position of the lens.

Move the screen 10 cm. farther away, and then do exactly as before.

If there is time, move the screen two or three more times, adjusting the lens, measuring, and recording each time. It is better to make a moderate number of settings and readings well than a large number carelessly, but an error of one or two millimeters in these readings will be of no great consequence.

140. Discussion of Exercise 23.—The distance from object to image in any case of Exercise 23 is $D_o + D_i$, and we may call this D_{oi}. This distance was shortest in the first case recorded. Let each member of the class divide the D_{oi} of this case by the focal length of his lens. Is there any general agreement between the quotients thus found?

When the screen was farther away, was there usually more than one position of the lens that would give a distinct image, the screen remaining unmoved?

If you were told that in a given case the D_o was 20 cm. and the D_i 60 cm., could you tell what the other possible D_o and D_i would be for the same positions of object and screen? Look at your record-columns for Exercise 23, and see whether they help you to answer this question.

Let each member of the class call F the focal length of the lens which he used, and let him test the truth of the formula,

$$\frac{1}{F} = \frac{1}{D_o} + \frac{1}{D_i},$$

or, what means the same,

$$D_o \times D = F(D + D_i),$$

for all cases tried and recorded by himself in Exercise 23.

PROBLEMS.

(1) D_o for a certain case is 50 cm. and D_i is 100 cm. How great is F?

(2) If D_i is 80 cm. and F is 20 cm., how great is D_o?

(3) If $D_o = D_i$, we will call each D.

(*a*) What in this case is the relation between F and D?

(*b*) How does this agree with your observations in Exercise 23?

141. Real Image Formed by a Lens.—In the preceding Exercises the object presented to the lens has been small, or has been at such a distance as to give a rather small image. It is now desirable to study larger images, and to study them with especial reference to their *shape* and *size*, rather than their distance from the lens. We shall in the next Exercise find the shape and size of an image of an arrow placed at right angles with the principal axis of the lens and not far from the lens. We shall not attempt to find the whole image at once, but shall find separately the images of several points of the arrow, and then make an approximate image of the arrow by connecting these points.

EXERCISE 24.

SHAPE AND SIZE OF A REAL IMAGE FORMED BY A LENS.

Apparatus: The lens (No. 31). Measuring-stick (No. 3). Block (No. 21) carrying in the narrow slot on its top a piece of wire (No. 34) extending first horizontally and then downward (see Fig. 108). A ruler (No. 24). Block (No. 25). A sheet of paper about 30 cm. wide and 1 m. long, having near one end an arrow 8 cm. long, drawn at right angles with a pencil-mark about 30 cm. long, and marked, or numbered, as shown by Fig. 107. Weights (No. 19) to hold the corners of this sheet in place on the table.

Arrange the apparatus as shown by Fig. 108, the centre of the lens over a point on the long pencil-mark, at a distance from the centre of

the arrow about equal to one and a half times the focal length of the lens, and block No. 25 in such a position that the vertical mark upon its face points straight down to point No. 3 of the arrow. This vertical mark will now cross the principal axis of the lens, if the lens is *accurately* placed.

Place the other block near the other end of the paper in such position that the vertical part of the wire it carries shall be near the

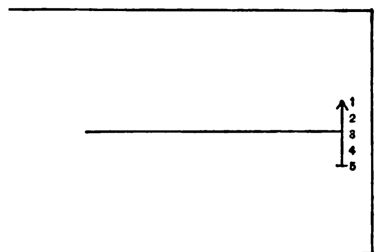

FIG. 107.

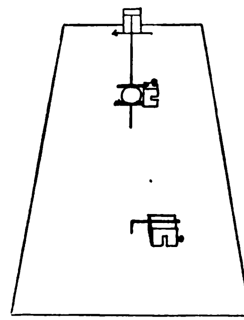

FIG. 108.

principal axis of the lens. Keep the eye 20 or 30 cm. distant from this part of the wire, on a level with the centre of the lens and in line with the centre of the lens and the vertical part of the wire. Look at this part of the wire so as to see it *distinctly*, and note whether you can see at the same time, near the wire, the image of the pencil-mark on the farther block. If so, find out by moving the eye to the right or left, as in Exercise 23, whether this image is more or less distant from the eye than the vertical wire is. Then move the block carrying the wire into such a position that the image and the wire seem to keep close together when the eye is moved a considerable distance to the right or left. When this adjustment is made, put a dot on the paper just beneath the vertical wire and mark this dot 3. It represents the image of object-point No. 3.

Find in a similar manner the image-points 1, 2, 4, 5, corresponding to the object-points 1, 2, 4, 5. The experimenter must take care not to let any idea he may have as to the position where an image-point *ought* to be affect his judgment in deciding where it is.

After all the five image-points are found, connect them, No. 1 to No. 2, No. 2 to No. 3, etc., by means of straight lines, thus getting a rough representation of the whole image.

Draw from each object-point toward the corresponding image-point a straight line as long as the ruler (No. 23), and note the point where these lines cross each other.

142. Formation of the Image in Exercise 24.—The formation of the image-points in Exercise 24 is illustrated by Fig. 109. One ray from the object-point A follows a secondary axis (§ 137) passing through the centre of the lens, and its direction after leaving the lens is the same as before entering it. (Its direction *inside* the lens is not quite the same, but the figure does not show this.)

Another ray from A runs parallel to the principal axis (§ 137) before entering the lens, and will therefore pass

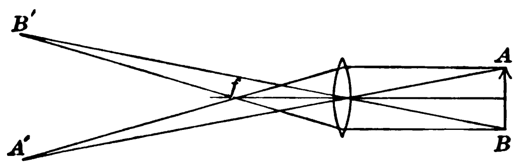

FIG. 109.

through the principal focus, f, on the farther side of the lens. The crossing of these two rays at A' shows the position of the image of A.

In a similar way B', the image of B, is located.

143. Size and Shape of Image.—If a straight line is drawn from A' to B' in Fig. 109, we may call this the length of the image, although the images of points between A and B will not lie on this line. It is evident from Fig. 109, and also from the figure obtained in Exercise 24, that the distance $A'B'$ is to the distance AB as the distance of $A'B'$ from the lens is to the distance of AB from the lens.

The curved shape of the image obtained in Exercise 24, if the work has been correctly done, is due to the fact that the ends of the object-arrow are farther from the lens than the centre of the arrow, and to the further fact

that the focal length along a secondary axis is less than the focal length along the principal axis. This latter fact can easily be shown by direct experiment with either method of Exercise 22.

144. Virtual Image Formed by a Lens.—We see in Exercise 24 and in Fig. 109, where the object-point is farther from a lens than its principal focus is, that the rays going from this object-point to the lens are bent by the lens in such a way that, after leaving it, they converge to a point again. We know, too, that if the object-point were placed at the principal focus the rays going from it to the lens would emerge from the lens parallel to each other.

It is not difficult to see that, if the object-point were placed *between* the lens and its principal focus, the rays going from it to the lens would be divergent still, after leaving the lens, though less divergent than before entering it. In the next Exercise we shall have a case of this kind.

EXERCISE 25.

VIRTUAL IMAGE FORMED BY A LENS.

Apparatus: The same as for the preceding Exercise, except that the sheet of paper need not be more than one half as long, and that the arrow upon it should be 4 cm. long and about 20 cm. distant from one end.

Place the lens between the arrow and the nearer end of the sheet of paper, at a distance from the arrow equal to about two-thirds of its focal length, and in such a position that its principal axis extends over the middle point of the arrow. Place the small block (No. 25) with vertical pencil-mark pointing straight down at the middle point, No. 3, of the arrow. Turn the vertical part of the wire on the other block so that it will point up instead of down, and place this block some distance behind the other one.

Holding the eye 20 or 30 cm. from the lens, look *through* the lens at the image of the vertical pencil-mark, and at the same time *over* the lens at the vertical part of the wire. Bring the wire into line with the image, and then by the usual test find which of them is the more distant. Move the wire back and forth until it coincides in

position with the image. Then mark with a figure 3 the point just under the vertica lpart of the wire. This represents the image of object-point No. 3.

In a similar manner locate the images of points 1, 2, 4, and 5.

Connect the image-points by straight lines, from 1 to 2, from 2 to 3, etc., thus forming an image of the arrow.

Draw a straight line from each image-point to its corresponding object-point, and note where these lines will cross each other if continued.

145. Formation of the Image in Exercise 25.—The images observed in Exercise 25 were *virtual* images. They could not be shown upon a screen, and were not formed by the actual crossing of light rays. Fig. 110 will serve to illustrate the way in which virtual image-points are formed.

Let AB be the object, placed between the lens LL' and the principal focus F'. To find the position of the virtual image of the point A, draw AI parallel to the principal axis of the lens. This ray will, after leaving the lens, pass toward F, the principal focus * on the farther side, and so will appear to have come along the path MF.

Draw another ray, AC, passing through the *centre* of the lens This ray will, after leaving the lens, have the same direction as before entering it, and will be represented by the line CN. If, then, we carry back the line CN till it crosses the line MF, also carried backward, the point A', where the crossing occurs, is a point from which both of the rays appear to come. A' is, then, the virtual image of A.

By a similar process B' is found to be the virtual image of B.

P', the image of the point P, is here represented as lying in the straight line between A' and B'. It is usually so

* The dotted lines drawn from M and N to F in Fig. 110 are not intended to show the actual course of the rays within the eye.

represented in books. Exercise 25 shows that it does not lie there.

The image $A'B'$ is evidently larger than the object AB. Whenever a virtual image is forced by a convex lens, this image appears, to an eye placed in any ordinary position on the other side of the lens, larger than the real object would

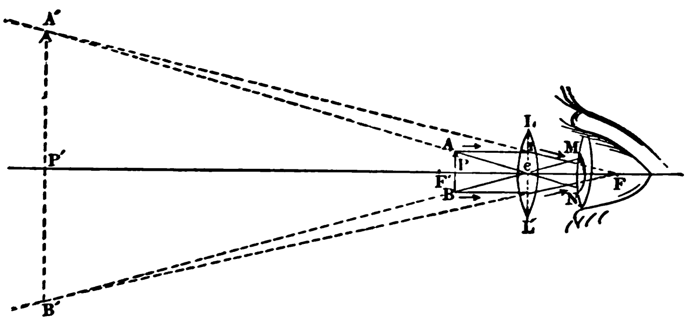

FIG. 110.

look if held at a comfortable seeing-distance from the eye. Hence the name *magnifying-glass*, so commonly given to a lens used as in Fig. 110.

146. Application of Formula.—The formula used in § 140 to express the relation betweeen focal length, object-distance, and image-distance in the case of real images, can be adapted to use with virtual images by merely changing the sign of one term, so as to make

$$\frac{1}{F}=\frac{1}{D_0}-\frac{1}{D}.$$

To illustrate the use of this formula it will be well to measure the distance from lens to object-point 3, and from lens to image-point 3, in the diagram made in Exercise 25, and try them in the formula, with the known value of F.

147. Spherical Aberration in Lenses.—*All* the rays going from a point O to a lens A do not, after passing through the lens, converge to a single point I. Those which go through the lens near its margin converge to a nearer point I'. This imperfection of a lens is called *spherical aberration.*

When a very clear-cut image is needed, it is customary to put a *stop* in front of the lens; that is, a thin metal

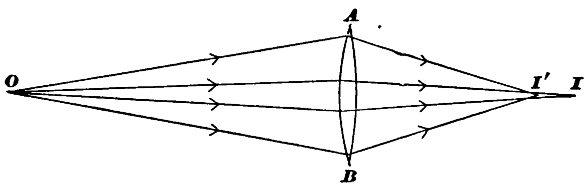

FIG. 111.

plate with a hole which permits only those rays to pass which are near the principal axis of the lens.

Lenses can be so constructed, with surfaces not quite spherical, as to do away with this defect in great part, for light of any one color, but such lenses are difficult to make and are uncommon except in large telescopes.

148. Chromatic Aberration in Lenses.—Ordinary lenses, made of a single piece of glass, give rise to colored fringes or borders about the images which they produce. The cause for this defect, which is called *chromatic aberration,* is this, that the objects looked at send more than one kind of light to the lens and that rays of different colors are not refracted equally by the lens, and so do not come to a focus equally near the lens.

But little trouble from this source is experienced in the use of lenses of slight convexity, whose images are not to be further magnified; as, for instance, in spectacles and ordinary magnifying-glasses. "Stopping out" the greater

portion of the surface of a lens with a circular diaphragm, which allows light to pass only through a small portion of the lens near its centre, improves its performance greatly. How much help such diaphragms give by reducing spherical and chromatic aberration, may be learned by taking out some or all of the diaphragms of an ordinary cheap spy-glass, and then looking with it at distant objects in bright sunlight.

149. Achromatic Lenses.—Fortunately for the manufacturers and users of optical instruments, it is possible to make an achromatic lens, or one, at any rate, which is practically achromatic. This is usually accomplished by uniting into one lens two separate lenses,* one, A, of flint-glass, and the other, B, of crown-glass, as shown in Fig. 112. A convex lens made in this way has, on the whole, a converging effect on parallel rays, while at the same time the superior *dispersive* power (§ 134) of the flint-glass enables the lens A, though of less *refractive* power than the lens B, just to counteract the dispersive tendency of the latter. Many of the lenses used in optical instruments of the best quality are achromatic. Eye-pieces (§ 164), however, of the ordinary pattern do not require achromatic lenses.

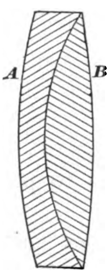

FIG. 112.

A large lens practically free from spherical and chromatic aberration is a marvel of skillful and patient work. Glass suitable for making a large lens of the best quality is very difficult to procure, as a very slight flaw or unevenness of quality may spoil a large block. The shaping and polishing and testing of the largest lenses, after the proper kind of glass is obtained, is a work of years, and men who are

* Sometimes more than two pieces are employed in making an achromatic lens.

skillful and patient enough to do it become known throughout the world.

For many years the largest and best lenses for great telescopes have been made by Alvan Clark and his two sons of Cambridge, Massachusetts; but now all of these famous men are dead. The largest lenses ever made, 40 inches in width, were placed in the great telescope of the Observatory of Chicago University by the last of the Clarks a few weeks before his death in 1897.

QUESTIONS AND PROBLEMS.

(1) An object is placed at a great distance from a converging lens and on its principal axis.

(*a*) What changes of position will the image of this object undergo while the object is moved along the principal axis up to the surface of the lens?

(*b*) In what part of this operation will the image be erect and in what part inverted?

(*c*) In what part will it be real and in what part virtual?

(2) In Exercise 25 the virtual image of a straight line was found to be a curve. How should a line be curved with respect to the lens in order to make its virtual image a straight line?

(3) The focal length of a certain convex lens is 15 cm.

(*a*) How far from the lens will the image be if the object is 30 cm. from the lens?

(*b*) How far if the object is 10 cm. from the lens?

(4) An object is 40 cm. from a convex lens and the image equally far from the lens. What is the focal length of the lens?

(5) If the object mentioned in problem 3 is 5 cm. long, how long will each of the images there mentioned be? (In answering this question disregard the curvature of the images.)

(6) A bright point, which is more distant from a converging lens than its principal focus is, sends white light to the lens. Which falls nearer the lens, the red image of the point or the blue image? Why?

(7) What would be the answer to the questions in (6) if the point were between the lens and its principal focus?

CHAPTER XI.

THE EYE: SIGHT AND COLOR.

150. Parts of the Eye.—The eye as an optical instrument consists of a liquid lens *A* (Fig. 113) called the *aqueous humor*, a solid lens, *B*, called the *crystalline lens*, a transparent jelly-like mass *C*, called the *vitreous humor*, and a screen *rr*, called the *retina*, upon which the image of the object looked at falls.

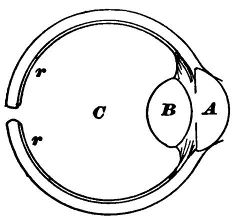

FIG. 113.

The aperture at the back of the eye is occupied by the optic nerve leading from the retina to the brain.

151. Accommodation.—Muscles attached to the lens *B* have power to change its form to some extent, thus adapting the eye to see distinctly near or distant objects at will. This is called the power of *accommodation*.

A *normal* eye, that is, an eye approved by physicians, has such shape as to give upon the retina distinct images of very distant objects without effort. In accommodating itself to see nearer objects such an eye has to make an effort, which grows greater as the distance lessens, but does not become painful until the object looked at is less than eight or ten inches from the eye.

152. Far-sight and Near-sight.—Some eyes lack the power of accommodation for near objects, and are called *far-sighted*, or *long-sighted*, although they cannot see distant objects any better than normal eyes can.

Some eyes are slightly egg-shaped, the retina being farther back than in normal eyes. These eyes are called *near-sighted*, or *short-sighted*, because they are well adapted for seeing near objects, while they cannot see distant objects distinctly.

For some purposes near-sighted eyes have a certain advantage over normal eyes, for they enable their possessor to hold an object very near, when there is need, and so make it look larger than it would look to the normal eye.

153. Eye-glasses.—Far-sighted eyes must wear convex lenses to help them converge the rays from a near object to an image upon the retina. Near-sighted eyes must wear concave lenses to prevent the rays sent by a distant object from coming to an image in front of the retina.

The Perception of Color.

154. The Color-sense.—In the retina are found the ends of the nerves through which we get the *sensation* of light and of color.

Although the eye can distinguish scores of different tints, it is believed that the sets of nerves operating in the perception of colors are very few, probably not more than three or four. Each set of nerves is supposed to give one peculiar color sensation and only one; but the combination of these few primary color sensations in various proportions is supposed to give all the other color sensations.

It is very generally believed that the primary color sensations are three—*red*, *green*, and *violet*.

155. Mixing Color Impressions.—The most convenient way to find the effect of mixing color sensations is to place variously colored pieces of paper on some body which can be made to spin rapidly before the observer's eyes. Tops or other whirling apparatus No. XXXV, for example, can

be used for this purpose, and indeed the whole outfit for this kind of experimentation is now readily obtained.

EXPERIMENT.

Place a red paper, a green paper, and a violet paper upon a whirling apparatus, and so vary the proportions of the visible parts of these papers that when rapidly whirled before the eye they will produce the effect of gray. (In the study of *color* all shades of gray, from brilliant white to dead black, must be classed together as white, the difference between them being merely a difference of brightness.)

156. Complementary Colors.—It has already been shown that ordinary white *light* is composed of many different colors, ranging from red to violet, but it is not necessary to put together all of these colors in order to get the *sensation* of white. There are many pairs of colors, any one pair of which will give the sensation of white when its elements are mixed in the right proportions. The two colors making such a pair are called *complementary* to each other. Thus, according to Rood,

red	is complementary	to green-blue,
orange "	"	" cyan-blue (between blue-green and blue),
yellow "	"	" ultramarine-blue,
greenish-yellow	"	" violet,
green	"	" purple.

EXPERIMENT.

Place blue and yellow disks upon the whirling apparatus, and so proportion the visible parts that when revolving rapidly they will produce the effect of gray.

Try the same experiment with other pairs of complementary colors.

157. Fatigue of the Retina.—If one looks steadily for a short time at some strongly colored object held against a background of gray or white, that spot of the retina upon which the image of the colored object falls loses in part, for

the time being, the power of giving the particular color sensation which it is furnishing, while its power of giving other color sensations may remain as great as ever. This temporary loss of power is called *fatigue* of the retina, and it may give rise to curious effects.

EXPERIMENT.

Hold a piece of bright green paper against a white background, and look very steadily *at one spot* on this paper for thirty seconds. Then look steadily at some one spot of the white surface for a few seconds and note any peculiar color effect that is observed. The color complementary to green will probably appear as a patch upon the white, the shape of this patch being exactly like that of the green paper.

Try the same experiment with other colors.

158. After-images.—The effects observed in the following experiment are still more curious than those of § 157.

EXPERIMENT.

Look steadily for half a minute *at some particular spot* on a window having the sky as a background. Then close the eyes and wait a few seconds for the figure on the window to show out against the darkness. Watch the changes of color the figure undergoes. Observe that details appear in this persisting image which were not noticed while the eyes were open.

"After-images" like the one here mentioned, cannot be the work of memory. They must be due to some change of state in the retina, some real *impression* made there, which lasts for a considerable time but gradually passes away.

CHAPTER XII.

OPTICAL INSTRUMENTS.

159. Importance of Optical Instruments.—Much of the progress of science during the nineteenth century has been due to improvements in the construction of optical instruments and their more general use in scientific investigations.

Improvements in telescopes and the invention and perfection of the spectroscope have enabled the astronomer to discover, and even to measure, objects and motions whose existence was unsuspected by the observers of two generations ago. The chemist is to-day able by means of the spectroscope to ascertain in a few minutes the presence, in a substance of unknown composition, of elements which it would have taken him days to detect by purely chemical means.

To the physician, the food-analyst, the manufacturing druggist, and to those engaged in many other professional or technical occupations, the microscope is a necessary piece of apparatus, a tool of daily, almost hourly, use.

Optical instruments comprise a great variety of combinations of mirrors, lenses, and prisms. Only some of the simpler ones can be referred to in an elementary book on physics.

160. The Photographer's Camera. — This instrument consists essentially of a box, in the front of which is fastened a convex lens or a combination of lenses, *L* (Fig. 114), the distance of which from a ground-glass screen, *P*, at the

other end of the box, may be varied at will. An inverted and usually diminished real image of any outside object not too near L may be formed on P. When this adjustment has been precisely made, the lenses are covered with an opaque cap; a plate of ordinary glass, coated with a film of gelatine made sensitive to light by the presence in it of certain compounds, usually of silver, is substituted for P; the cap is then removed, and the light is allowed to act for a

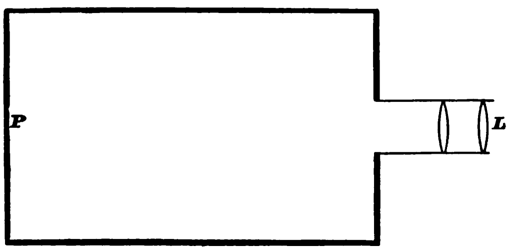

FIG. 114.

sufficient time upon the sensitive plate, after which the cap is replaced and the plate removed and "developed" into a photographic "negative."

Those who are interested in practical photography will find in Exercise 24 some explanation of the difficulty experienced in making all parts of the ground-glass screen show clear images at the same time; and in § 147 there is a suggestion as to the effect of "diaphragms" with larger or smaller holes.

161. The Magic-lantern.—This instrument, known also by various other names, *stereopticon*, for instance, requires a powerful source of light, such as a large kerosene-flame, or some form of calcium-light A (Fig. 115), in which a cylinder of quicklime is heated by a flame formed by burning together oxygen and coal-gas, or, best of all, the electric arc-light. By means of a large lens B (Fig. 115), called

the *condenser*, a powerful beam of light from this source is thrown upon the painted or photographed "slide," the

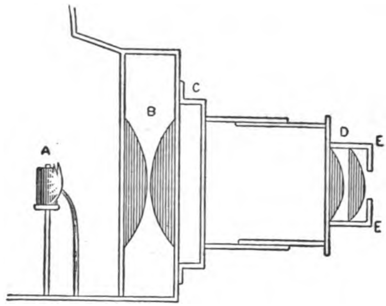

FIG. 115.

image of which is to be exhibited. This slide is pushed into the opening *C*, a little outside the focus of a smaller convex lens or a pair of such lenses, *D*, and a greatly enlarged real image of the slide is thrown upon the screen.

The throwing of large images upon a screen is called *projection* of these images and apparatus used for this purpose is called *projecting apparatus.*

162. Projecting a Spectrum.—A kind of spectrum has been shown in the experiment of § 89, but a better dispersion of the colors can be obtained by means of some device like that described in the following experiment. If sunlight is not available, the stereopticon, if provided with a calcium light or an electric arc-light, can be successfully used, the prism being placed in the path of the rays after they have traversed the projecting lens.

EXPERIMENT.

By means of a porte-lumière (No. XXX) throw a beam of sunlight through a narrow slit at *S*, Fig. 116 Place a lens, *L*, in the path of the beam, and adjust it so as to throw a distinct image of the slit on a screen at *I*. Now introduce a pr.sm, *P* (No. XXXII), in the

position shown in the figure, and then place the screen at RR', making the distance PR equal to PI. The prism used may be of flint-glass, or, better, may be hollow and filled with the highly dispersive liquid bisulphide of carbon.

Examine the spot of colored light on the screen. (1) How many colors can be distinctly seen? (2) Do they blend, or are they sharply

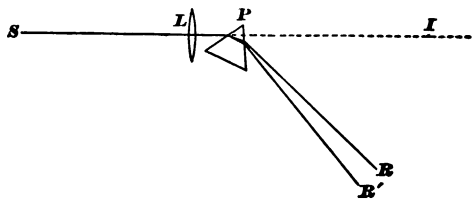

FIG. 116.

separated from each other? (3) Which color is most refracted? least refracted? Try the effect of passing the emergent pencil through a second prism similar to the first, and placed so as to refract the light in the same direction as the first.

Try the effect with a second prism so placed as to refract in the opposite direction from the first.

163. The Simple Microscope.—In its least complicated form the simple microscope, or magnifying-glass, consists of a convex lens used, as explained in § 145, to form an upright magnified image of any small object. When much magnifying-power is required, two or even three convex lenses, mounted one over the other with their surfaces only a few millimeters apart, are often used. Such combinations are called *doublets* or *triplets*, according to the number of lenses composing them. They have certain advantages over single lenses of equal magnifying-power.

The discussion in § 145 will help the student to see that the magnifying-power of a simple microscope is greater as its focal length is less.

164. The Compound Microscope.—For viewing objects under any but the lowest magnifying-powers, that is, in all

cases where the apparent diameter of the image is to be anywhere from 50 to 5000 times the actual diameter of the object, the *compound microscope* is employed. The essential optical parts of this instrument, as usually constructed, are (see Fig. 117), an *eye-piece*, *LL'*, here represented as

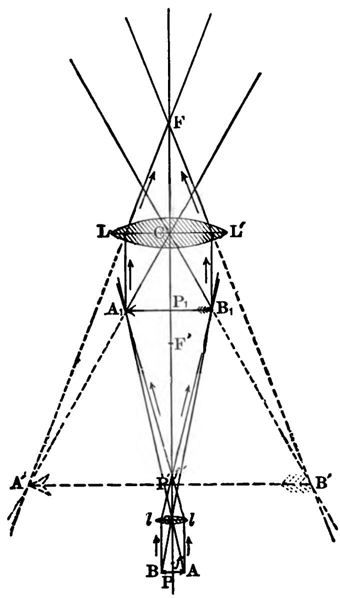

Fig. 117.

single, but generally consisting of two convex lenses, and an *objective*, *ll*, frequently consisting of from two to six pieces. These lenses are fixed in a brass tube so arranged that the distance between the eye-piece and the objective can be varied at will, within certain limits. A mirror, not here shown, which is adjustable to any desired angle, is usually employed for throwing light upon the object.

The object to be viewed is placed on a platform beneath

the objective, and is strongly illuminated by light reflected from the mirror. A real, inverted, magnified image, A_1B_1, of the object is formed within the tube of the instrument at a position somewhat nearer to the eye-piece than its principal focus. This real image is therefore magnified by the eye-piece, which forms an enlarged virtual image, $A'B'$, of it at a position not far from the object.

The foci of the object-glass are at f and f', those of the eye-piece at F' and F.

The total *magnifying-power* of the instrument is that of the objective multiplied by that of the eye-piece. In general, the shorter the focal length (see Appendix I) of a microscope objective, the greater its magnifying-power.

An objective of one inch focal length will, on a tube 10 inches long, give, with the lowest power eye-piece in common use (the "A" eye-piece), a magnification of about 50 diameters; with an eye-piece of double the magnifying-power ("B" eye-piece) the total magnification will be about 100 diameters, and so on.

EXPERIMENT.

Fasten a page of fine print, *P* in Fig. 118, upright on a table in a good light. Set up in front of it a short-focus convex lens, *L*, at a distance from the page somewhat greater than the focal length.

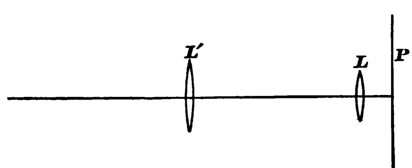

FIG. 118.

Hold another short-focus convex lens, *L'*, in various positions farther from the page until one position is found in which an eye close to *L'* sees through it an inverted, magnified image of the print, this being a *virtual* image of the *real* image formed by the lens *L*. This apparatus is a rude model of the compound microscope.

165. The Astronomical Refracting Telescope.—This instrument consists essentially of the long-focus *object-glass*, or *objective*, *L* (Fig. 119), mounted in one end of a tube, at

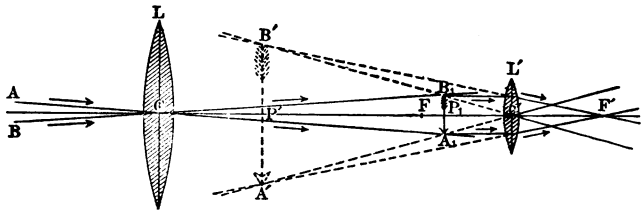

Fig. 119.

the other end of which is placed an *eye-piece*, *L′*, precisely similar to that of the compound microscope. The eye-piece can be moved toward or away from the object-glass in order to make the image appear most distinct.

The real image of any distant object is, of course, always formed by the objective very near its principal focus. The foci of the eye-piece are at *F* and *F′*.

Astronomical telescopes are always furnished with achromatic object-glasses (§ 149).

EXPERIMENT.

Mount upon blocks two convex lenses, one of 30 or 40 cm. focal length, the other of about 5 cm. focal length. Set them up on the table with their principal axes coincident—that is, with their centres on the same straight line at right angles to the centres of their faces. Mount a bit of tracing-paper or greased writing-paper, and place this screen in such a position between the lenses that the one of greatest focal length shall throw upon it a distinct image of some distant bright object. Look at this image on the translucent paper through the 5-cm. lens. Choose such a position and distance as to give a clear virtual image, as much magnified as possible, of the real image on the screen. Now remove the screen, and observe that the virtual image of the real image is still visible.

166. Efficiency of the Telescope.—The usefulness of the telescope as an aid to vision depends upon the following

points: (*a*) the clearness and sharpness of the image, or what is called the *definition* of the instrument; (*b*) the brilliancy of the image; (*c*) the amount of allowable magnification.

Good definition depends upon the accuracy with which the lens is shaped and finished, and upon the quality of the glass, which should be free from flaws.

Brightness depends upon the amount of light which can be concentrated in the different parts of the image. Hence a large objective will, other things being equal, give the best illumination. In some recent telescopes the objective has a diameter of 3 feet or more.

The magnification, with a given eye-piece, is evidently very nearly proportional to the focal length of the objective; but unless the objective is large, and furnishes much light, it is useless to give it great focal length, for the reason that the much-magnified image would be too faint to be seen to advantage.

QUESTIONS AND PROBLEMS.

(1) How could you find the weight of a body that will float, if you had no balance but had a vessel filled with water and a "graduated" glass flask—that is, a flask with marks upon it showing the number of cu. cm. required to fill it to certain depths?

(2) If a liter of hydrogen weighs .0896 gm. and if the sp. gr. of oxygen as compared with hydrogen is 16, what is the weight of 1 cu. m. of oxygen?

(3) A certain volume of mercury of density 13.6 weighs 216 gm., and the same volume of another liquid weighs 14.8 gm. Find the density of the second liquid.

(4) A piece of iron weighs 200 lbs. in air and 172.5 lbs. in water. How great is its sp. gr.?

(5) A given body weighs 500 gm. in air and 400 gm. in water.

(*a*) How great is its volume?

(*b*) How great is its sp. gr.?

(6) A board $12 \times 6 \times 1$ in. weighs 1.5 lbs. What is its density in lbs. per cu. ft.?

(7) A cubical block of wood 15 cm. along the edge weighs 1125 gm. What is its density?

(8) A 30 cu. cm. body weighs 10 gm. in water. How great is its sp. gr.?

(9) What is the volume of a body which weighs 25 gm. in air and 20 gm. in water?

(10) A body weighs 180 lbs. in water and 120 lbs. in a liquid that is 1.8 times as dense as water. Find the volume and the sp. gr. of the body?

(11) How much will a kgm. weight of sp. gr. 7 weigh in a liquid which is 0.8 as dense as water?

(12) A cubical box, 3 ft. square on a side, made of 2 in. plank of sp. gr. 0.5, has a bottom but no top. It contains a body weighing 100 lbs. To what depth will this box sink, upright, in water?

Ans. 6.7 in. nearly.

(13) The sp. gr. of air, as compared with water, is about .00129 at 0° C. under ordinary atmospheric pressure. How many grams would equal the buoyant force exerted by air in this condition upon a cu. m. of any substance?

(14) If the sp. gr. of a certain block is 0.3 and its volume 100 cu. cm., how much of it would be submerged if it were floating in a liquid of sp. gr. 2.

(15) A rod floats one-half submerged in a liquid of sp. gr. 0.9. How much of it would be submerged in a liquid of sp. gr. 3?

(16) There is a uniform rod 6 ft. long and 4 in. square, of sp. gr. 0.5. What must be the sp. gr. of a cubical piece of metal 4 in. on the edge which, when attached to the rod, would just hold it submerged in water?

(17) If a diver with his suit weighs 200 lbs. and it takes $\frac{1}{40}$ of a cu. ft. of lead, sp. gr. 11.4, to keep him submerged in fresh water, how many cu. ft. of water does he, in his suit, displace?

(18) Two boys are pulling at a rope in opposite directions, each with a force of 25 lbs.

(*a*) How great is the tension on the rope?

(*b*) How great would you call the tension if the rope were tied to a beam and supported a weight of 25 lbs.?

(19) A uniform beam, 12 ft. long and weighing 300 lbs., rests, horizontal, on a fulcrum 2 ft. from one end. How much weight must be applied at this end to make the beam balance in its present position?

(20) (*a*) Find the direction, position, and magnitude of the equil-

ıbrant (§ 74) of two forces, parallel and in the same direction, one of which is 10 lbs. and the other 12 lbs., their lines of action being 8 ft. apart.

(*b*) Find the direction, position, and magnitude of the resultant (§ 75) of the same two forces.

(21) One end of a horizontal beam 20 ft. long and weighing 50 lbs. rests upon a wall, and the other end is supported by a rope that will bear only 85 lbs. A boy weighing 100 lbs. walks slowly along the beam from the wall toward the rope. How far from the rope will the boy be when it breaks?

(22) A hammer is used to draw out a nail from a board. The head of the hammer rests against the board at a distance of 3 in. from the nail. A force of 50 lbs. is applied at right angles with the handle at a point 12 inches from the board. How great is the force exerted by the hammer on the nail? (This case is similar in principle to some of those discussed in connection with the pulley. See Experiments under § 58.)

(23) If a force of 50 lbs. is applied at the end of the handle of a "jack-screw" 18 in. from the centre of the screw, and if one revolution of this screw lifts a weight 0.5 in., how great is this weight, if there is no friction?

(24) A sled weighing with its load 50 lbs. rests on the side of a hill rising 1 ft. in a distance of 5 ft. along the incline.

(*a*) How great a force acting parallel to the incline is needed to keep the sled from sliding downward if there is no friction?

(*b*) If the crust on the snow is just strong enough to bear the sled under these conditions, how much would the load on the sled have to be lightened in order that a similar crust might bear the sled on a level?

(25) An inclined plane rising at an angle of 45° has a load of 50 lbs. resting upon it. How large a horizontal force will be needed to keep this load moving up the incline if there is no friction?

(26) A horizontal force of 10 lbs. is required to keep a certain body moving along a horizontal surface with which its coefficient of friction is 0.2. How great is the weight of the body?

(27) A mass of 100 lbs. rests upon an inclined plane 10 ft. long and 4 ft. high.

(*a*) How great must be the resistance of friction to keep the body from sliding down the incline?

(*b*) How great must the coefficient of friction be?

(28) If a simple pendulum 1 m. long vibrates 58 times a minute

what is the length of a simple pendulum that vibrates 116 times in a second?

(29) The length of a simple pendulum vibrating once a second in the latitude of New York is about 39.1 in. How many seconds a day would a clock lose if controlled by a simple pendulum 40 in. long?

(30) Two lights, A and B, are placed 20 ft. apart. The power of A is to that of B as 4 to 9. At what point between them must a screen be placed in order to be equally lighted up on both sides?

(31) The distance of the planet Neptune from the sun being 2,800,000,000 miles, nearly, how long does it take a wave of light to go from the sun to Neptune?

(32) What is the height of a tree which casts a shadow 100 ft. long, when an upright rod 5 ft. tall casts a shadow 7 ft. long?

(33) The image of an upright stake 8 ft. tall, and 10 ft. from a window-shutter appears on a screen 4 ft. beyond the shutter. The aperture in the shutter through which the light passes from the stake to the screen is very small. How great is the length of the image?

(34) The clock on a wall indicates 9.30. What time will it appear to indicate if the observer sees the reflection of the clock in a mirror on the opposite wall but does not distinguish the numerals?

(35) A plane mirror lies upon a table and a pencil 6 in. tall stands upright on one edge of the mirror. How wide must the mirror be in order that a person whose eyes are 5 in. above its surface and 20 in. distant from the pencil may just see the whole length of the pencil reflected in the mirror? (To be solved by drawing and measuring. The thickness of the *glass* is to be neglected.)

(36) Prove that if an object is placed in front of a plane mirror and the mirror is moved either toward or from the object, without turning, the image will move twice as far as the mirror.

(37) Prove that if a candle is placed in front of a vertical plane mirror and the mirror is turned 45° about a vertical axis, the image of the candle will move through an arc of 90° around the axis of the mirror.

(38) Two plane mirrors, A and B, are placed 12 cm. apart, facing each other and parallel. A small object is placed between them 4 cm. distant from A. Calculate the distance from A to the first and second images seen in it. Do the same for B.

(39) Two plane mirrors, placed vertical, make with each other an angle of 60°. A candle is placed between them, but nearer one than the other. Draw a figure showing the positions of the various images of the candle.

(40) If the radius of curvature of a concave spherical mirror is 50 cm., and if a candle is placed 40 cm. distant from the mirror,

(*a*) How far from the mirror will the image of the candle be?

(*b*) Will this image be real or virtual?

(*c*) Will it be erect or inverted?

(*d*) If the candle-flame is 2 cm. long, what will be the length of its image?

(41) If the candle mentioned in the preceding problem was 10 cm. from the mirror, what would be the answers to the questions there stated?

(42) What would be the answers in problems 40 and 41 if the mirror were convex?

(43) Have you ever seen curved mirrors used except in a class-room or laboratory? If so, for what purposes were they used?

(44) Define the term *index of refraction.*

(45) The index of refraction of the earth's atmosphere is little greater than 1 with respect to the space outside this atmosphere. Does this fact delay, or does it hasten, the first glimpse of the rising sun?

(46) For which of the colors here named is the index of refraction of glass the greatest—red, green, yellow, blue? For which of them is it least?

(47) How could you find by experiment the color complementary to any given tint?

(48) Show that the image formed by a convex lens may be either larger or smaller than the object.

(49) Prove algebraically, and also graphically (after the manner of § 142), that when the distance of an object from a convex lens is twice the focal length, the image is at the same distance on the other side.

(50) A rod 5 cm. long held in front of a convex lens, at right angles with the principal axis, has an image 25 cm. long upon a screen distant 100 cm. from the lens. How great is the focal length of the lens?

(51) An object 4 cm. long, placed 20 cm. from a certain lens and at right angles with the principal axis, has a real image 10 cm. distant from the lens. If the same object were placed 5 cm. distant from the same lens,

(*a*) Would the image be real or virtual?

(*b*) How far from the lens would the image be?

(*c*) How great would the length of the image be?

APPENDIX I.

FOCAL LENGTH, ETC., OF LENSES AND COMBINATIONS OF LENSES.

It is customary to define the focal length, F, of a single lens as the distance from the focus to the nearest point of the surface of the lens, and in the formula $\frac{1}{F}=\frac{1}{D_0}+\frac{1}{D_1}$ to consider D_0 and D_1 as measured from the object and image, respectively, to the nearest point of the lens. With this interpretation of the letters, the formula is not exactly fulfilled by any actual lens. It holds strictly true only for the ideal case of a lens of zero thickness, but it is sufficiently near the truth for common purposes in the case of ordinary lenses. The formula is about equally accurate, for a double convex lens, at least, when all the distances, F, D_0, and D_1, are measured to the *optical centre* of the lens (§ 137).

When a combination of lenses is used, as in a microscope-objective or a photographic camera, a formula similar to that just given can be applied, but the F, D_0, and D_1 occurring in it are not now measured either to the nearest point of the combination or to the optical centre. They are measured to certain other points determined by the radii of curvature, thickness, and refractive index of each lens, and the distance between the two lenses. In the ordinary use of such a combination, its magnifying power is substantially equivalent to that of a single ideal thin lens having a focal length equal to what is called the focal length of the combination. The calculation of the focal length of the combination is frequently very laborious.

Dealers in photographic objectives very frequently state as the focal length of a combination of lenses the distance from the principal focus to the nearer surface of the nearest lens. They sometimes call this the "back focal length," or, rather, the "back focus" of the com-

bination. It is a convenient quantity to use in the description of a lens, but it is not intended for use in the formula $\frac{1}{F} = \frac{1}{D_o} + \frac{1}{D_i}$.

The term "equivalent focal length," or "equivalent focus," is sometimes applied, in the case of a combination of two equal lenses, to the distance from the principal focus to a point midway between the two lenses.

APPENDIX II.

INDICES OF REFRACTION OF VARIOUS SUBSTANCES AS COMPARED WITH A VACUUM. (See § 128.)

Agate	1.540	Selenium (crystals)	2.98
Canada balsam	1.53		
Diamond	2.5	Alcohol	1.36
Fluor spar	1.434	Petroleum (heavy)	1.45
Glass (ordinary crown)	1.53	" (light)	1.4+
" (" flint)	1.61*	Water	1.333
Ice	1.31		
Quartz	1.544	Nitrogen	1.000298
Rock salt	1.544	Oxygen	1.000271

* The *dispersive power* (§ 134) of flint glass is nearly twice as great as that of crown glass.

APPENDIX III.

ALL the articles in the first list here given should be furnished to each member of the laboratory section.

LIST OF ARTICLES REFERRED TO BY NUMBER IN THE "EXERCISES" OF THIS BOOK.

No. 1. A 10-cm. section of a meter-rod.

No. 2. A meter-rod, marked on one side in feet and inches.

No. 3. A 30-cm. bevel-edged measuring-stick, marked on one side in inches.

No. 4. A waterproofed wooden cylinder about 8 cm. long and 4.5 cm. in diameter, loaded internally with shot so that it will float nearly submerged in water.

No. 5. A brass can about 14 cm. tall and 7 cm. in diameter, having a slightly declining, straight, overflow-tube, about 6 cm. long and 0.8 cm. in internal diameter, extending from a point about 1.5 cm., clear, below the top of the can (see Fig. 6). To prevent *dribbling* the junction of tube and can should be covered, internally, with a coat of paraffin melted on.

No. 6. A brass catch-bucket with a wire handle, capable of holding about 175 gm. of water, and weighing not more than 50 gm.

No. 7. An 8-oz. spring-balance graduated to 0.5 oz. (There is now in the market an improved balance, graduated on one side in 10-gm.

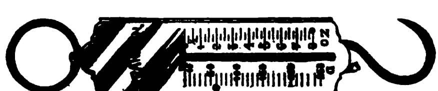

FIG. 120.

intervals and on the other side in 0.25-oz. intervals. It is, moreover, especially adapted for use in the horizontal position. This improved balance is desirable for this course.)

No. 8. A rectangular waterproofed block of wood, about 7 cm. long and 4.5 cm. square on the end, so loaded internally with shot that it will sink in water, but not enough to make it weigh more than 225 gm.

No. 9. A rectangular waterproofed cherry block about 7.5 cm. × 7.5 cm. × 3.8 cm. This block should be smooth, and therefore the waterproofing should be done by soaking it in *very hot* paraffin. For the best results this soaking should be done in a vacuum. Excess of paraffin should be scraped off before the block is used.

No. 10. A one-gallon glass jar of *good* quality. (It is poor economy to buy a poor jar and have it break with a liquid in it.)

No. 11. A lump of roll sulphur weighing about 175 or 200 gm. It is not worth while to cast these lumps into regular cylindrical form.

No. 12. A lead sinker with wire handle, weighing about 175 gm.

No. 13. A waterproofed wooden cylinder about 1 cm. in diameter and 20 cm. long. Doweling-rod, furnished by hardware dealers, serves well when waterproofed.

No. 14. A holder for keeping No. 13 upright in water. It consists of a waterproofed wooden rod about 12 cm. long and 1.3 cm. square on the end, provided with a clasp for attaching it to the side of a jar, and with two screw-eyes projecting from one side, the rings of which are large enough to let the cylinder No. 13 slip easily through them, but not large enough to allow the cylinder to tip far from the vertical position (see Fig. 10).

No. 15. A cylindrical glass jar, about 14 cm. tall and 10 cm. in diameter, with level top.

No. 16. A broad-mouthed bottle with ground-glass stopper, standing not much more than 11 cm. tall with stopper, and weighing, when filled with water, about 175 or 200 gm.

No. 17. A lever and supporting-bar. The lever is a 30-cm. section from a meter-rod, pivoted upon the *smoothed* cylindrical body of a brass screw which is driven horizontally into the end of a bar of hard wood about 25 cm. long, 5 cm. wide, and 3 cm. thick. A brass plate projecting from this bar and overhanging the middle of the lever prevents the lever from tipping far, while it allows sufficient freedom of motion. The lever itself, except for a distance of 2 cm. each side of the middle, is cut away so that its top is level with the upper part of the hole through the centre. There should be a

screw-hole running downward through the middle of the supporting-bar, to facilitate in attaching it, as shown in Fig. 21.

No. 18 (*A* and *B*). Two brass scale-pans about 6.5 cm. square, each with its suspending threads weighing accurately 1 oz. (that is, not differing from this weight by more than .01 oz.). Each pan is suspended by four strong linen threads meeting in a knot about 20 cm. above the pan, two of them continuing in a loop about 4 cm. long above this knot. (Fig. 21.)

No. 19. A set of iron weights, 8 oz., 4 oz., 2 oz., and two 1 oz., making a total of 16 oz. No weight should be in error more than .01 oz.

No. 20. A flat pine board about 50 cm. long and 15 cm. wide for use in the Exercises on Friction.

No. 21. A cubical block of wood about 3.7 cm. on each edge. A groove about 1 cm. wide and 2 cm. deep extends through the lower part of the block with the grain of the wood. An ordinary short screw extends through one side of the block into this groove, and serves to fix the block in position upon a meter-rod. *Across* the grain at the top of the block is a slot about 0.1 cm. wide and 0.5 cm. deep. (Figs. 26 and 105.)

No. 22. Two bits of wood, each about 8 cm. long and 1 cm. square on the end, for supporting the spring-balance in a horizontal position. (Fig. 41.)

No. 23. A plate-glass mirror about 15 cm. long, 3.8 cm. wide, and 0.2 cm. thick, the coating on the back protected by paint or varnish.

No. 24 (*A* and *B*). Two straight-edged rulers of some wood that will keep its shape well—white pine, for instance—each about 30 cm. long, 5 cm. wide, and 1 cm. thick.

No. 25. A block like No. 21, but without the large slot and the screw. One side of this block is coated with white paper, and a vertical pencil-mark or ink-mark is made across the middle of this paper. (Fig. 108.)

No. 26. A Walter Smith "school square," or other equally good protractor.

No. 27. A cylindrical mirror of nickel-plated brass, about 5 cm. tall and 8 cm. wide, cut from seamless tubing 4 inches in diameter and $\frac{1}{8}$ inch thick, mounted upon a semicircular base-board of wood of the proper radius of curvature. The base-board should be about 1.5 cm. thick.

No. 28. A piece of plate-glass about 7 cm. square and 0.6 cm. thick,

for Exercise 20 on Index of Refraction. Two opposite edges or narrow sides of the glass should be ground tolerably plane and polished sufficiently to allow seeing readily through the whole width of the plate (see Fig. 89).

No. 29. A brass partition made to fit the small glass jar (No. 15) and to extend downward into the jar a distance equal to about one-third the diameter of the jar. It should be made of sheet brass

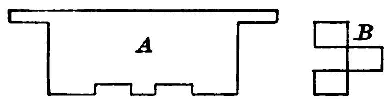

FIG. 121.

about .07 cm. thick. The method of shaping and adjusting the partition is suggested by Fig. 121, where *A* shows a side view and *B* an end view of the partition. The flanges shown in *B* are bent more or less in adjusting the partition to fit the jar closely, but without too much pressure.

No. 30. An index of thin sheet brass made to clasp the side of the jar (No. 15). This index is a strip about 15 cm. long, before bending, and 1 cm. wide, tapered to a point at one end. To enable it to clasp the jar, about 3 cm. at the untapered end is bent over. (See *pb* in Fig. 91.)

No. 31. A circular (not elliptical) double-convex spectacle-lens, having a focal length not less than 12 cm. and not more than 16 cm. The lens is mounted on a block similar to No. 21. (See Fig. 105.)

No. 32. A white cardboard screen about 8 cm. square, of such thickness as to be held firmly in the narrow slot of the small block No. 21. (Fig. 105.)

No. 33. A small kerosene lamp of such size and shape as to fit it for the use shown in Fig. 106. The lower partof the chimney is surrounded by a thin sheet of asbestos paper, having a hole 3 or 4 mm. in diameter at the height of the flame.

No. 34. A wire, of the right size to fit into the narrow slot of No. 21, bent at a right angle, one arm about 6 cm. long, the other about 4 cm. (Fig. 108.)

ARTICLES USED BY THE TEACHER, BUT NOT TO BE FURNISHED TO STUDENTS.

(*Most of them are referred to by number in the "Experiments" of this book.*)

No. I. A gauge for testing pressure at various points and in various directions in a jar of water. In Fig. 122, P is a pillar of wood or metal about 25 cm. tall; C is a small glass thistle-tube about 1.7 cm. wide; m is a thin rubber membrane fastened water-tight across the mouth of C; p and p are hard-rubber pulleys about 1.7 cm. in diam-

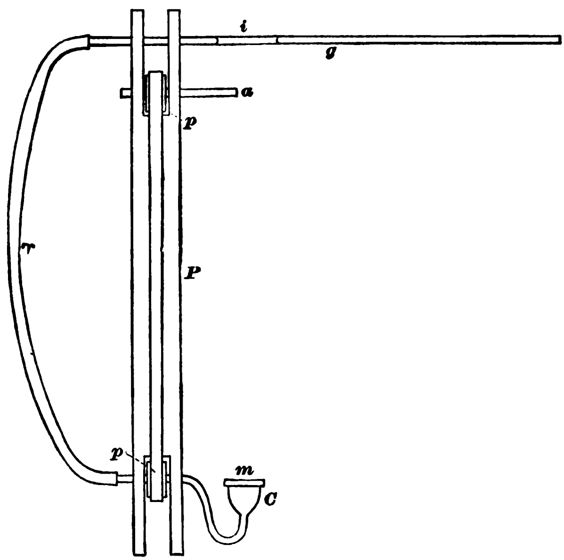

FIG. 122.

eter, fitting closely on their axes; r is a small rubber tube; g is a glass tube; i is a short column of water serving as an index. A band of strip-rubber, such as toy stores supply, connects the two pulleys p and p, so that by turning a, the axis of the upper pulley, between the thumb and finger, the gauge-face m may be turned upward, downward, or sidewise, without changing level. A student-lamp chimney, with stopper for one end, accompanies this gauge.

No. II. Apparatus for bursting a bottle by an attempt to compress water within it.

The essentials are a glass bottle, with a perforated rubber stopper which fits the bottle well when driven in its full length; a strong frame for holding the bottle and keeping the stopper in place; a rod, with convenient handle, to be driven water-tight down through the hole in the stopper.

No. III. Glass tube about 1 m. long, closed at one end, connected by a strong rubber tube 25 cm. long with another glass tube 20 cm. long. (See Fig. 12.)

No. IV. Strong *thistle-tube* (Fig. 13) about 2.5 cm. wide, covered at the mouth with strong sheet rubber and furnished with a thick-walled rubber tube about 20 cm. long.

No. V. Small air-pump suitable for both exhaustion and compression.

Such a pump is frequently sold without base, but it is well to have a base, bell-jar plate, and one or two bell-jars. For many purposes a larger pump is desirable.

No. VI. Bent glass tube for Boyle's law, the whole tube about 1.5 m. long (Fig. 14).

No. VII. Common large rubber foot-ball, with a rubber tube about 30 cm. long attached to the key. (Fig. 15.)

No. VIII. Small bottle provided with rubber stopper fitted with two glass tubes as in Fig. 16.

No. IX. Glass model of lifting-pump (Fig. 17).

No. X. Glass model of force-pump (Fig. 18).

No. XI. Hydrometer for liquids less dense than water.

No. XII. Hydrometer for liquids more dense than water.

No. XIII. Glass U tube (Fig. 20) about 60 cm. long before bending.

No. XIV. Some form of the Cartesian Diver.

No. XV. Eight-inch and four-inch wooden disks combined in one piece for use as a pulley. This piece is fitted with various pins (removable) for suspending weights. It is mounted much like the lever of No. 17. (See Fig. 34).

No. XVI. Centre-of-gravity board, with suspension and plummet. (Fig. 24.)

No. XVII. Platform balance weighing from 1 kgm. to 0.1 gm., provided with a set of brass weights.

No. XVIII. Well-made small brass pulley with a hook or loop. (Fig. 37.)

No. XIX. Well-made small double brass pulley with hook or loop. (Fig. 38.)

No. XX. An inclined plane,* shown about one-fourth natural size in Fig. 123. The roller should be of brass, accurately turned. It weighs with its frame just 16 oz. The graduations of the scale may be in millimeters. The apparatus should be made with care.

No. XXI. Pendulum-support and pendulum-balls (Figs. 56 and 57).

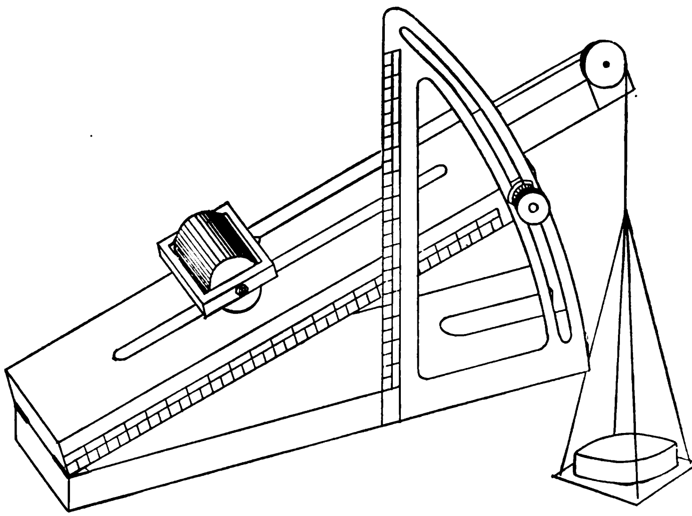

FIG. 123.

No. XXII. Three small packages of dyestuffs soluble in water, various colors.

No. XXIII. Three glass plates, red, green, and blue, about 10 cm. square.

No. XXIV. Camera obscura consisting of two pasteboard tubes each about 25 cm. long. The larger, about 5 cm. in diameter, is closed at one end save at the centre, where there is a hole about 0.1 cm. in diameter in a thin partition. The smaller tube, about 4 cm. in diameter, is closed at one end by thin tracing-paper. (See § 94.)

* A number of excellent features in this apparatus are due to Mr. Sweet, formerly of the Rindge Manual-Training School in Cambridge.

No. XXV. Make according to the following directions: On a board about 35 cm. square (Fig. 73) lay off a circle 30 cm. in diameter. Bore 12 holes, 1, 2, 3, etc., dividing the circumference in 30° parts. From the centre draw radii, making the angle α of 90°, β of 60°, and γ of 30°. Provide pegs, about 15 cm. tall, to fit in all the holes.

Mount two strips of thin "silvered" glass, each about 20 cm. long and 10 cm. wide, on two boards hinged together in such a way that the angle between them may be varied from 30° or less to 90° or more, the longer edges of the mirrors being horizontal.

No. XXVI. An inexpensive kaleidoscope.

No. XXVII. A concave spherical mirror 12 or 15 cm. in diameter.

No. XXVIII. A "granite-ware" basin 15 cm. or more in diameter.

No. XXIX. Thin waterproofed board, pierced by knitting-needle for experiment on the critical angle. (Fig. 96.)

No. XXX. A porte-lumière.

No. XXXI. Right-angled glass prism about 5 cm. long, for showing "total reflection." (Fig. 95.)

No. XXXII. A pair of equilateral prisms, for experiment on projecting the solar spectrum. (§ 162.)

No. XXXIII. Set of about half a dozen lenses of various shapes 4 or 5 cm. in diameter. (Fig. 101.)

No. XXXIV. Set of about half a dozen convex lenses varying from 2 cm. to 50 cm. in focal length, the largest 6 or 8 cm. in diameter.

No. XXXV. Rotating apparatus* suitable for carrying Maxwell's color-disks, etc.

No. XXXVI. Set of color-disks, *e.g.*, those made by Milton Bradley.

MISCELLANEOUS ARTICLES.

Two pounds of clean mercury.

Two pounds of assorted soft glass tubing, from 2 mm. to 8 mm. inside diameter.

Six feet of rubber tubing, about 5 mm. inside, that will not collapse when connected with the air-pump.

An ounce or two of very small rubber tubing.

Piece of thin sheet rubber about 6 in. square, for use with the gauge. (No. I.)

Set of cork-borers.

Three-cornered file for cutting glass tubing.

* The well-known little tops with color-disks serve very well if larger forms of XXXV and XXXVI are not available.

Screw-driver.
Pair of wire-cutting pliers.
One-half pound of naked copper-wire about 1 mm. in diameter.

LABORATORY TABLES.

The laboratory tables used in the Cambridge grammar-schools are well suited to the work of this course. They are about 10 ft. long, 4 ft. wide, and 2 ft. 10 in. tall. They have white-pine tops about $1\frac{1}{2}$ in. thick, and heavy white-wood legs. Extending from end to end over each table are two horizontal bars, about 2 in. by 3 in., adjustable at various heights (which should range from $1\frac{1}{2}$ ft. to $3\frac{1}{2}$ ft. by 3-in. intervals) above the table-top, their ends, which are cut in tenons, sliding in grooves in the supporting posts. These posts are fastened to the frame of the table and rise through slots in the table-top, being *flush* with the ends of this top and about 10 in. distant from the sides. Pins of iron or wood placed in holes in these posts support the ends of the horizontal bars. To adapt these tables to Exercises 29 and 30 (Second Part of book), holes about $1\frac{1}{2}$ in. in diameter should be bored through the top.

For Exercises in heat (see Second Part) this table should have a gas-pipe running along the middle of the top from end to end, with three stop-cocks leading to the right and three leading to the left. This pipe should be readily detachable, as its presence would be inconvenient in many experiments.

Each table is intended to accommodate six independent experimenters.

SECOND PART.

CHAPTER XIII.

PROPERTIES OF SOLID BODIES.

167. Introductory.—Very many bodies, such as pieces of wood, stone, and metal, have a shape of their own which they will keep under all ordinary conditions. These are called *solid* bodies, or, for brevity, *solids.*

Plainly distinguished from this class is another, made up of bodies such as air, water, oil, etc., which adapt their shape to the vessel they may happen to be contained in, showing no tendency to keep any form which can be called their own. Bodies of this second class are called *fluids,* a name which includes *gases,* as air, and *liquids,* as water.

The same *substance* may be at times solid and at times fluid. We are all familiar with at least one substance which occurs commonly in each of the three states here named—the solid state, ice; the liquid state, water; the gaseous state, steam.

The student already knows considerable about liquids and gases, and farther on he will learn more about them; but just now we are to study some properties of solid bodies, the properties which distinguish them from fluids. In doing this we shall get an idea of the kind of knowl-

edge of materials which engineers and other builders must have in putting together framed structures, like houses, bridges, and boats, upon the strength and stability of which we are dependent every day of our lives.

The properties which we shall study are *tenacity*, that is, resistance to breaking apart under a straight pull, and *elasticity* of various kinds.

Tenacity.

168. Illustration and Definition. — When an engineer wants to know how large a rod of a given material he must use to suspend a given load, he first inquires how heavy a load could be borne by a rod of the same material and quality 1 sq. in. or 1 sq. cm. in cross-section. We will suppose that he finds the load required to break a rod 1 sq. cm. in cross-section to be 4,000 kilograms. Then he sees that, if the load to be sustained is 20,000 kgm., he must use a rod at least 5 sq. cm. in cross-section. In fact, he would, to make perfectly sure, use a rod two or three times as large as that.

It is evident that experiments made on the force required to break one rod would be of no use in regard to another rod, unless the sizes, or at least the relative sizes, of the two rods were known. Accordingly, people who make it their business to get the information needed by engineers *measure the force, F, required to break a rod of given material, and also the area of cross-section, S, of the rod, and then record the ratio $F \div S$ as the tenacity, or "breaking-strength,"* of this material. The experiments may be made on fine wires, instead of rods, but even in such cases the tenacity found may be expressed as so many kilograms per square centimeter of cross-section. In the following exercise the pupil will find F and S for a piece of spring-brass wire and calculate the tenacity $F \div S$;

that is, the number of kilograms it would take to break a bar of brass 1 sq. cm. in cross-section if it had the same quality as the fine wire actually used.*

EXERCISE 26.

BREAKING-STRENGTH OF A WIRE.

Apparatus: Several pieces of spring-brass wire, No. 27 of Brown and Sharpe gauge, each about 1 m. long. Articles 50, 51, 52, 53, 54 from the list at the back of the book.

The wooden cylinder No. 53 is fastened upright on the top of the table (see Fig. 124), and a tack is driven half-way into the table or into the cylinder near the base.

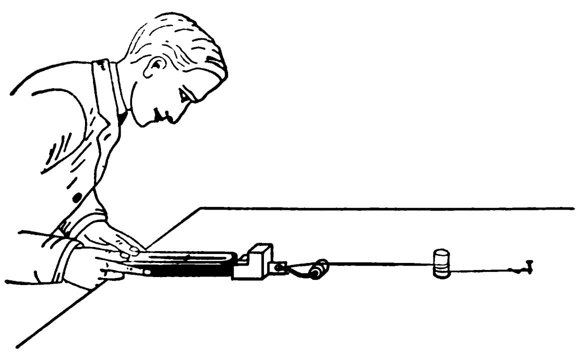

FIG. 124.

Take one of the pieces of wire and fasten one end securely to the tack and the other end to the bar of the balance, where the hook is attached. The sharp bends made in the wire at the points of fastening weaken it, and care must be taken to relieve these points in part from the pull that is to be applied to the wire. Therefore wind the wire at least three times around the upright cylinder and an equal

* In fact, the quality of a rod is not the same as the quality of a fine wire. The wire is much stronger in proportion to its size than the rod. The working over which the material gets while being fashioned into wire seems to knit the particles more firmly together. It certainly brings them more closely together, for the specific gravity of a given material in the form of wire is usually greater than it is in larger forms.

number of times around the other wooden cylinder, which is slipped on the hook of the balance, taking care to avoid sharp kinks and making the turns upon each cylinder lie close together without overlapping.

Now, holding one foot well back, pull steadily with the spring-balance, gradually increasing the force, looking *vertically* down upon the index all the time (see Fig. 124), and taking care to avoid friction within the balance. Note the position of the index when the wire breaks, which it will do very suddenly at the last.*

Repeat the experiment a number of times, using each time a new piece of wire.

Take the average of the readings † made at the several breakings as the true strength of the wire, rejecting, however, any trials, if there are such, in which the wire breaks at either fastening.

Measure the diameter of an unbroken piece of the wire in thousandths of a centimeter, by means of the screw-calipers, and calculate the area of its cross-section as a fraction of a square centimeter.

Calculate the number of kilograms of force that would be required to break a rod of brass 1 sq. cm. in cross-section, if it were of the same quality and condition as the brass of the wire.

169. Introductory to Exercise 27.—The preceding Exercise illustrates what is meant by tenacity, or breaking-strength, and how it may be measured. The next Exercise will show, in one or two simple cases, the variation of tenacity with the nature or condition of the metal.

It will serve also to direct attention to the different ways in which wires break under a steady pull, some break-

* If the wooden guard (No. 51) on the *bar* of the balance is not used, care must be taken to avoid injury to the hands from the recoiling hook of the balance when the wire breaks.

† When a spring-balance, graduated for use in the vertical position, is placed unstrained in the horizontal position, the index rests behind the zero-mark. The force required to bring the index out to the zero-mark when the balance is in this position can be found with sufficient accuracy by means of another much more sensitive spring-balance, or by attaching to the hook one end of a string reaching over a pulley and bearing a known weight at the other end. This force is to be added to the actual readings of the balance in order to find the true force exerted by it in the horizontal position. (See Exercise 12.)

ing suddenly, almost without warning, others stretching very considerably before the final rupture comes, thus showing not only strength, but *toughness*, which is a very valuable quality for many purposes. The steel of which armor-plates for ships is made is not only strong, but it is so tough that a big solid shot fired against the armor may bed itself in the steel, as a stone beds itself in moist clay, without seriously cracking the plate.

The distance that a piece of metal will stretch before breaking, divided by the length before stretching, is called its *elongation*. Some specimens of manganese steel (§ 172) are said to have an elongation as great as 50 per cent.

EXERCISE 27.

COMPARISON OF WIRES IN BREAKING TESTS.

Apparatus: Two pieces of spring-brass wire just like that used in the preceding Exercise and two pieces of annealed soft iron wire of the same diameter, and of about the same length, as the brass wire. A candle. Other apparatus as in the preceding Exercise. The lengths of the guards used on the bar of the balance should vary with the strength of the wire used.

BRASS WIRE.—Measure the diameter of a piece of the brass wire. Light the candle and hold one end of the wire in the flame* for a few seconds. Wipe off the coating of soot deposited by the flame, and measure the diameter of the wire again to make sure it has not been made smaller by the flame. Now hold the two ends of the wire across each other, pinching each about 1 cm. from the tip, and try to bend one by means of the other. Which is the stiffer?

Now attach one of the brass wires to the balance and to the upright cylinder, precisely as in the preceding Exercise, and before breaking heat the wire, by means of a match-flame, at a point midway between the balance and the post. Then pull and note the force required to break the wire.

Note whether the wire broke at the point that had been heated. Measure the diameter at the point of breaking and see whether it

* A very hot flame like that of a Bunsen burner will melt the wire.

is the same as before breaking. Test the stiffness of one of the broken ends, comparing it with an end that has just been heated, and see whether the breaking pull has increased or decreased the stiffness of the wire.

If satisfactory experiments have been made with one piece of brass wire, the other need not be used.

IRON WIRE.—Measure the diameter of the iron wire before breaking.

Note the force required to break this wire.

Measure the diameter of the iron wire at one of the broken ends.

QUESTIONS.—Does heating (or "annealing") spring-brass wire increase or decrease its stiffness?

Does it increase or decrease its strength?

Does annealed brass wire stretch perceptibly before breaking?

Does the breaking pull increase or decrease the stiffness of the annealed part of the wire?

Is the particular piece of soft iron wire used stronger or weaker than the annealed brass wire of the same diameter? Is it stronger or weaker than the unannealed spring-brass, as tested in the preceding Exercise?

How great is the *elongation* of the iron; that is, the ratio of the increase of length to the original length? (The increase of length is to be calculated from the decrease of diameter caused by the breaking pull.)

170. Cohesion.—The preceding Exercise illustrates some of the changes which may easily be made in the properties of metals. There are many such changes, and no one can tell just why they occur as they do. In fact, no one can at present *explain* tenacity. Why does it take force to break a wire? We may say, of course, that there is an *attraction* of neighboring particles of metal for each other, which we call *cohesion;* but that, after all, is merely saying that it requires force to pull them, or keep them, apart when they are near each other. If cohesion explains tenacity, what explains cohesion?

No satisfactory answer to this question can now be made, in spite of all the attention that has been given to

the subject, and perhaps none ever will be given, but that should not discourage us from trying to find out all we can about the properties of bodies; partly because the facts we discover may be of great practical use, and partly because so many wonderful, and seemingly unattainable, things have been found out, that it is rash to speak of any fact of nature as hopelessly out of our reach.

171. Order of Tenacity.—Arranged in the order of their tenacity, the following metals must be placed thus: steel, iron (see § 172), copper, zinc, tin, lead. Steel may be twenty times as strong as zinc and fifty times as strong as lead. Silk fibre may be one third as strong, and some kinds of wood, lance-wood for instance, one sixth as strong as steel of the same size.

[§ **172. Iron and Steel.**—Iron commonly contains small quantities of other substances, such as carbon, sulphur, and phosphorus, left in it by the process of manufacture from iron ore, or added to produce certain changes of quality. The first operation in the manufacture leaves considerable carbon, perhaps 2 or 3 per cent, in the iron. The metal in this condition can be melted with comparative ease, and when poured into moulds in the liquid condition readily takes the form of the moulds. This operation is called *casting*, and iron thus treated is called *cast iron*. Many articles of household use—kitchen stoves, pots, and kettles, for instance—are of cast iron.

When cast iron is in the molten condition, much of its carbon can be burned out by exposing it to a stream of air. The decarbonized iron thus obtained is *worked* while hot by pressing, hammering, etc., and thus becomes *wrought* (that is, *worked*) *iron*, which is softer and very much tougher and stronger than cast iron. The iron used by blacksmiths is wrought iron. It can be readily welded, which is not true of cast iron.

Steel, in the ordinary sense of the word, means iron containing less carbon than cast iron and more carbon than wrought iron. The amount of carbon in ordinary steel may vary from a small fraction of 1% to 1.5%.

Sometimes steel is made by slowly burning out a part of the car-

bon from cast iron, occasional tests of the progress of the operation being made by taking out and cooling a small quantity of the molten metal. When the quality of the tested piece is satisfactory the operation is stopped. *Open-hearth* steel is made by this, the Siemens-Martin, process and sometimes contains so little carbon that it is very much like wrought iron.

For making the rails of car-tracks and for other rough purposes great quantities of *Bessemer* steel are used. This is made by burning out the carbon from melted cast iron very rapidly, streams of air being forced through the liquid metal in immense pots called *converters*. No attempt is made to stop the operation at the instant when the metal contains just the right proportion of carbon. It is easier to burn the carbon all out, or nearly so, and then add the desired amount in a certain mixture or compound of iron and carbon called *spiegeleisen*.

Steel containing from 0.5% to 1.5% of carbon is stronger than wrought iron. One specimen of such steel in the form of wire is reported to have had a tenacity of nearly 345,000 lbs. per square inch. Such steel is in its softest state harder than wrought iron; and when it has been heated to redness and cooled suddenly, it becomes very hard indeed, some pieces being hard enough to cut or scratch glass. This process of hardening steel is often called *tempering;* but, strictly, tempering is the partial softening that the steel is subjected to after hardening, the degree of softening being adapted to the purpose for which the metal is to be used.

Other substances are used with carbon to give peculiar properties to steel or iron. *Mushet's steel*, which is exceedingly hard and brittle, contains *tungsten*. Steel containing 10 or 15% of *manganese* is strong and exceedingly tough. A mixture of steel and *nickel*, called *nickel-steel*, is so strong and tough and elastic that it has been used for the armor-plate of heavy war-ships.

Ordinary wrought iron cannot be cast in moulds with success, because of the "blow-holes," caused by gas-bubbles, which form in the cooling metal; but the addition of a small quantity of *aluminium* (more commonly called *aluminum*) to the iron prevents this difficulty. Iron treated in this way is called *mitis* iron. It is used in place of wrought or forged iron in making the magnets of dynamos.

The demand for the very best kinds of iron and steel for the manufacture of armor-plate and large cannon has led to great improve-

ments in the processes and products of manufacture during the last twenty-five years.]

173. Definitions.—We shall often find the scientific use of a word to be somewhat different from its popular meaning, and, unless this fact is recognized and remembered, much confusion of ideas, and much vain discussion, is likely to occur. Illustrations may be found in one or two words to be used in the next few paragraphs. Thus the word *strain* means, in scientific language, a change in the shape or size of a body; while the word *stress* is applied to the force which produces the strain. More strictly still, *strain* means the *ratio* of the change of dimension or size to the original dimension or size; and *stress* means the *ratio* which the whole force applied bears to the area of the cross-section through which it is applied. Thus, if a rod L cm. long is stretched until it is $(L + l)$ cm. long, or compressed until it is only $(L - l)$ cm. long, the strain, as to length, is $l \div L$. And if the area of cross-section of the rod is s sq. cm., and a force of F grams is applied to lengthen or shorten it, the *stress* is $(F \div s)$; that is, the force per unit area of cross-section.

In popular usage *strain* has nearly the same meaning as *stress*. Again, in popular speech, a body is said to be highly *elastic* when it can suffer and recover from *great changes* of size or shape. India-rubber is the type of such elastic substances. But in the scientific sense the elasticity of shape possessed by steel and glass is much greater than that of india-rubber.

174. Elasticity: of Volume, of Figure.—The power which a body has of recovering, more or less perfectly, its original volume, after the force which has changed the volume is withdrawn, is called *elasticity of volume.* This property exists in perfection in liquids and gases, which recover completely from the effect of any compression,

however long it has been continued, when the forces producing it have ceased to be applied.

The power which a body has of recovering, more or less perfectly, its original shape after the force which has changed the shape is withdrawn, is called *elasticity of shape, or figure.* Liquids and gases are usually regarded as entirely lacking in this property, although any body of liquid or gas left entirely to its own forces and at rest would finally assume a sphérical shape (see Chap. XIV). Elasticity of figure, then, is peculiarly a property of solids. It is called also *rigidity.*

So-called inelastic, or *plastic*, solids are those which, like lead, wet clay, or putty, yield readily to a slight distorting force, manifesting little or no ability to resume their original shape when that force ceases to act upon them. Highly elastic solids, like tempered steel, spring-tempered brass, and ivory, offer a constant resistance even to very long-continued distorting forces, and will immediately resume their original shape upon the removal of any such force, provided the strain does not exceed a certain value, called the *limit of elasticity*, beyond which the substance will act like a plastic solid.

EXPERIMENT.

Take some pieces of No. 25 or No. 27 (B. & S. gauge) copper wire and some spring-brass wire of the same diameter, each about 30 cm. long. Wind a piece of each wire closely about any smooth cylinder —for example, a stout glass tube or student-lamp chimney; upon letting go the ends notice whether the wires uncoil at all.

Try the same experiment with the brass wire after heating it almost to redness in a flame.

175. "Modulus" of Elasticity.—If several bars of various sizes and shapes, but made of the same material in the same, state of purity, hardness, etc., are subjected to longitudinal tests that do not exceed the limit of elasticity, it is found that the quantity, *stress* ÷ *strain*, is the same for all of

them. It is called the *modulus* of elasticity* for the given material under the kind of strain used. A different material has a different modulus, that is, a different value for *stress* $\div$ *strain*. For instance, if force is reckoned in grams and area of cross-section in square centimeters, this modulus for steel bars of a certain grade is 2,500,000,000, or 25×10^8, and for copper wire of a certain grade about 1,200,000,000, or 12×10^8.

Tables of such moduli, derived from experiments, are printed in many books used by physicists and engineers, but different grades of the same general substance will differ so much in their tenacity and elasticity that materials which are to be put to any severe and important use—steel for making heavy cannon, for instance—are often subjected to special tests to prove their qualities.

The particular *modulus* which we are concerned with in tests of stretching or shortening is called *Young's modulus*, from the English scholar, Dr. Thomas Young,† who suggested and used it.

We shall define the *stress* as the force, F, in grams, divided by the cross-section of the wire, S, in square centimeters. The *strain* will be the increase of length, l, divided by the original length, L, both in centimeters. Young's modulus is

$$\frac{F}{S} \div \frac{l}{L} = \frac{F \times L}{S \times l}.$$

If we use M to signify this modulus, we have

$$M = \frac{F \times L}{S \times l}.$$

* Latin : a little measure.

† Young was born in 1773 and died in 1829. He was a man of much learning and ability, and one of the founders of the wave-theory of light; though Fresnel, a Frenchman, who took up the matter somewhat later than Young, was more active and efficient in establishing the theory.

The use of this equation may be shown by the following:

Illustration

An engineer wishes to know how much a rod of a certain kind of steel 3 m. long and 5 sq. cm. in cross-section will be stretched by a load of 500 kgm. If he knows the modulus of elasticity of this kind of steel, from his own experiments or from those of any one else, he can without further experiment answer the question by a simple calculation.

Let us suppose the modulus to be (reckoned in grams and square centimeters) 25×10^8. He writes the known quantities in the equation above given, reducing lengths to centimeters and force to grams, and gets

$$25 \times 10^8 = \frac{500000 \times 300}{5 \times l};$$

whence

$$l = \frac{15 \times 10^7}{125 \times 10^8} = .012 \text{ cm.}$$

So, if the numerical values of any four of the letters in the equation containing M are known, the numerical value of the remaining letter can be found at once.

If we write the equation in this way,

$$l = \frac{F \times L}{S \times M},$$

we see that l, the increase of length, is proportional to the total force, F, proportional to the original length, inversely proportional to the area of cross-section, and inversely proportional to the modulus, M.

EXERCISE 28.

ELASTICITY: STRETCHING.

Apparatus: A piece of No. 27 spring-brass wire 4 m. or more long, soldered at one end to a screw driven into a table-top or other convenient fixed object, and having, about 10 cm. from the free end, a piece of paper fastened crosswise, as in Fig. 125, with sealing-wax. (This paper serves as an index, or marker, and is best put on after the wire is attached to the balance.)

FIG. 125.

Nos. 2 and 50, and a short focus lens to be used as a magnifying-glass.

Attach the free end of the wire securely to the hook of the balance. Let one experimenter draw the wire straight by means of the balance, while another measures its length from the screw to the marker on the wire. (This measurement need not be made with extreme care, as an error of a few mm. would make practically no difference in the final result of the Exercise.)

Lay the meter-rod under the marker, parallel to the wire, at such a height that one end of the marker touches the rod, and then, making sure that the rod does not move, note with the magnifying-lens the change in position of the edge of the marker, when the pull on the balance is increased to 1, 2, and 3 kgm. in succession, beyond the first straightening pull, coming back after each increase to this original pull to see whether the marker returns, as it should, to its original position.

The wire being rigidly fastened to the screw at one end, the movement of the marker shows the increase in length of that part of the wire which lies between the screw and the marker.

Record thus :

$p =$ straightening pull $= \ldots\ldots$ kgm.

$F =$ increase of force.	$l =$ increase of length.	$F \div l$
From p to $p + 1$ kgm. $= 1$ kgm.	......	....
" " " " $+ 2$ " $= 2$ "	......	....
" " " " $+ 3$ " $= 3$ "	......	....

Find the mean value of F/l, and then, knowing L, the length, of the stretched part of the wire, and the area of cross-section, S, of the wire (see Exercise 26), calculate *Young's modulus* for this wire by means of the formula

$$Y.\ M. = \frac{F}{S} \div \frac{l}{L} = \frac{F \times L}{S \times l}.$$

The tests thus far described in this Exercise are not intended to exceed the elastic limit of the wire; that is, the wire is not expected to show any permanent increase of length as a result of the stretching. It would be well to continue the experiment by increasing the stretching force, 1 kgm. or less at a time, until it becomes great enough to produce permanent stretching. The ratio of the force at this stage to that required to *break* the wire (see Exercise 26) should then be noted.

After the wire has become permanently stretched it should not be used again.

176. Compression.—The same general laws hold for compression as for stretching, and, moreover, the *modulus* for the compression of any substance is numerically the same as the modulus for its stretching, provided the limit of elasticity is not exceeded. The limit of elasticity in compression is, however, not necessarily the same as the limit of elasticity in stretching. Thus, cast iron will bear a much greater compression strain than extension strain before breaking, although the force required to produce a minute decrease of its length is practically the same as the force required to produce an equal increase.

177. Strains Produced by Transverse Forces.—The preceding considerations lead naturally to Exercise 29, on bending, for it will be found that usually when a body is bent some parts of it are stretched and some parts compressed.

EXPERIMENT.

Take a long thick india-rubber "eraser" and measure carefully the length of the two opposite sides. Then bend the eraser sharply, in such a way that one of the measured sides will be on the inner arc and the other on the outer arc, taking care not to stretch or compress the rubber unnecessarily. Measure the length of the two arcs thus formed and determine for each side whether it is longer or shorter than before bending.

Draw a straight line along the middle of one of the other sides from end to end, and when the rubber is bent exactly as before, measure the length of this line, which will be midway between the concave and convex sides of the bent eraser, and see whether it has increased or decreased in length.

A strip of paper will be found convenient for making the measurements.

Countless instances of flexure or bending produced by some kind of transverse force acting upon more or less

elongated objects will occur to every one. A sagging telegraph-wire, the limb of a pear-tree bending under its load of fruit, the bent springs of a loaded carriage, the sagging rafters of an old building, are only a few familiar examples of bodies strained by transverse forces. Exercise 29 is intended to enable the student to answer for himself the question, what general quantitative relation exists between the amount of transverse force applied and the amount of bending which it produces.

EXERCISE 29.

ELASTICITY: BENDING. EFFECT OF VARYING LOAD.

Apparatus: Articles 55A, 56, 57, 58, 59, 60 (100 to 400 gm.).

Mark lightly two points on the rod 100 cm. apart and mark the point midway between them. Support the rod on two of the wooden prisms placed 100 cm. apart (see Fig. 126), each prism being just

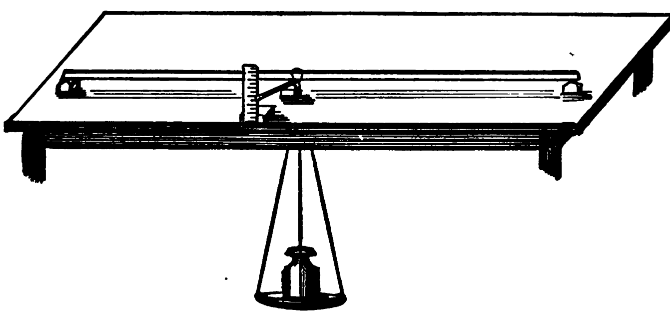

FIG. 126.

under one of the marks on the rod, the central mark on the rod being just over one of the holes in the table-top.

Suspend the scale-pan from the middle of the rod, so that it may hang beneath the table-top, taking care to prevent the suspending string from touching the table.

Near one end of the thin slip of wood (No. 57) make a light cross-mark, and call this mark *1.* Make a similar mark 5 cm. from the

first, and call this second mark *2*. Make a third similar mark 25 cm. from mark 2, and call this last mark *3*. This slip of wood is to be used as an index.

Place the third prism parallel to the rod and opposite the middle, and rest the index on the top ridge of the prism as a fulcrum, the mark 2 being exactly over this ridge and mark 1 being exactly under the nearer edge of the rod. Place the upright scale alongside the index, so that the face of the scale, upon which the divisions are marked, shall be just as distant from the rod as the mark 3 is.

It is now evident that, if the centre of the rod is pushed down any distance, the height of the point at which the index passes the face of the scale will be raised five times this distance.

Everything being carefully adjusted,* read the point on the scale where the upper edge of the index passes it, and record this reading in the third column of a table, made like that below, under the heading "Readings without Load."

Then place 100 gm. in the pan, watching the apparatus to see that nothing is deranged in this operation ; take the reading, and record it in the column headed "Readings with load," placing alongside in the column headed "Load" the number 100. Then carefully remove the weight and make another reading, to be recorded in the column "without load."†

Then put 200 gm. in the pan, read and record as before, and make another reading "without load." Then take 300 gm. and 400 gm. in turn.

In this way the observations are made and recorded in the first three columns of the table. The other columns explain themselves. The last column, which shows how much the rod bends per gram in each case, should be particularly noted.

* The rod is sometimes warped so that it will not rest fairly on the prisms at the ends. In this case one end of one of these prisms should be elevated as much as may be necessary to give a good bearing at both ends of the rod. Thin, flat slips of cardboard serve very well for making this adjustment.

Sometimes a rod has such a bend that its centre is considerably higher than the fulcrum, and the index does not touch the front *edge* of the rod. In such a case the fulcrum should be raised by means of the cardboard slips.

† Slight differences may be expected in the "readings without load," but these differences should not be as great as 0,1 cm.

Load.	Readings with load.	Readings without load.	Mean.	Rise of pointer on scale.	Deflection of rod.	Deflection per gm.
gm.	cm.	cm.	cm.	cm.	cm.	cm.
100	3.40	3.00 } 3.00 }	3.00	0.40 ÷ 5 =	0.080	0.00080
200	3.75	2.96	2.98	0.77 ..	0.154	0.00077
....	....	....	....	....	....	....
....	....	....	....	....	...	....
						Mean ....

From this Exercise the student should be able to make out the general relation between amount of load and amount of *flexure*, or bending; that is, if he is told that a load of 100 gm. bends a rod .01 cm., he should be able to tell how much a load of 375 gm., for instance, will bend it, if the elastic limit is not exceeded.

EXERCISE 30.

ELASTICITY: BENDING. EFFECT OF VARYING DIMENSIONS.

Apparatus: As in Exercise 28, with the addition of No. 55B and the remaining weights of No. 60.

Three conditions are to be tried, which will be called the Second, Third, and Fourth Cases. Exercise 28 is regarded the First Case.

SECOND CASE.—Use the narrow rod with supports 50 cm. apart. Use first 1000 gm , and then double this load, following the same method of procedure and record as in Exercise 28.

THIRD CASE.—Use the broad rod on its broad side with supports 100 cm. apart. Use 400 gm., and then double this load.

FOURTH CASE.—Use the broad rod on edge with supports 100 cm. apart. Use 1000 gm., and then double this load.

Find for each case the deflection per gram.

As the grain of the wood may differ considerably in different rods, it is necessary, in deducing from these two Exercises the laws relating to dimensions, to make use of the observations made with a considerable number of rods, all as much alike in grain as may be. The effect of length, of width, and of thickness upon stiffness can in this way be made out with considerable clearness.

178. Laws of Stiffness.—(1) Comparing the First Case of bending (Exercise 29) with the Second Case (Exercise 30), find what general relation * exists between length of bent portion and amount of bending, other things being equal.

PROBLEM.

If a load of 150 grams bends .01 cm. a rod 40 cm. long, how far would it bend a rod similar in all other respects but 80 cm. long? 120 cm. long?

(2) Comparing the First Case and the Third Case, find what relation exists between width of rod and amount of bending, other things being equal.

(3) Comparing the First and Fourth Cases, if they are entirely satisfactory, one can find the relation between thickness of rod and amount of bending; but as it is difficult to have rods of very accurate dimensions, owing partly to shrinkage in the dry air of the laboratory, and as slight inaccuracies in the thickness affect the result a good deal, it may as well be said at once that the relation in question is this: *the amount of bending with a given load is inversely proportional to the third power of the thickness;* or, more briefly,

$$bending \propto \frac{1}{(thickness)^3}, \quad \text{or} \quad \frac{1}{T^3},$$

T standing for thickness.

The student should make comparison to see how closely his observations agree with this law.

Stiffness is inversely proportional to bending, so that we should write

$$stiffness \propto T^3.$$

* Of course, several different lengths should be used and the experiments should be very carefully conducted in order to enable one to deduce with complete confidence the relation between length and flexure. In this case, and in all others where general laws are to be deduced, the Exercises of this book are to be regarded as suggestive rather than conclusive.

The student should now try to put into one compact expression all of the relations here suggested between the stiffness and the dimensions of a rod. The general form of the expression will be similar to that for the *breaking-strength* in the next article, but the exponents attached to the letters will not all be the same as in that expression.

The general expression for the relation between *bending*, *load*, and *stiffness* is

$$bending \propto \frac{load}{stiffness}.$$

179. Breaking-strength.—It is important to notice that the formula for stiffness suggested in § 178 will not express the dependence of *breaking-strength* upon dimensions in the case of rods submitted to the action of transverse forces. For instance, adding to the thickness of a rod increases its strength less than it increases its stiffness. A thick rod is hard to bend, but a little bending will break it. We have the following relations: *strength* $\propto$ *width; strength* $\propto$ *thickness*^{2}*; strength* $\propto \frac{1}{length}$. Combining these three relations we get the formula

$$strength \propto \frac{width \times thickness^2}{length}, \quad \text{or} \quad \frac{W \times T^2}{L}.$$

Experiments on the breaking-strength of rods are too wasteful of material to be available for class work, but a few tests made by the teacher in the presence of the class will serve to enforce the law just stated.

SUGGESTIONS FOR EXPERIMENTS.

The long round rods, $\frac{1}{4}$ to $\frac{1}{8}$ inch in diameter, sold by hardware dealers to make dowel-pins, serve well for such tests. It is most convenient to apply the force by means of a spring-balance. Two stout pegs set vertically into a plank about 15 cm. apart, and projecting an inch above the plank, make good bearings for $\frac{1}{4}$-inch rods, to be broken by a horizontal pull. The pins should be whittled to an edge

where the rods touch them. A second pair of pegs may be 30 cm. apart. Care should be taken to select straight-grained rods for breaking tests.

180. Cross-section of Iron Beams.—It has already been noted (see § 177) that the outer arc of a bent rod is stretched and the inner arc compressed. It has also been stated that in a given substance the limit of elasticity in stretching is not necessarily the same as its limit in compression. Cast iron, for instance, will bear much more compression than stretching before giving way. Accordingly, cast iron beams or rails are made much wider on the side which is to be stretched than on the side which is to be compressed; a cross-section like that shown in Fig. 127 being sometimes used.* When such a beam, placed horizontal, is loaded in the middle, the broad under side is stretched less than the narrow upper side is compressed.

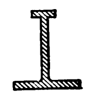

FIG. 127.

181. Elasticity of Torsion, or Twisting.—The elasticity of torsion, twisting, is most familiarly manifested in the alternate twisting and untwisting to be noticed when a weight is suspended by an ordinary (not a braided) cord or rope. It is much better shown by a wire than by any rope, on account of the great elasticity of the wire and the consequent promptness with which it springs back to and beyond its original position after a torsional stress has been removed.

EXPERIMENT.

Hang by one end a piece of No. 27 spring-brass wire, not less than a meter long, from a firm suspension (for example, a small hand-vise), and attach to the lower end a weight of fifty or a hundred grams, to which a pointer of cardboard or a straw has been fastened at right angles to the wire. Turn the weight through a considerable angle, then release it and notice the rapid circular movements of the pointer.

We are, in the affairs of every-day life, less familiar with twisting strains than with stretching or bending, but when

* See Goodeve's Mechanics.

we reflect that all of the revolving rods, or "shafts," used to transmit power through machinery are subject to twisting, we see that engineers, at least, must make a study of the laws of torsion. The frequency with which main-shafts of great steamers break in mid-ocean, sometimes endangering the lives of hundreds of people, is not due to any lack of knowledge as to the general laws of torsional elasticity and strength, but rather to defects in the *manufacture* of the shafts.

EXERCISE 31.

ELASTICITY: TWISTING.

Apparatus: Articles 7 (two), 50 (two), 61, 62, 63. Strings for attaching the balances to the cross-bars.

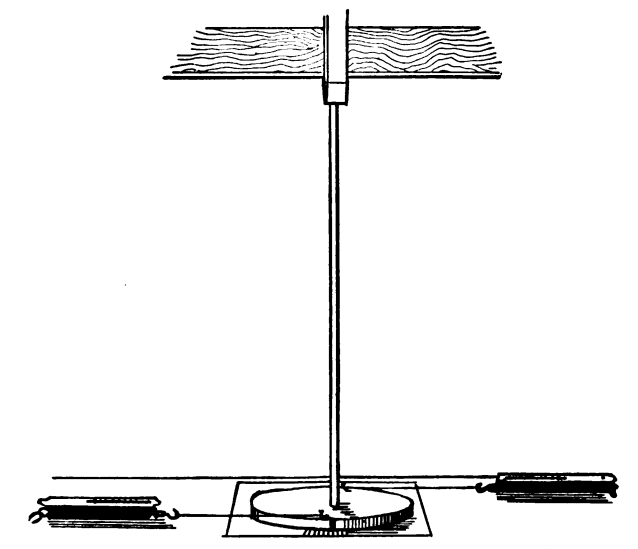

FIG. 128.

FIRST CASE.—Fasten the small rod to the horizontal bar over the laboratory table in a vertical position, so that the top of the cross-bar (shown as a disk in Fig. 128) will be 80 cm. beneath the fasten-

ing, the brad beneath the cross-bar entering a small hole made to receive it in the table, the cross-bar clearing the table-top by two or three millimeters. Adjust and fasten the graduated arc upon the table, so that the centre of the circle of which it makes a part shall coincide with the centre of the brad at the bottom of the rod, the 0° at one end of the scale being just beneath a vertical index-mark made on one end of the cross-bar.

Attach one of the small spring-balances to each peg upon the cross-bar by means of a string of convenient length, and pull in opposite directions upon these balances, keeping the line of pull at right angles with the cross-bar at each end, and applying to each balance a force of 50 gm. Observe and record the number of degrees * moved over by the vertical index-mark on the end of the cross-bar, which will indicate the number of degrees of torsion produced by the forces applied. Note whether the index returns, as it should, to its original position when the pull ceases.

Then pull with a force of 100 gm. upon each balance, observe and record the amount of torsion. (After this and every other application of twisting force look to see whether the index returns to its initial position. If the rod is twisted too far, it will not entirely recover.)

Then apply 200 gm. to each balance, observing and recording as before.

SECOND CASE.—Lower the horizontal supporting bar until a length of 40 cm. only remains between the fastening and the top of the cross-bar. Then proceed as in the first case, using the same forces as before.

THIRD CASE.—Replace the thin rod by the thick one, using 80 cm. as in the First Case. Apply forces of 500 and 1000 gm. in turn.

All the observations made in this Exercise should be recorded in a tabular form similar to that used in the two preceding Exercises. Thus for the

FIRST CASE.

Force at each balance.	Readings under stress.	Readings without stress.	Torsion per 1 gm. at each end.
		0°.0	
50 gm.	8°.2		0°.164
		0°.0	
100 "	....		....
		....	
150 "	....		....
		....	
		Mean =........	

* Read, if practicable, to one tenth of a degree.

182. Laws of Torsion.—Exercise 31 affords a means of studying the relation between the torsional forces applied to a rod and the amount of twisting produced, as well as the effect of varying the dimensions of the rod. From his observations the student should try to answer the following questions:

(1) What is the relation between the forces * applied and the amount of torsion? (For example, if a force f produces a torsion of 5°, how many degrees of torsion will a force $2f$ produce?)

(2) What is the relation between the length of the rod and the amount of torsion produced by a given force?

(3) How nearly do the observations agree with the statement, *torsion* $\propto \frac{1}{D^4}$, where D is the diameter of the rod?

Try, as in discussing the result of Exercises 29 and 30, to reduce the laws found to a single formula.

183. Breaking-strength.—As in bending, so in twisting, we must distinguish carefully between stiffness and strength. Force, applied at a given distance from the axis, required to break a rod by twisting is independent of the length of the rod, but is proportional to the third power of the diameter.

184. Best Use of Material in Beams.—We have now seen that increase of thickness is very effective in adding to the stiffness and the strength of rods, whether subject to bending or to twisting. The same amount of material is more effective when it is placed some distance away from the centre of cross-section than when it is very near this centre. Accordingly, beams that need to be made as stiff and strong as possible in proportion to their weight, to resist bending

* The effect of a force in producing torsion depends upon its point of application. The forces in Exercise 31 are all applied at a distance of 15 cm. from the axis. If they were applied at a distance of 7 5 cm. from the axis, they would produce only one-half as great an effect.

or twisting, are not made of compact form, but rather of a cross-section like some one of those in Fig. 129, two of which are hollow.

This fact sometimes leads people to the conclusion that a hollow beam is stronger than a solid one of the same external dimensions, but this is a mistake. If saving of weight and material were no object, beams would always be

Fig. 129.

made solid, but where, as in bicycles, flying-machines, etc., it is necessary to combine lightness with great stiffness and strength, the tubular form is almost always used. Witness the lightness and stiffness of quills.

185. Inductive Reasoning: Need of Caution in Drawing Inferences.—The kind of reasoning by which general inferences are drawn from the study of a group of observations upon an object or phenomenon is called *inductive reasoning.* The value of its conclusions depends upon the accuracy, the number, and the variety of the observations upon which they are based.

If a savage who is familiar with no metals except gold and copper should argue, from his acquaintance with these, that metals will sink in water, his opinion would be of little value, because of his very limited knowledge of metals. Any chemist or physicist of the last century, however, would have drawn from an acquaintance with all the metals then known the same conclusion, namely, that metals will sink in water; and it is only since the discovery, early in the present century, of sodium, potassium, and other such metals which will float, that it has become possible to dissociate the idea of comparatively great density from the necessary characteristics of metals.

The student must be extremely careful, in the course of his experiments, never to infer more than his observations warrant. If a certain conclusion seems probable from the data obtained, state that it seems probable, not that it has been proved true.

A number of general inferences in regard to elasticity have been suggested by the preceding Exercises, and these inferences, or laws, are accurate enough to be of great service, but we must not suppose that they would hold true under all circumstances. If the strain to which the body is subjected is too great, if the "elastic limit" is exceeded, the laws cease to be strictly applicable. Now the *elastic limits* of any particular body are difficult to determine accurately. The more sharply we look, the more we refine our methods of examination, the narrower becomes the space included between such limits.

186. "Fatigue" of Metals.—Moreover, the strain produced in a body by any application of force may depend upon the length of time, or the number of times, the force is applied. In Germany experiments have been made, in which bars of metal have been loaded and relieved alternately by machinery day after day and week after week, until the number of repetitions has reached into millions. We are told, for instance, of a rod of iron that "was torn apart . . . after 10,100,000 tensions," and of another rod that was sound after 48,000,000 bendings. The general result of such experiments is to show that a bar of metal may be broken by a very long application, or by very many applications, of a force that applied once for a short time would do no apparent harm. This fact is recognized in the phrase the *Fatigue of Metals*, which is the title of a little book * from which the quotations just made are taken.

* Van Nostrand, New York.

It is well known that cannon may be gradually weakened and finally burst by repeated discharges.

187. Characteristics of Solids.—In fact, the more closely we examine the characteristics of so-called *solids*, the less distinct appears the division between solids and liquids. Many substances ordinarily classed as solids will flow slowly under a slight pressure.* For instance, a stick of sealing-wax supported at its ends in a moderately warm room will gradually sag in the middle. And, on the other hand, all known liquids offer considerable resistance to rapid changes of form. Clerk Maxwell showed that a certain thick liquid, Canada balsam, exhibits properties resembling those of a strained solid, when it is inwardly disturbed by the movement of some body through it. Much importance has been attached to such experiments by Maxwell and by Lord Kelvin (Sir Wm. Thomson) in their discussions of the nature of the luminiferous ether.

QUESTIONS AND PROBLEMS ON CHAPTER XIII.

(1) Define elasticity and limit of elasticity. Illustrate your definitions.

(2) If No. 27 brass wire breaks with 15 lbs. pull, calculate the breaking-strength of No. 25 † brass wire.

(3) If No. 27 spring-brass wire breaks with 15 lbs. pull, and No. 30 annealed-iron wire with 5 lbs., find the relative tenacity of spring brass and annealed iron.

* The following story was told me by one of the sons of the late Alvan Clark, Sr., the great telescope-maker: A file was carelessly laid on the top of a large mass of pitch. After a time, the weather being warm, it was noticed that the file was nearly swallowed up by the pitch, and to pull it out by main force seemed impracticable. The elder Clark, upon being appealed to, remarked: *It went in slowly, and it will have to come out slowly.* So they rigged a contrivance to exert a steady, moderate, pull upon the file, and in the course of a week it came out.—E. H.

† See table of wire-gauge numbers and diameters, Appendix. In these calculations it is to be assumed that wires of the same metal are of exactly the same quality and condition. In practice this is not always the case. A thin wire is likely to be stronger, relatively to its size, than a thick wire.

(4) What pull would a wire of each kind, mentioned in Problem 3, 1 sq. mm. in area of cross-section sustain?

(5) What must be the diameter of a spring-brass wire that shall just sustain a load of 50 kgms.? Use data obtained in Exercise 26.

(6) A certain kind of wire breaks with 40 kgms. tension; it takes 10 cm. of this wire to weigh 1 gm. Find how many kilometers of such wire will break with its own weight.

(7) If in Exercise 27 the wire were 8 m. long instead of 4 m., how would the change affect the readings in the column headed "Increase of length"? readings in column headed $F \div l$?

(8) If a force of 1 kgm. stretches 1 mm. a wire which is 1 m. long and 0.1 sq. mm. in cross-section, how great a force is required to stretch 5 mm. a wire of like material 10 m. long and 1 sq. mm. in cross-section?

In Problems (9)–17 it is to be understood that the beams or rods are horizontal, supported at the ends and loaded at the middle.

(9) What is the ratio of the stiffness of a rod 50 cm. long to that of another rod 100 cm. long, but similar in all other respects?

(10) What is the ratio of the breaking-strength of the two rods in (9) under simple bending?

(11) It is found that an 8-in. floor-joist (i.e., one eight inches thick) is bent 0.5 inch by a certain load. What would be the thickness of one that would be bent only 0.1 inch by the same load?

(12) A certain beam 4 ft. long is bent downward 0.5 in. by a load placed at the middle. How far would it be bent by the same load if it were 8 ft. long?

(13) If a beam 3 m. long, 8 cm. wide, and 9 cm. thick is depressed 0.5 cm. by a certain load, how much would a similar beam 4 m. long be depressed by the same load?

(14) How much would the same load (Problem 13) depress the first beam if its thickness were increased to 12 cm.?

(15) If a beam 10 ft. long, 4 in. wide, and 6 in. thick is bent 1 in. by a load of 500 lbs., how much would a beam 5 ft. long, 2 in. wide, and 3 in. thick be bent by a load of 250 lbs.? Ans. 1 inch.

(16) If a certain beam 10 ft. long, 4 in. wide, and 6 in. thick, supported at the ends, is bent 1 in. by a load of 500 lbs., how great a load would bend 1 in. a beam 20 ft. long, 8 in. wide, and 12 in. thick?

(17) Is the stiffness of two similar rods, laid one on the other, as great as if the two were really one?

(18) (*a*) If one end of a rod is firmly fastened, and certain forces

applied to a cross-piece at the other end twist the rod 10°, how many degrees would it have been twisted by forces twice as great and similarly applied?

(b) How many degrees would these latter forces twist a rod twice as long as this one, but similar in other respects?

(19) A rod of the same material and length as the smaller one used in Exercise 31 is twisted 16 times as much as the latter rod for each unit of pull upon the balances. What is the diameter of the new rod?

(20) How much would the new rod have to be shortened to be twisted the same amount per unit of balance-pull as the smaller rod of Exercise 31?

CHAPTER XIV.

LIQUIDS AND GASES.

188. Introductory.—Fluids, including liquids and gases, are distinguished from solids by the ease with which they may be made to change shape.

Gases are distinguished from liquids by their small specific gravity, and still more by the fact that they suffer marked changes of volume with moderate changes of pressure.

Liquids range from the peculiarly fluent, or *thin*, like ether, alcohol, and water, to the peculiarly viscous, or *thick*, like glycerine and molasses; but none of them show the same kinds of elasticity of shape that we have been studying in solids, though it is true that the surface film of a body of liquid possesses a property somewhat like elasticity, which we shall consider farther on under the head of *surface tension*.

Molecular Attractions: Surface-tension, etc.

189. Molecular Attractions.—In a certain sense liquids have tenacity. We shall see later, in our experiments upon heat, good evidence for believing that there is a strong attraction of the particles, which we will now call *molecules*, for neighboring particles in water; for the amount of energy (§ 267) required to separate the molecules of a kilogram of water and turn it into a vapor is very great. If a column of some liquid, mercury, for instance, is inclosed in a tube to the sides of which it adheres, so that it cannot *dwindle* before pulling apart, it shows a very considerable tenacity,

or *cohesion.* Newton showed that a column of mercury * much taller than the barometric column can be sustained vertically in a glass tube, provided the air is excluded and the mercury is in contact with the closed top of the tube. The mercury is, in fact, suspended by its top, and must be in a somewhat *stretched* condition.

When two layers of molecules are very close together the attraction between them is probably very great, but if they are not very close together the attraction is imperceptible. Quincke found that the attraction of water or of mercury for glass acted through a very thin layer of some third substance placed between them, but if the thickness of the layer was made as much as .00001 cm., or even somewhat less, the action apparently ceased.

190. Surface-tension. — Within a body of liquid the attraction of molecule for molecule presses them hard together, but is equally great in all directions and does not tend to change the *shape* of the liquid body; for no change in the shape of the body would allow the interior molecules to come nearer together. But the molecules at the surface, not being surrounded on *all* sides by attracting molecules, may be drawn inward, thus coming into close relations with more of their fellow molecules. This operation is necessarily accompanied by a contraction of the surface. The surface of a body of liquid therefore tends to become smaller and smaller, acting somewhat like a film of stretched india-rubber. This fact suggests the name *surface-tension*, which is used in connection with many curious phenomena, all of which are of great interest to the student of physics, and some of which are of great practical importance.

When a body of liquid is left free to follow its own tendencies, unaffected by forces from without, surface-tension

* See Maxwell's article on *Capillarity* in the Encyclopædia Britannica.

brings it to the form having the least surface, that is, the spherical form. Drops of melted lead, falling from a great height in a "shot-tower," take the spherical form as they descend, and cool sufficiently to retain that shape on reaching the bottom, where they fall into water. Thus shot are made. A very small drop of mercury on a table-top assumes a form which is very nearly spherical, in spite of its weight, which tends to flatten it out. If the amount of mercury is larger, the drop is perceptibly flattened, though it is still convex at its edge. Water, on surfaces which are not *wetted* by it, on leaves and feathers, for instance, under certain conditions, acts like mercury on wood. If a liquid wets the body on which it rests, the attraction between the two acts, with the weight, to prevent the liquid from taking the spherical form.

191. Liquid Films.—A liquid film attached to a solid boundary takes that shape which makes the surface the smallest possible under the circumstances.

EXPERIMENTS.*

(All the wire frames and the pipe described in the following experiments will be grouped in the list of apparatus as No. L.)

(1) Blow a small soap-bubble with a common pipe, holding the mouth of the pipe downward. Watch the bubble for a little time, and see whether it contracts, taking care not to close the mouth end of the stem.

(2) Take a wire frame about 4 cm. wide, like that shown in Fig. 130, the light cross-wire *s* being free to slide on the guides *g* and *g*. After dipping the parts *s* and *e* into a soap solution, pull *s* some distance away from *e*, hold *g* and *g* horizontal, and then release *s*.

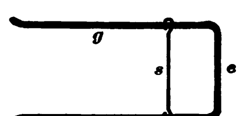

FIG. 130.

(3) Take a wire frame of the shape shown in Fig. 131, the ring being about 4 cm. in diameter. Dip this ring into the soap solution, and then blow gently against the film stretched across it.

* See Plateau's "Statique des Liquides."

(4) Use two rings like Fig. 132, dip both into the solution, and then, after putting them flat together, draw them two or three cm. apart, observing the shape of the film formed by this operation. Touch the central diaphragm of this film with the heated end of a wire.

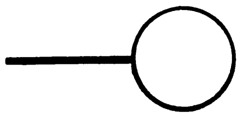

FIG. 131.

(5) Take a cubical frame of wire, like Fig. 132. Dip the whole of the cube into the soap solution, withdraw it, and observe the behavior of the film upon it.

Touch various parts of this film with the hot wire.

(6) Make a mixture of alcohol and water of such proportions that drops of olive-oil will *float submerged* in it, and note the shape which these drops take when so floating. (It is convenient to make two mixtures, one having a little too much water, the other a little too much alcohol, and then pour the second mixture gently upon the top of the first mixture. The oil will then float near the junction of the two.)

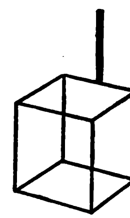

FIG. 132.

192. Surface Action on Larger Bodies of Liquid.—However great the body of a liquid may be, we must think of its surface as tending to contract. The exposed surface of the water in a large vessel is in a condition somewhat like that of a film of india-rubber stretched across the vessel. We see no evidence of this fact under common conditions, but if we lessen the tension at any one spot, the result is very striking, the adjacent parts of the surface film drawing quickly away from the affected spot.

EXPERIMENTS.

(1) Place a small cork stopper upon the surface of water in a large vessel, and then with a small tube let fall upon the water near the cork a drop or two of alcohol.

(2) Drop some alcohol upon a thin layer of water on a flat plate.

193. Oil Films on Water.—The experiment just described, with the cork stopper on water, may be repeated

with success a number of times with the same body of water, if only a small quantity of alcohol is used each time. If a drop of oil be used instead of a drop of alcohol the effect is about the same, *the first time.* A second drop of oil produces little or no effect.

The fact is that the alcohol quickly mixes with the main body of the water, and so produces very little permanent effect upon the surface. But the single drop of oil covers the whole exposed surface of the water in an instant with a very thin layer, and the second drop makes little or no change in the surface condition. The film of oil upon the water may be detected by the iridescence, play of rainbow colors, which it shows. The visible formation of this film with the first drop is very interesting.

194. Use of Oil in Stilling Waves.—A few years ago people who thought themselves well informed used to ridicule the notion that a gallon or two of oil poured upon the surface of a stormy sea could serve as an efficient protection to ships in distress; but it is now admitted that under certain conditions this seeming miracle is wrought. A ship driving before a gale, and in danger of being overwhelmed by the pursuing waves, trails out astern canvas bags filled with oil, which gradually finds its way into the water and, spreading over the surface, visibly abates the violence of the ocean. The theory is that the film of oil, being more viscid than the water surface, is less easily shaken up into the microscopic ripples which give the wind a hold upon the surface. *Practically* there is less friction of wind upon an oil surface than upon a water surface.

195. "Capillary" Action.—It is well known that if one end of a small open tube is dipped into water, or any one of many other liquids, the liquid rises in the narrow bore of the tube somewhat above the general level of its surface outside the tube.

If the tube is very slender indeed the bore may look like a hair, and this fact suggests the name *capillary* tube, which is commonly used for any very narrow tube. Accordingly, the elevation of liquids in such tubes is called *capillary* action. It is explained as follows: The wall of the tube attracts strongly the adjacent layer of water, and so tends to spread this layer out over a larger part of the wall surface, in opposition to the surface tension of the liquid itself, which resists such an extension. In most cases the spreading tendency of the wall's attraction prevails, and the liquid close to the wall creeps up a little, both inside and outside the tube, thus taking a concave shape.

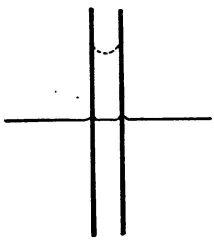

FIG. 133.

Inside the tube the curvature reaches completely across, and the top of the liquid is brought into the shape of a shallow cup. If we now think of this surface as held and supported at its edge by the glass, and remember that it is all the time tending to contract, we shall see that it tends to lift the liquid column beneath it, either directly by cohesion (§ 189), or indirectly by relieving the column from some portion of the atmospheric pressure upon its top. The indirect action is the ordinary one; but the direct action may occur, if the experiment is tried in a vacuum.

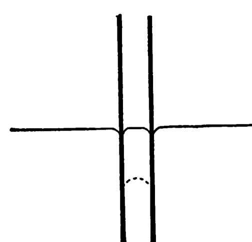

FIG. 134.

When mercury, instead of water, is used in a glass tube, the attraction of the glass for the mercury, although it is greater than the attraction of glass for water, does not succeed in *squeezing out* the layer of mercury so as to make it cover more of the glass surface. On the contrary, the surface tension of mercury, which is very

great, prevails, and the edges are drawn down, as in Fig. 134. The contraction of the *convex* surface thus formed within the tube forces the liquid column down. That is, mercury stands lower in the tube than in the vessel outside the tube.

So-called capillary action is perceptible in tubes of considerable size, as we shall see in a number of the following Exercises. It occurs also in narrow spaces which are not tubes; in the meshes of a lamp-wick, for instance, thus raising the oil to the flame.

EXPERIMENT.

1. Dip one face of a cube of sugar into water, and note how quicklythe whole lump becomes wet through.

2. Place two rectangular pieces of glass vertical in water, and let them meet at a very acute angle. Note the shape of the water surface near the angle.

Pressure in Fluids at Rest: Hydrostatics.

196. Review.—Before going farther it will be well to recall and perhaps to repeat the experiments made with the pressure-gauge in Chapter III.

The facts or principles established by these experiments are, for liquids at rest:

1. Pressure increases with descent in the liquid and decreases with ascent;

2. Pressure at a given point is equally great in all directions;

3. Pressure is equally great at all points on the same level in any one body of a single liquid, provided the density is everywhere the same.

The laws of fluids at rest are called the laws of *hydrostatics*.

We shall now make use of the facts just mentioned in studying some cases of fluid pressure that may appear more complex or difficult than those considered in Chapter III.

197. Mariotte's Bottle.—Admirable practice in applying the principle of hydrostatics is afforded by a study of the phenomena presented by "Mariotte's bottle."

EXPERIMENT.

Take a bottle (with three lateral openings No. LI), each stopped as shown in Fig. 135; fill the bottle with water and insert in the mouth a perforated cork, through which a glass tube, 30 or 35 cm. long, extends nearly to the bottom of the bottle. Take out each of the stoppers, *a*, *b*, and *c*, in turn, noting in every case whether water runs out freely, and returning the stopper again to its place. Note, after the removal of each stopper, the level at which the water stands in the vertical tube.

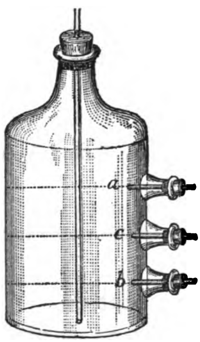

FIG. 135.

Refill the bottle, raise the tube until its lower end is on the level half-way between *a* and *b*, and allow the water to run from *a* and from *b* (for a short time) in turn. Repeat with the lower end of the tube half-way between *b* and *c*, allowing the water to flow from *a*, from *b*, and from *c*, in turn.

Try to explain the observed results by considering the bottle and the tube as communicating vessels. The *free* surface of the water in the *bottle*, as distinguished from the vertical tube, is at the level of any opening, *a*, *b*, or *c*, that may at the time be unstopped.

"Free surface" here means a surface exposed to the direct pressure of the outer air. In unravelling hydrostatic puzzles it is best to begin at a free surface; for at this surface we know how great the pressure is. It is simply the pressure indicated by the barometer.

198. Pressure at a Point.—In our experiments with the pressure-gauge, we have tested the pressure upon the gauge-face, taken as a whole. It is evident, however, that when this face is not horizontal, different parts of it, being at different depths, are subject to pressures of different intensity. To find the meaning of the phrase ***pressure at a***

point, imagine the face to shrink continually without change of shape, the centre being kept at a particular point. The total pressure upon the face will decrease continually during this process, but the ratio, *total pressure* $\div$ *area*, will change very little, if at all. The limit which this ratio approaches, as the circle approximates to a mere point, is called the *pressure at the point*. It is equal to the total pressure upon a unit surface, at every part of which the pressure is as intense as at the point in question.

199. Average Pressure.—If we find the total pressure upon a given surface and divide this total by the area of the surface, we obtain what is called the *average pressure* over the given surface. If the pressure is equally great at all parts of the surface, the average pressure is equal to the actual pressure per unit-area at every part of the surface. If the pressure is not equally great at all parts of the surface, the average pressure is evidently greater than the smallest pressure per unit-area, and less than the greatest pressure per unit area, to be found on any part of the surface.

The average pressure on certain surfaces, e.g., a circle or a rectangle placed vertically, is easily seen to be equal to the pressure at the central point, the liquid being supposed of equal density throughout; for pressure in such a liquid increases regularly with the depth, and any given element of the surface below the centre is offset by any perfectly similar element placed at an equal distance above the centre. In Fig. 136, which represents a section of a rectangular box with vertical sides, the depth of the water at A is 0 and at D is $\overline{AD}$ cm. Therefore the average depth is $\frac{1}{2}\,\overline{AD}$ cm., or $\overline{AE}$ cm., and the average pressure per sq. cm. against the end of the box is equal to the pressure at the middle point, which is $\overline{AE}$ grams.

200. Calculation of Total Pressure on a Non-horizontal Surface.—By definition of average pressure the total press-

ure upon a surface is equal to the area multiplied by the average pressure. Thus the total pressure, expressed in grams, against one end of the tank, Fig. 136, is equal to its *area* $\times \overline{AE}$, or *area* $\times \frac{AD}{2}$.

If some other liquid of density δ were used, the pressure at depth $\overline{AE}$ cm. would be $\overline{AE} \times \delta$ grams per sq. cm., and

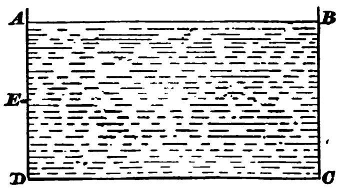

FIG. 136.

the total pressure against the end would be equal to the *area* $\times \overline{AE} \times \delta$.

In any except very simple cases the calculation of the total pressure on a surface is too difficult for the purposes of this book.

201. Pressure at Great Depth.—Since in water each 10 meters of depth will give a pressure of 1 kgm. per sq. cm., the pressure encountered in deep soundings, in the lakes or sea, becomes immense. During the days of sailing navigation on the Great Lakes a curious use was made of this pressure. No ice being carried on board vessels, it was difficult in summer to procure cold water for drinking, but this was often secured by lowering an empty, tightly-stoppered jug, heavily weighted with lead, to a depth of some hundreds of feet. When this was hauled up the cork would be found forced in and the jug full of water, at a temperature not far from 40° Fahrenheit.

QUESTIONS.

(1) An upright cylindrical jar 20 cm deep, the base of which is 100 sq. cm. inside, is filled with water.

(*a*) What is the weight of water resting upon each sq. cm. of the base?

(*b*) If the pressure of the atmosphere is 1000 gm. upon each sq. cm. of the upper surface of the water, what is the total pressure upon each sq. cm. of the base?

(*c*) What is the total pressure upon a horizontal sq. cm. at the depth of 3 cm.? at a depth of 10 cm.? at a depth of 15 cm.?

(*d*) What is the total pressure upon a sq. cm. of the vertical wall, the centre of the square being at a depth of 5 cm. in the water? at a depth of 10 cm.? at a depth of 15 cm.?

(2) Let the jar be closed by a flat cover touching the water, having an open tube 1 sq. cm. in cross-section rising from its centre and extending upward 30 cm. above the top of the jar. Let both jar and tube be full of water. Let the atmospheric pressure be disregarded.

(*a*) What is the total weight of water in the jar and tube?

(*b*) What is the total pressure of the water upon the whole base?

(*c*) What is the total upward pressure of the water against the whole cover?

(*d*) Subtract the total upward pressure against the cover from the total downward pressure against the base and compare the result with the weight of all the water in the jar and tube.

(3) Suppose now that in any way, by means of a piston, for instance, a pressure equal to the weight of 50 gm. is brought to bear upon the top of the water in the tube.

(*a*) What will now be the pressure upon the sq. cm. which lies at the top of the jar, just beneath the tube?

(*b*) How much will the total pressure against the bottom of the jar be increased by the action of the piston.

(*c*) What is the total pressure upon 1 sq, cm. of the vertical side of the jar before the piston is made to act, the centre of the square being at a depth of 5 cm. beneath the cover of the jar? at a depth of 10 cm.? at a depth of 15 cm.?

(*d*) What is the pressure upon each of these vertical squares after the piston is made to act?

202. Pascal's Principle.—A part of what is taught by the preceding experiments and questions is summed up in a principle announced by the French physicist Pascal about the middle of the seventeenth century: "If a vessel full of water, closed in all parts, has two openings of which

the one is a hundred times the other, placing in each a piston which fits it, a man pushing the small piston will equal the force of a hundred men who push that which is a hundred times as large, and will surpass that of ninety-nine. Whatever proportion these openings have, and whatever direction the pistons have, if the forces that one applies on the pistons are as the openings, they will be in equilibrium."

The principle is usually stated nearly as follows:

Pressure exerted anywhere upon a mass of liquid is transmitted undiminished in all directions, and acts with the same force upon all equal surfaces in a direction at right angles to those surfaces.

This principle when properly applied is extremely useful, but, unless it is carefully illustrated and discussed, students often get from it the false notion that pressure is equally great at all parts of a liquid.

203. The "Hydrostatic Bellows."—This apparatus in one of its simplest forms consists of two boards, each some 20 cm. square, connected by a piece of leather tacked closely around the sides. A tube, bent at right angles at the bottom, opens into the interior of the bellows at the upper surface of the lower board. When water is poured into the upright portion of the tube until the bellows is filled, the upper board of the bellows is forced up with a pressure proportional to its area and to the height of the water in the tube above the level of the water in the bellows. Each centimeter in height of the column in the tube will give rise to a pressure of 1 gm. per square centimeter in the bellows ("Pascal's Principle"). If the area of the cross-section of the tube were 1 sq. cm. and that of the top of the bellows were 400 sq. cm., it is clear that each cubic centimeter (or gram) of water poured into the tube would exert a pressure of 400 gm. against the

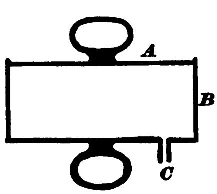

FIG. 187.

bellows-top, if the latter were prevented from rising. Since the leather sides of this apparatus soon become hardened and crack, the metal cylinder (No. LII),* shown in Fig. 137, may conveniently be used to illustrate the principle which we are discussing.

EXPERIMENT.

Wire a stout rubber tube firmly on to the short brass tube *C*, force the piston *A* to the bottom of the cylinder, and then, holding the free end of the tube as high as possible above the cylinder, pour in water through a funnel, while the upward movement of the piston is resisted by the application of heavy weights.

204. The Hydrostatic Press.—In the case of the apparatus just described the pressure which moves the piston is due to the weight of the water in the upright tube. The *hydrostatic press* usually depends for its action on the pressure imparted to water in one cylinder by a piston or plunger working in another cylinder, often at a considerable distance from the former one. The ram or larger plunger (*A*, Fig. 138) is so arranged as to work water-tight in the

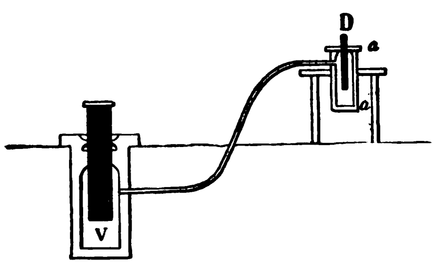

FIG. 138.

cylinder *V*, and the plunger *D* works similarly in the small cylinder *a*. Small quantities of water are successively forced from *a* into *V* by the movements of *D*, and *A* is raised very slowly, but with great force, carrying with it

* Devised by Mr. A. P. Gage.

whatever weight may have been placed upon it, or compressing strongly any material, for example a bale of cotton, confined in an open-bottomed chamber above the ram, into which the latter rises.

Since the pressure per square centimeter exerted by the small piston is transmitted undiminished to every part of the body of water in both cylinders, the total upward pressure exerted by the ram *A* will be to the total downward pressure applied to the plunger *D* as the area of the cross-section of *A* is to the area of the cross-section of *D*, loss of power through friction being disregarded.

205. Balancing Columns.—The principles of hydrostatics with which we have now become familiar enable us to determine the specific gravity of liquids with much accuracy by the method of *balancing columns.*

We know that communicating columns of equal height and of the same liquid will balance each other even when, as in Fig. 139, they are not of equal cross-section, if capillary effects can be neglected. If the communicating columns are of different liquids, of different specific gravities, a given column of the heavier liquid will balance a taller column of the lighter liquid, as in Fig. 140, where a short column of mercury, *ml*, balances a tall column of water, *wl*, the height of each column being measured from the level of the surface *ll*, on which the two liquids meet. In this case also the height of each column would remain unchanged if its area of cross-section were varied. A little consideration will show that the heights of the columns in this case are inversely proportional to their densities. That is, the density of mercury being 13.6 times as great as that of water, the water-column, *wl*, must be 13.6 times as tall as the balancing mercury-column, *ml*.

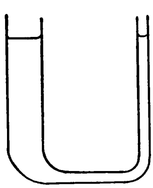

FIG. 139.

The method of determining specific gravities which is suggested by Fig. 140 can be used to advantage only when the two liquids to be compared do not readily mix with each other. The method used in Exercise 32 depends upon the same principles, but is more convenient and more generally applicable. The two columns used in this Exercise (see Fig. 141) do not directly balance each other, but as the pressure per unit area on the tops of the two columns is the same, and as the pressure per unit area at the bottom of each column (inside the tube at the level of the outer surface) is equal to that caused by the atmosphere, the pressure per unit area produced by the *weight* of either column must equal the pressure per unit area produced by the weight of the other column. The two columns would therefore balance each other directly if they met at the bottom, and the calculation may be made as if they actually did so.

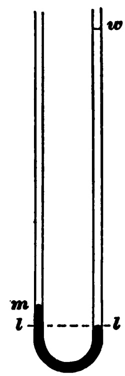

FIG. 140.

The tubes need not be of the same size, and the level of the liquid need not be the same in the two tumblers. The capillary effects should be carefully observed, although in this case they do not greatly influence the final numerical result of the experiment.

EXERCISE 32.

SPECIFIC GRAVITY OF A LIQUID BY BALANCING COLUMNS.

Apparatus: Articles 64, 65, and 66. Water for one of the tumblers and a nearly saturated solution of sulphate of copper for the other.

Place the free ends of the glass tubes one in the water and the other in the sulphate of copper, and fix them in a vertical position against the upright (see Fig. 141). Observe and record the height to which each liquid rises in its tube by capillary action, the pinchcock being open.

Applying the lips * to the open rubber tube at the top, draw out some of the air, thus raising the two liquids in their respective tubes. When the taller column reaches nearly to the top of the glass tube, pinch the rubber tube near the lips, and then close the pinch-cock. Watch the columns to make sure that there is no falling of their surfaces, as there will be if the connections are not air-tight (It is well to moisten the connections occasionally with a little glycerine to keep them air-tight.)

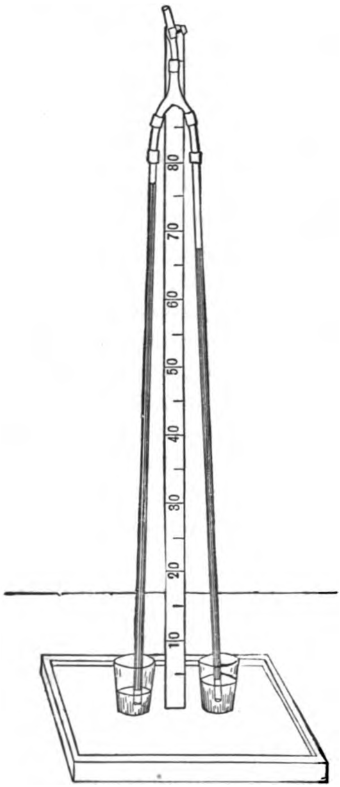

FIG. 141.

When assured that the columns are not sinking, measure carefully the height of each from the *present* level of the liquid surface in the corresponding tumbler.

Subtract from the height of each column as thus measured the height to which it was raised by capillary action at the beginning.

From the data now obtained find the specific gravity of the solution of sulphate of copper.

206. Pressure in Gases.—Gases obey the same hydrostatic laws as liquids. Pressure in a gas at rest increases with descent, and is equally great in all directions at any given point. In a single body of any one gas at rest pressure is equally great at

* This use of the lips can be avoided by using a small air-pump. To prevent too sudden action of the pump a large bottle should be inserted between it and the upright tube.

all points on the same level, provided there are no differences of density that need be considered.

Various experiments illustrating atmospheric pressure and the principle of the barometer have been shown in Chapter III. But it is well, before taking up the next Exercise, to give other experiments of a similar kind.

207. Atmospheric Pressure: Magdeburg Hemispheres, etc.

EXPERIMENT.

Push the piston *A* of Fig. 137 to the bottom of the cylinder, clamp the rubber tube tightly, fasten the ring at one end of the apparatus to a stout hook in the wall or other suitable attachment, and try to pull the piston out to the open end.

A similar experiment is frequently performed with two hollow hemispheres. "The experiment with the hemispheres was made for the first time by Otto von Guericke, burgomaster of Magdeburg, who repeated it, in 1654, at Ratisbon, before Ferdinand III. and the assembly of the College of the Empire. The hemispheres were 24 *pouces* 8 *lignes* * in diameter. Although much air remained within them, it required 12 horses to pull them apart. The gas (air), rushing into the vacuum, at the moment of separation, produced a violent explosion. This celebrated experiment helped to spread the doctrine of the weight of the air, and to popularize the air-pump, which Otto von Guericke had invented." †

Hundreds, and perhaps thousands, of years before the date of this experiment the difficulty of maintaining a vacuum, the tendency of fluids in communication with a vacuum to rush into it, had been noticed. The statement that "nature abhors a vacuum" had been accepted as a principle, or, perhaps we should say, as a proverb, among those who dabbled in science during the middle ages, and

* That is, about 26 inches.

† Daguin's *Traité Élémentaire de Physique.*

this statement, though unscientific, according to modern ways of thinking, and inaccurate, was no doubt extremely useful in its time. But sometimes it led to wrong conclusions. "Thus Mersenne, in 1644, speaks of a siphon [see § 44] which shall go over a mountain, being ignorant then that the effect of such an instrument was limited [with water] to a height of thirty-four feet. A few years later, however, he had detected this mistake; . . . and in his third volume . . . he . . . speaks correctly of the weight of air as supporting the mercury in the tube of Torricelli. It was, indeed, by finding this horror of a vacuum to have a limit at the height of thirty-four feet, that the true principle was suggested. It was discovered that when attempts were made to raise water higher than this, Nature tolerated a vacuum above the water which rose. In 1643, Torricelli tried to produce this vacuum at a smaller height, by using, instead of water, the heavier fluid, quicksilver [mercury]; an attempt which shows that the true explanation, the balance of the weight of the water by another pressure, had already suggested itself. Indeed, this appears from other evidence. Galileo had already taught that the air has weight."*

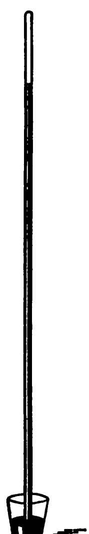

Fig. 142.

The experiment of Torricelli referred to in this passage is precisely like, in principle, to one already given (§ 34), but a more common form of it will presently be shown.

EXPERIMENTS.

(1) Into one end of a piece of stout glass tubing of about 1 cm. internal diameter and 1 m. long insert a good cork or, better, a close-fitting rubber stopper. Fill the tube with water, close the open end with the forefinger, place this end under water, after inverting the tube, and remove

* Whewell's *History of the Inductive Sciences*, Vol. I.

the finger. Note whether the water in the tube sinks at all. Remove the stopper and note the result.

(2) Fill a strong glass tube about 50 cm. long, closed at one end, with mercury, stop the open end with the forefinger, and, after inverting it, open it under mercury in a small dish, for example a small tumbler. Note whether any fall in the mercury in the tube can be observed.

(3) Repeat experiment 2 with a barometer-tube nearly a meter long (Fig. 142); note the decided fall of the mercury in the tube, and measure as accurately as you can the height of the mercury remaining in the tube above the level of that in the tumbler. This last experiment, known from the inventor as Torricelli's experiment, serves to give a tolerably accurate measure of the value of the atmospheric pressure.

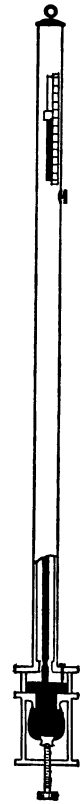

FIG. 143.

208. Construction and Use of Barometer.—Although the principle of the barometer is precisely that of the tube of Torricelli just described, it is well to give a description of the instrument as ordinarily made, and a few words of explanation in regard to its use.

A good form of barometer is shown in Fig. 143. It consists essentially of a cistern for mercury, covered to prevent loss of mercury when the instrument is carried about or jostled, and a glass tube, closed at the top and open into the cistern at the bottom, alongside which is placed a scale divided either into inches and tenths of inches, or into centimeters and millimeters.

The scale is marked as if it began at a certain level in the mercury cistern, and the indications of the barometer are correct only when the surface of the mercury in the cistern is maintained at this fixed level. This result is attained by raising or lowering the flexible leathern bottom of the mercury-cistern by turning a screw worked by a milled head until the upper surface of

the mercury in the cistern just touches the point of a pin reaching down from the top of the cistern. In the use of a barometer of this class the level of the mercury in the cistern should first be adjusted, and after this the height of the column is to be read on the scale alongside the tube.

It has already been stated (§ 195) that the top of a mercury column in a glass tube is usually convex, and that capillary action tends to depress such a column. The depression thus produced in a tube of considerable size, 1 cm. diameter for example, is very small. Sometimes a column that has recently fallen a good deal has a flat or concave top. Errors arising from this source may be obviated by carefully jarring the barometer before taking a reading. In every case the extreme height of the convex surface of the mercury in the tube should be taken as the height of the column.

A much less expensive barometer than that shown in Fig. 143 can be made useful in this course, if it has once for all been calibrated, that is, tested over a wide range of pressures by comparison with a standard instrument in the neighborhood.

209. Variations of Atmospheric Pressure—Measurement of Heights by Barometer.—Two more points remain to be noticed; namely, that the atmosphere varies extremely in its density at different heights, and that the atmospheric pressure, and consequently the barometric readings at any given place on the earth's surface, vary considerably from time to time.

The unequal density of the various portions of the atmosphere at various heights above the earth's surface is due to the sensibility of air to changes of pressure and temperature. As a cubic foot of hay from the bottom of a haystack or haymow, when it is pressed upon by many feet of hay above, contains more matter than a cubic foot from the

upper part of the pile, so a cubic foot of air at the earth's surface at sea-level contains about twice as much matter as a cubic foot taken from a point 3.4 miles above sea-level.

An extremely useful application of the barometer depends upon the decrease in atmospheric pressure with ascent from the sea-level. Since, for small changes of elevation near the sea-level, a fall of 0.1 inch of the barometer corresponds to an elevation of 87 feet, or about 1 millimeter to 10 meters, the heights of hills,* etc., may be directly ascertained by the observer carrying a barometer up the hill to be measured, and noting the fall of the barometer as he ascends, due allowance being made, however, for barometric variations not due to the change in elevation. Aneroid barometers are commonly used for such work (see § 38).

The daily and hourly changes in the height of the barometer may be considerable. These barometric changes must be noted when any careful experiments upon pressure of gases are being performed, in order that the experimenter may know to just what pressure the gas under examination is really exposed. Since the usual height of the mercurial column in the barometer at sea-level is about 30 inches, or 76 cm., if the barometer at any given time stands at 74 cm., for example, the pressure of the atmosphere indicated is about $\frac{74}{76}$ of its average sea-level value.

210. Behavior of a Gas under Changes of Pressure.—It has already been stated that gases are very sensitive to changes of pressure. We have had a number of experiments in Chapter III illustrating compressibility or elasticity of air. It is well, however, for the student to go through by himself the experiments leading to *Boyle's law;* for this law is very important, and the mechanical

* Complicated calculations are necessary for considerable heights, as those of mountains.

manipulations will be novel and instructive, giving the student his first opportunity to see how easy it is to spill mercury and how hard it is to pick it up.

EXERCISE 33.

COMPRESSIBILITY OF AIR: BOYLE'S LAW.

Apparatus: No. 67, and No. 66 to support it. Sufficient mercury to fill at least the long arm of the tube. A barometer (No. 68).

Pour into the bent glass tube sufficient mercury to fill the bend and rise to the straight part in each arm. At first the mercury will stand a little higher in the long arm, but by tipping the tube and letting out a little of the imprisoned air the level can be made the same in both arms, as in Fig. 144, or left only one or two mm. higher in the long arm than in the other. This operation may well be performed before the tube is attached to its support.

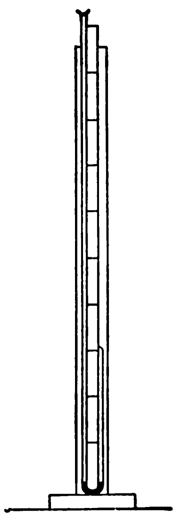

FIG. 144.

Now measure the length of the imprisoned air-column, from the curved top of the mercury-surface to the curved top of the bore of the tube, taking care not to handle the short arm much, lest the warmth of the hand expand the air, and write the length thus found under the letter V * in a table like that here indicated.

V	P	$V \times P$
....	....	....
....	....	....
....	....	....
....	....	....

The pressure upon this air is now, if the mercury-level is the same in both arms, equal to that upon the unimprisoned air. It is as great as would be exerted by a column of mercury as tall as that in the barometer. Take, then, a reading of the barometer, and record this reading under the letter P †

* The *length* of the air-column is equal to *its volume*, if we take for our unit of volume in this experiment the space contained in unit length, 1 cm., of the tube. The bore of the tube is supposed to be of the same size throughout the short arm.

† It is common practice to measure pressure in terms of centimeters of depth of mercury.

in the table. But if the level is a little higher in the long arm than in the short arm, write under P the height of the barometer column *plus* the excess of height in the long arm.

Pour in more mercury till the difference of level in the two arms is about 20 cm., then measure again the length of the confined air-column. Record this length under V, and record under P the barometric height *plus* the present difference of mercury-level in the arms of the tube.

Proceed in this way, by fairly uniform stages; measuring and recording at each stage, till the volume of the confined air-column is about one-half what it was at first.

Multiply each number under V by the corresponding number under P, and write the products in the column headed $V \times P$. An examination of this column will probably show a pretty close confirmation of *Boyle's* law.

211. Discussion of Boyle's Law.—The law in question was announced by Robert Boyle, an Englishman, in 1662, and by Mariotte, a Frenchman, about sixteen years later. It is often called Mariotte's law. It may be stated thus: *The volume of a given portion of air, at a fixed temperature, is inversely proportional to the pressure to which it is subjected.* Or thus:

$$V \propto \frac{1}{P}; \text{ that is, } PV = \textit{a constant quantity.}$$

Strictly speaking, Exercise 33 can establish this law only for a given amount of air at a particular temperature. A vast number of experiments were needed to enable physicists to state the law, with its slight necessary modifications, for all known gases at all observable temperatnres.

Boyle, in announcing the law, spoke of the "*Spring of the Air*," and this phrase is very apt; for no matter how much or how long a gas has been compressed, it will always return to its original volume as soon as the original pressure and temperature are restored. In other words, gases have perfect elasticity of volume.

212. Graphical Representation of Results of Exercise 33. —Let each division of the ruled paper, Fig. 145, which we will suppose marked off in tenth-of-an-inch squares, represent, along a vertical line, a pressure of 1 cm. of mercury, and along a horizontal line 1 cm. length of the confined air-column. The letter h will stand for atmospheric pressure as indicated by the barometer column, and, if we deal with no pressures less than h, we may as well mark the lowest horizontal line of the paper h, or $h + 0$.

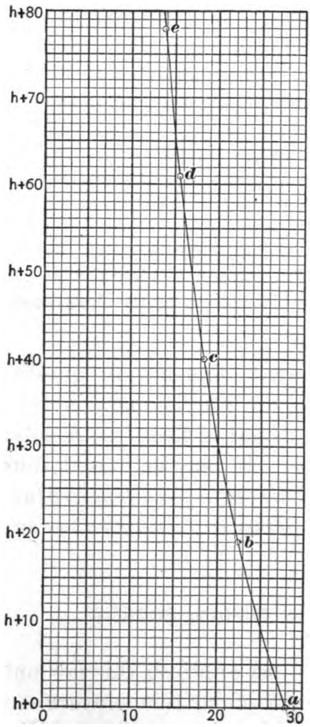

FIG. 145.

If the confined air-column under mere atmospheric pressure was 28 cm. long, make a dot at the point a, 28 divisions from the left end of the base-line $h + 0$. If at the next observation the pressure was $h + 19$ cm. and the length of the air-column 22.4 cm., put a second dot at the point b, and so on.

After all the dots are placed, bend a narrow saw-blade, or a rubber tube, or any other conveniently flexible object, so that its outline shall pass, as nearly as may be, through all the dots, a, b, etc., in a smooth curve, and then, with a hard well-sharpened pencil, trace this outline upon the paper.

By studying the diagram thus obtained one can readily

find what volume would correspond to some pressure intermediate between those actually observed, or what pressure would produce a given volume intermediate between those observed.

QUESTIONS.

(Capillary effects, unless expressly mentioned, will be disregarded in these questions.)

(1) In the Exercise on Boyle's law is it necessary that the two arms of the glass tube should be of the same diameter ?

(2) Would a small solid body adhering to the inside of the long arm, in such a way as to make the mercury-column at that point smaller than elsewhere, affect the accuracy of the results ?

(3) Do the numerous air-bubbles which cling to the inside of the long arm affect the accuracy of the results ?

(4) Would an air-bubble extending completely across the bore of the long arm, so as to make a real break in the mercury-column, need to be considered in measuring the height of the column ?

(5) Would it be a correct statement of Boyle's law to say that equal successive additions to the pressure on a body of air cause equal successive subtractions from the volume of the air ?

(6) If Fig. 145 represents an actual case, what volume would correspond to a pressure of $h + 25$? of $h + 45$?

(7) What pressure in Fig. 145 corresponds to volume 20 ? to volume 15 ?

(8) Calculate the value of h in Fig. 145 by means of Boyle's law, taking for one volume 15 and for the other 20, and using the corresponding values of the pressures. (The value of h thus found may not be exactly 76, and a somewhat different value may perhaps be found by taking different points in Fig. 145. In fact, this figure probably does not correspond exactly to Boyle's law.)

(9) The lower end of an open tube rests in mercury, and the upper end is connected with a bottle. If the barometer reads 76 cm., how high will the mercury stand in the tube after half the air has been pumped out from the bottle ? after one-quarter has been pumped out ? after three-quarters ?

(10) A tube bent into the form of an elongated U is placed with its ends uppermost, one open to the air, the other connected with a bottle. Mercury is poured into the tube, and stands at the same level in the two arms.

(*a*) How does the air-pressure in the bottle compare with that outside?

(*b*) What will be the difference of level in the two arms when one-half the air has been pumped from the bottle?

(*c*) What fraction of the original amount of air in the bottle will remain there when the difference of level in the two arms is 68 cm., the height of the barometric column being 75 cm.?

213. Density of Gases.—The density of air, or any other gas, depends on its temperature and the pressure to which it is subjected, but it is customary to give that value which holds when the temperature is that of freezing water, and the pressure is equal to that of 76 cm. of mercury. These are the so-called *standard conditions*, but in the following Exercise we shall determine the density of the air at the temperature and pressure which hold in the laboratory at the time of the experiment.

Since the air cannot be weighed without some containing vessel, it will be necessary, as in obtaining the specific gravity of liquids by the "specific-gravity bottle," to obtain the weight of the bottle empty, as nearly as may be, and then, by subtracting this from the weight of the bottle filled with air, to find the weight of the air itself. Strictly, we cannot get all of the air out of the bottle, and we have to make allowance for what remains in it.

EXERCISE 34.

DENSITY OF AIR.

Apparatus: Nos. 68, 69, 70 (supported by 66), 71, 72, and 73.

Set up the apparatus according to Fig. 146, after moistening with glycerine the junctions at which leakage might occur, and pour mercury into the U tube till both arms are about half full.

Work the pump and draw air from the bottle until the difference of mercury-level in the arms of the U-tube gauge is as much as 70 cm., then pinch the rubber tube that leads from the pump very hard and watch the mercury in the gauge for a short time to see whether it is changing level. If it is changing, there is some leak which should be looked for and stopped. If there is no leak, read

and record the difference of level in the arms of the mercury-gauge, and then immediately close the pinch-cock near the bottle, making it thoroughly tight.

Disconnect the bottle, with the attached tube and pinch-cock, from the rest of the apparatus and place it on the platform balance, after making sure that this balance is in good condition. Weigh the bottle and attachments *very carefully*, to the nearest tenth of a gram if possible, and then, without removing the bottle from the balance, open the pinch-cock and let the air enter. Then weigh again, as carefully as before.

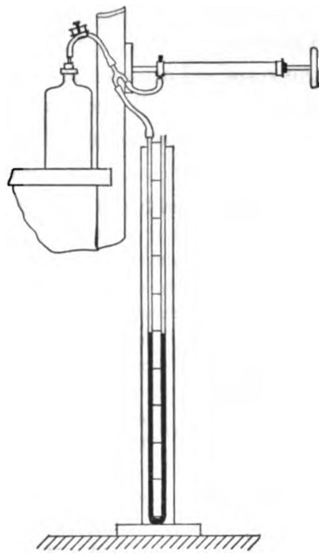

FIG. 146.

The difference of the two weights thus found is the weight of the air originally pumped out from the bottle. The reading of the mercury-gauge, already recorded, compared with the height of the barometric column, which should now be read, will show what fraction this was of the whole amount of air contained by the bottle when open to the atmosphere, and so the weight of all the air in the bottle can be found.

It is now only necessary to find the capacity of the bottle, in cu. cm., in order to be able to calculate the density * (§ 22) of the air. The capacity of the bottle may be found by finding how many grams of water it will hold.†

* Unless this Exercise is performed with care and intelligence the results will be absurdly inaccurate. No balance available for elementary laboratory purposes is quite satisfactory for such weighing as that here described. Nevertheless, the Exercise, if carefully done, is instructive and profitable.

† It is well for the teacher to determine the capacity of the bottle once for all and mark it on a label, in order to save frequent wetting of the interior. Drying out bottles is a very tedious and thankless task.

214. Height of "Homogeneous Atmosphere." — The "*height of the homogeneous atmosphere*" is a common phrase which means the height that would be necessary to produce the standard atmospheric pressure, of 76 cm. of mercury, if the air had at all heights the same density which it has at the sea-level under standard conditions (§ 213).

QUESTIONS.

(1) If the density of mercury is 13.6 and that of standard air is .00129, each in gm. per cu. cm., how great is the height of the "homogeneous atmosphere"? *Ans.* 8012 m. nearly.

(2) How much, approximately, is the difference between the readings of two accurate barometers, placed one at the bottom and the other at the top of a Chicago building having 20 stories averaging 9.5 ft. high?

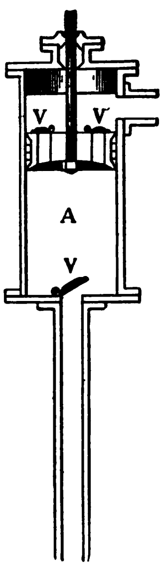

FIG. 147.

(3) If water were used in a barometer instead of mercury, how many cm. tall would the column be under ordinary atmospheric pressure?

(4) How tall would the mercury barometric column be at a depth of 20 m. in water?

(5) What is the weight, in kgm., of the air contained under standard conditions (see Problem 1) in a room 20 m. long, 15 m. wide, and 8 m. high?

215. Pumps for Liquids.—Most pumps for raising liquids depend upon atmospheric pressure as one necessary condition for their operation.

In that form of the *lifting-pump* (often called suction pump) which is shown in Fig. 147, the piston is perforated by two openings, covered at the top by the upward-opening valves *VV* (shown closed in the figure). The piston slides closely, but with little friction, in the barrel, or *cylinder*, *A*. Water is supplied from below by the "suction-pipe," which leads from a reservoir. At the top of this pipe is an upward-opening valve. As constructed for actual use the

valves often have a form very different from that of the simple "clack-valve" shown in the figure.

As here represented the piston is in the act of rising; hence the lower valve is opened by the upward-flowing water and the upper valves are closed by the pressure of the water and atmosphere above them. The height to which water can be lifted, above the lower valve, depends only upon the strength of the pump and the power applied to work it. The lower valve, however, cannot be successfully placed at a height of more than 10.33 m. (about 34 ft.) above the level of the water in the reservoir, when ordinary atmospheric pressure prevails. In practice the valve is not set so high as this.

In *force-pumps* the water is not merely lifted, but it is forcibly driven out of the cylinder. The piston is solid, and the upper valve opens outward, or away from the cylinder, as shown in Fig. 148, into the pipe through which the pump discharges.

With the simple form of pump shown in Fig. 148 the water is thrown out in successive jets. This defect may be remedied by delivering the water through an air-chamber, as in Fig. 149. The

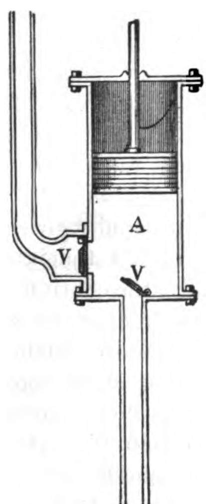

FIG. 148.

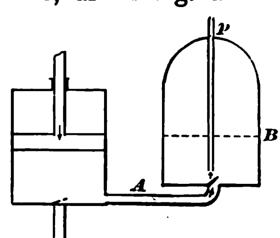

FIG. 149.

water forced in through the tube *A* at each impulse com-

presses the air above the water-level B in the chamber, and this compressed air gradually expanding continues the stream of water through the tube P while the piston is ascending.

216. Air-pumps.—The principles of construction of the air-pump are essentially the same as those of the water-pumps just described; and, indeed, the great blowing-engines, that supply the intense blast of air to many kinds of furnaces for smelting metals, work force-pumps which drive air through tubes, much as an ordinary force-pump drives water. The ordinary air-pump of the laboratory, however, is more nearly like a lifting-pump (Fig. 147), and differs from the latter mainly in the shapes and proportions of its parts and the construction of the valves.

If the valves of a pump used to exhaust the air from some vessel are worked merely by the difference of air-pressure on their opposite faces, the effective action of the pump ceases when the air-pressure in the vessel becomes too small to open the first valve. This often happens. Fig. 150 shows a pump with an *automatic* lower valve at the bottom of the cylinder. This valve is opened by a rod which is lifted a short distance by the ascending piston.

Used in the ordinary way the air-pump cannot give a perfect vacuum. Even if it is so constructed that the valves will be opened and shut automatically, the continued action of the pump can only reduce the fraction of an atmosphere remaining in the exhausted vessel to smaller and smaller values without ever diminishing it to zero. For each upward stroke of the piston can at most remove only the air in the cylinder, leaving the air in the vessel (E, Fig. 150) and connecting-pipe to expand and fill the entire space contained by the cylinder (below the piston), the connecting-pipe, and the vessel.

To make the case as simple as possible, suppose the

cylinder and the vessel to be of equal capacity, and the pipe to be so small that its contents may be neglected. If the volume of air before the first up-stroke were 2 liters, the

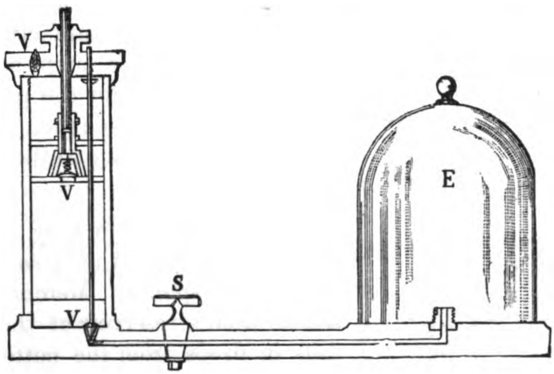

FIG. 150.

volume after that stroke, in vessel and cylinder, would evidently be 4 liters, and the tension would have fallen to one-half what it was at first. The down-stroke would make no change in the pressure within the vessel. After the second up-stroke the tension would become $\frac{1}{2}$ of $\frac{1}{2}$, or $\frac{1}{4}$, and so on.

Pumps for "High Vacua."—To produce the most perfect attainable, or "highest," vacua, mercury-pumps are commonly used, that is, pumps in which a quantity of mercury acts as a piston, driving the air before it. Sometimes the mercury is used in a large slow-moving mass, sometimes as a small stream, or rapid succession of small columns through a tube.

It is obvious that a liquid piston fits better and moves more readily than a solid piston. The special qualities which recommend mercury above other liquids for use in pumps are its density, its peculiarly slight tendency to

evaporation, and its great surface tension, which prevents it from wetting the glass tubes or vessels in which it flows.

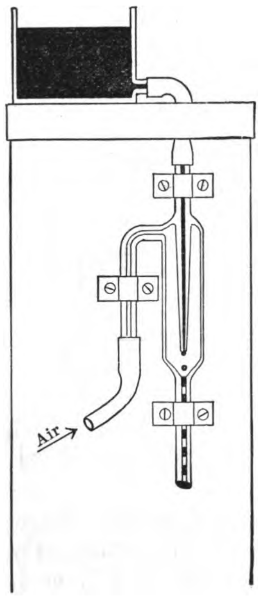

FIG. 151.

Twenty years ago mercury-pumps for high vacua were laboratory luxuries. Now the manufacture of incandescent electric lamps and of "Crookes tubes" for the production of X-rays has made such pumps familiar and necessary to the business world.

Fig. 151 shows in very simple form the upper part of a so-called Sprengel air-pump. Mercury descends from a reservoir at the top, falls in drops from the bottom of the small glass tube within the bulb-shaped space, and as these drops descend through a narrow tube below, they carry between them small quantities of air which have flowed in from the vessel which is being exhausted. The vertical tube through which the drops descend from the bulb must be taller than the barometric column.

QUESTIONS.

(1) The upper surface of the water in a reservoir is 40 m. above a city street. What is the pressure per sq. cm. in the water-main at the surface of the street? *

(2) A man just uses his whole weight, of 70 kgm., to force a smoothly-fitting plug into a hole in the water-main of Problem (1). What is the cross-section of the plug?

(3) A tank 3 m. deep has its sides composed of boards each 20×300 cm. When the tank is full of water what is the total pressure upon one of the boards? *Ans.* 900 kgm.

* In this and the following problems the pressure of the atmosphere is to be neglected unless it is expressly mentioned.

(4) A dam 50 feet long has water standing against its upper side to the depth of five feet. What is the total pressure of the water against the dam, reckoning the weight of a cubic foot of water at 62.43 lbs.?

(5) A cubical block of stone 30 cm. on a side is lowered to the bottom of the tank 3 m. deep, which is full of water. Calculate the total pressure upon each face of the block.

(6) A cubical vessel, each side of which is 10 cm. square, has a tube 1 sq. cm. in cross-section and 20 cm. tall rising from the middle of its top. The tube is open at both ends, and both vessel and tube are full of water. Neglecting atmospheric pressure and weight of vessel and tube, find—

(*a*) How great is the total pressure which the vessel as now filled exerts upon its support.

(*b*) How great is the total pressure exerted against the bottom of the vessel by the water within it.

If these pressures are not equal, explain the difference.

(7) The water-pressure upon an object submerged in water is 433.54 lbs. per sq. cm. How many feet deep is it submerged?

(8) With the barometer at its ordinary height at sea-level (76 cm.) how deep in fresh water must an object be sunk to be exposed to a total pressure, from the atmosphere aad the water together, equal to two atmospheres?

(9) A large cylinder 13 m. long is filled with water and is closed air-tight at each end. It is then placed upright with one end just submerged in water and this end is opened. The barometer reads 76 cm. What will now be the height of the water-column in the cylinder?

(10) A barometer which in the open air reads 76 cm. is introduced into the cylinder and held vertical—

(*a*) With the mercury-surface in its cistern at the level of the reservoir water-surface;

(*b*) 400 cm. above reservoir-level;

(*c*) At the top of the cylinder-column.

Give the barometer-reading in each case.*

(11) A pressure-gauge is arranged so as to register the highest pressure to which it is subjected. The barometer-reading is 76 cm. The gauge is lowered into a lake until it registers a total pressure

* No allowance is made for any pressure of water-vapor at the top of the column. In fact, if such experiments were tried, there would be some air at the top, given up in bubbles from the water.

(from atmosphere and water together) of 5000 gm. per sq. cm. To what depth was it lowered?

(12) A barometer-tube 100 cm. long, filled with air at 76 cm. barometer, is immersed vertically, mouth downward, in mercury (under standard atmospheric pressure) until the air-column is only 40 cm. long. What is the difference of level of the mercury inside and outside of the tube?

(13) A straight uniform tube 1 meter long, open at both ends, is pushed vertically downward into a mercury-tank till all but 10 cm. of its length is submerged. The upper end is then tightly covered and the tube is raised till the air-column in its top becomes 30 cm. long. What is now the height of the mercury column in the tube above the general level of the mercury-surface in the tank, the barometer column being 75 cm. tall at the time?

(14) A tube of uniform cross-section 152 cm. long, closed at one end, is plunged, open end downward, into a mercury-well, and is pushed down until just one half its length is submerged. The tube retains all the air which is held before entering the mercury, and the barometric pressure is 76 cm. How far is the final level of the mercury in the tube below the level of the general mercury-surface?

Solution: Let $x =$ distance required. Then we have final volume $= 76 + x$, and also, as it happens, final pressure $= 76 + x$. By Boyle's law we have $(76 + x) \times (76 + x) = 76 \times 152$; whence

$x = 31.48$. *Ans.*

(15) A quantity of air occupying 5 cu. cm., at the atmospheric pressure prevailing, is admitted to the space above the mercury-column of a barometer, and there expands until it occupies 30 cu. cm. The mercury-column under it is now 62.5 cm. tall. How tall was it before the admission of the air?

(16) In Exercise 33 (the compression of an air-column) the mercury stood at first 4 cm. above the base-board in both parts, and the top of the air-column was 28 cm. above the base-board. Mercury was then poured in until its level in the closed tube was 7 cm. above the base-board. What was the new mercury-level in the open part? (Barometer 76 cm.).

(17) A bubble of air liberated at a depth of 20 meters under water has a volume of 5 cu. cm. What will be its volume at the moment of reaching the surface, if the barometer stands at 76 cm.?

Ans. 14.68 cu. cm.

(18) A balloon contains 1000 cu. ft. of coal-gas (which expands

with diminution of pressure nearly at the same rate as air). It rises 87 ft. from sea-level, the barometer at the time reading 30 inches. What is the new volume of gas in the balloon?

Ans. 1003.3 cu. ft. nearly.

(19) A glass tube of 1 cm. cross-section and 25 cm. long, closed at one end, is fitted with a frictionless piston which slides in it air-tight. The tube is filled with air at 76 cm. barometer, and the piston is then forced in to a point 7 cm. from the closed end. What pressure in grams was applied? *Ans.* 2656 gm.

(20) Soundings have been made at sea to a depth of 7100 m.

(*a*) What would be the pressure per square centimeter at this depth, the density of sea-water being 1.026?

(*b*) If a body of air occupying 400 cu. cm. at the surface were lowered to this depth in a tube closed at one end and kept with the open end down, what would its volume there be?*

Ans. 0.566 + cu. cm.

(21) A vessel is filled with water to a depth of 40 cm. A cylinder of wood 30 cm. long and 100 sq. cm. in area of cross-section, the specific gravity of which is 0.5, extends upward through a hole in the bottom of the vessel, the top of the cylinder being 20 cm. beneath the surface of the water. Show whether the cylinder tends to rise or to fall, and how great a force is required to hold it in its present position.

(22) Let a Mariotte's bottle have two lateral apertures, and let an open tube extend vertically through the stopper at the top of the bottle to a point between the level of the two lateral apertures. The bottle and the tube being originally full of water—

(*a*) State and explain what will take place when the higher lateral aperture only is opened.

(*b*) State and explain what will take place when the lower aperture only is opened.

(23) The inside diameter of the cylinder in Fig. 137 being taken at 14.5 cm., what weight can be supported by the piston when the rubber tube attached contains a column of water 2 m. above the water-level in the cylinder? *Ans.* 33.0 + kgm.

(24) Suppose the same cylinder to be used as a hydrostatic press,

* If the solution of this question were to be given with precise accuracy, it would be necessary to allow for increase of density in the sea-water (due to compression from its own weight) at the rate of about $\frac{1}{100}$ of itself for every 2300 meters.

the rubber tube attached being laid horizontally, and let a plunger of 1 sq. cm. area of cross-section be forced into the open end of the tube by a pressure of 0.5 kgm. in excess of friction. What weight could be sustained by the piston? *Ans.* 82.5 + kgm.

(25) What would the answer be in Problem (23) and in Problem (24) if oil of sp. gr. 0.9 were substituted for water?

(26) (*a*) How high could sulphuric acid of sp. gr. 1.84 be raised above its level in the containing vessel by means of a siphon?

Ans. = 561.4 cm.

(*b*) How high could the lower valve of a lift-pump, used in pumping up this acid, be placed above the level of the acid to be raised?

(27) If the water has all run down to its natural level out of the cylinder and suction-pipe of a lift-pump, it may usually be pumped up again by filling the cylinder with water and working the pump rapidly. Can you explain this?

(28) In an air-pump the capacity of the cylinder is 0.5 liter, and that of the vessel to be exhausted 4 liters. The capacity of the connecting tube may be neglected. How much of the air originally present will remain in the receiver after five double (complete) strokes?

Ans. 55.5 per cent nearly.

CHAPTER XV.

COMPOSITION AND RESOLUTION OF FORCES.

217. Force and Equilibrium: Newton's First Law of Motion.—The word *force*, as commonly used in physics, means a *push* or a *pull*. In the phrases "forces of nature," "natural forces," etc., the word has a different and often a more vague meaning.

The effect of a single force applied to any body is to set the body in motion, or to change in some way the motion that the body already has. But if two or more forces are applied to the body at once, they may neutralize each other in such a way that the body will act as if no force were applied to it, provided we may neglect any change of shape or size that the body undergoes under the action of the forces. Forces so neutralizing each other are said to be in *equilibrium* with each other.

For a very long time it was supposed that a body under the influence of no force, or of a set of forces equivalent, taken together, to no force, must come to rest and remain at rest. It was supposed that motion could be maintained only by a continual application of force. It was supposed, for instance, that the planets must be *driven* or *carried* around the sun. All this was an error.

It is true that the moving bodies with which we are most familiar do tend to come to rest. The error lay in supposing that no force is acting upon them while they are coming to rest. Experiment shows that when better and better

means are taken for lessening friction, resistance of air, etc., moving bodies show less and less tendency to come to rest, and as a result of all experience and reasoning it is now believed that no body ever comes to rest except because of some obstruction, some application of force which stops it. In fact, *the natural behavior of any body, the behavior of a body not acted upon by forces outside itself, is to remain at rest if at rest, to move with unchanging motion if in motion.* This is the substance of Newton's *First Law of Motion.*

The subject of setting bodies in motion or changing their motion will be taken up in Chapter XVII. We shall in the present chapter be dealing with cases of *equilibrium;* that is, *cases in which bodies are at rest or, as nearly as may be, in uniform, unchanging motion.* We shall in studying these cases have in view several objects, one of which is to determine what relations must exist in a set of forces in order that they may neutralize each other in their action upon the body to which they are applied. These relations are called the *conditions necessary for equilibrium,* or, simply, the *conditions of equilibrium.*

218. Illustrations of Equilibrium.—Suppose a lead bullet to be suspended by a slender thread a meter or more in length, so as to hang at rest a few centimeters above the top of one of the laboratory tables. If the air is still and the building is not jarred, by footsteps or otherwise, the bullet will remain motionless with reference to any point on the table-top; for instance, the head of a pin stuck into the table immediately beneath it. There are, however, forces at work upon the bullet. Burn off the thread, and the bullet will fall, impelled towards the earth by what we call the force of gravity. As long as the bullet remains suspended and at rest the various forces acting upon it must antagonize each other, so as mutually to destroy each

other's effect and thus produce a balanced condition in the bullet. Many forces beside the pull of the thread contribute in greater or less degree to this result. The bullet, as we know, is acted on by the pressure of the air from all sides, and is also slightly attracted by surrounding objects in the room, by more distant objects on the earth's surface, and even by every one of the heavenly bodies, especially the moon and the sun.

Other illustrations of equilibrium of forces will readily suggest themselves to the student. Floating bodies owe their stability to the joint action of gravity and the upward thrust or buoyancy of the liquid. A picture hung against the wall is supported in position by the simultaneous action of gravity, of the two segments of the picture-cord on either side of the nail or hook from which it is suspended, and of the push of the wall against the lower part of the picture-frame. A balloon held fast to the ground by a rope is subject to the force of the rope and to that of gravity, on the one hand, and to the buoyant effect of the air, on the other hand. The spherical form of the balloon is maintained by the outward pressure of the contained gas acting against the silken envelope which contains it, and the resistance of the silk would not be sufficient to prevent the balloon from bursting, if it were not for the pressure of the surrounding air upon its outer surface.

219. Description and Representation of Forces. — In order to describe a force completely or to calculate what effect will be produced by it, we must know the *amount*, the *direction*, and the *position*, or *line of action*, of the force.

The directions of forces with reference to each other may be represented by lines, and the lengths of the lines may represent the relative magnitudes of the forces. One end of a line will represent the point of application of the force, and an arrow-head upon the line will indicate

whether the force is directed towards the right or towards the left along it. Any scale may be chosen, as, for instance, one centimeter to the kilogram.

220. Bodies to be Acted Upon.—It would be most desirable, in experimenting upon the effect of forces, to have as the object to be dealt with some body of matter entirely free from all external forces, except those to be applied by the experimenter. This, however, is impossible. The best that we can do is to arrange some body in such a way that it will move with as much freedom as possible in all directions, or, at least, in the directions which the forces to be applied would tend to make it follow. It is easy to secure considerable freedom of motion in directions nearly or quite horizontal. We can do this by hanging up the body by a long suspension, by floating it in some liquid, or by supporting it on easy-running rollers. For some purposes we may use a body so light that its weight and the resistance it encounters from friction may be disregarded in comparison with the forces to be applied.

In all our discussions of force we shall, unless the contrary is stated, suppose the body dealt with to be so rigid as not to suffer change of shape under the action of the forces.

221. Review.—The student already knows, from his every-day experience and from previous experiments (Chapters IV and V), many important facts concerning the equilibrium, or balancing, of forces. He knows, for instance,

a. That two forces neutralizing each other, without the help of any other force, must be equal, opposite in direction, and lying in the same straight line.

b. That, if the line of action of a force remain unchanged, the point of application may be moved to any new position in this line without changing the effect of the force (see § 48).

c. That when three parallel forces in equilibrium are applied to a lever or pulley, one of the three being exerted by

the fulcrum, or pivot, this force is equal to the sum of the other two and is opposite to them in direction, and the other two must be such and so applied that the power $\times$ *the power-arm* = *the weight* $\times$ *the weight-arm* (Chapter IV).

d. That if three forces, all applied to the same point, be in equilibrium, the lines of the forces must all lie in one plane, and the magnitudes and directions of the forces must be such that, if any two of the lines representing them be taken as the sides of a parallelogram, the third line will be equal to and in the same straight line with the diagonal of this parallelogram (see Exercise 13).

We need to discuss and illustrate some of these propositions more fully, but we can do this better after becoming familiar with a few words that we have used little or not at all before.

222. Definitions: Resultant, Components, etc.—It is a familiar fact that two or more forces acting upon a body may, in many cases, be replaced by a single force; that is, in many cases, a single force can be found whose unaided action upon the body would produce upon it the same effect that is produced by the joint action of the given forces. This one force is called the *resultant* of the combination to which it is equivalent.

The several forces of the combination are called the *components* of the resultant.

The process of finding the resultant of a given combination of forces is called the *composition* of forces.

The process of finding a set of components to equal a given resultant is called the *resolution* of forces.

The force which will exactly neutralize a combination of other forces is called the *equilibriant* of that combination.

It is evident from what precedes that the resultant and the equilibrant of a combination are exactly equal and opposite to each other.

These definitions may be illustrated by a diagram. Let the lines A, B, and C, in Fig. 152, represent three forces which are in equilibrium with each other. Then either one of the three is the equilibrant of the other two, since it neutralizes their joint effect.

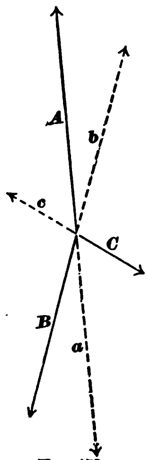

FIG. 152.

A is the equilibrant of B and C.
B " " " A " "
C " " " " " B.

The dotted lines a, c, b represent forces equal and opposite to A, B, C, respectively. It is evident that A, which exactly neutralizes B and C together, would exactly neutralize a alone, which is therefore seen to be the exact equivalent, or resultant, of B and C. So b is the resultant of A and C, and c is the resultant of A and B.

The same definitions apply as well to parallel forces as to those which meet at a point.

Thus in Fig. 153, representing three parallel forces, A, B, and C, in equilibrium,

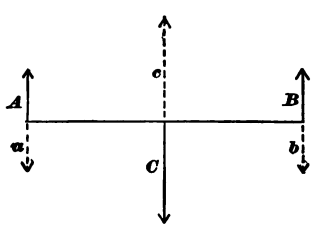

FIG. 153.

A is the equilibrant of B and C,
B " " " A " " .
C " " " " " B.

Moreover,

a	would	be	the	resultant	of	*B*	and	*C*,
b	"	"	"	"	"	*A*	"	"
c	"	"	"	"	"	"	"	*B*.

Parallel Forces.

The following problems make application of proposition *c* of § 221:

PROBLEMS.*

(1) To a rod extending east and west are applied two forces, one of 10 lbs., the other of 5 lbs., both due south, the points of application being 18 ft. apart. State the magnitude, direction, and point of application of their equilibrant; of their resultant.

(2) If the two forces applied to this rod were 6 lbs. due north and 15 lbs. due south, the points of application being 12 ft. apart, state what would be the magnitude, direction, and point of application of the equilibrant; of the resultant.

(3) Let two forces, one of 10 lbs. due north, the other of 10 lbs. due south, be applied to the rod, the points of application being 6 ft. apart. Find, if possible, the magnitude, direction, and point of application of the equilibrant; of the resultant. (A set of two equal forces like those just described, applied to the same body in exactly opposite directions, but not in the same line, constitute what is called in mechanics a *couple*. The student should bear carefully in mind what he here learns concerning the equilibrant and the resultant of a couple.)

Having now learned how to calculate the resultant of two parallel forces, we can find the resultant of any number of parallel forces, even when they do not lie in one plane. We have merely to find the resultant of any two of a given set and replace the two by this resultant, thus reducing the number engaged by one. This process is often

* In these and many other problems upon parallel forces conclusions reached by the student can often be put to the test by means of spring-balances attached by strings to the bar running across the top of Apparatus No. 74, each foot mentioned in the problems being represented by one centimeter on the bar.

tedious, but continued, it gives at last a single resultant for the original set, unless it happens that the set is equivalent to a *couple* such as we have already discovered. We shall presently (§ 226) find an easier method of dealing with such cases when the number of forces is large.

(4) To the rod already mentioned let three forces be applied, one of 5 lbs. due north, one of 10 lbs. due south, one of 10 lbs. due north, the points of application being respectively 4, 8, and 12 ft. from the western end of the rod. State the magnitude, direction, and point of application of the equilibrant of these three forces; of their resultant.

(5) Let the forces be 6 lbs. north, 8 lbs. south, 10 lbs. north, and 15 lbs. south, the points of application being respectively 4, 8, 12, and 16 ft. distant from the western end of the rod. State the magnitude, direction, and point of application of their equilibrant; of their resultant.

(6) A board one foot square is placed horizontal and is loaded at the northeast corner with 10 lbs., at the southeast corner with 10 lbs., at the southwest corner with 20 lbs. Find the position where a single supporting point must be placed and the load which this point must bear, the weight of the board being disregarded.

The six preceding problems are cases in the composition of parallel forces. It is easy to apply proposition *c* to the resolution of forces also.

(7) A force of 20 lbs. acts due north from a point 8 ft. from the western end of the rod. State the magnitudes, directions, and points of application of three pairs of forces, each one of which pairs would just neutralize this force.

Before going further we shall need a few more definitions, which will be given in the following articles.

223. Definitions of Translation and Rotation; Examples.—Motion of *translation* is motion from one place to another; *rotation* is spinning or whirling motion. An ice-boat, sailing straight forward on the ice, or an ordinary hotel-elevator moving vertically upward or downward on its

guides, illustrates simple motion of translation.* The revolution of a fly-wheel or of a circular saw illustrates well the properties of rotation uncombined with motion of translation, while a top which is spinning and meanwhile travels about, or a moving car-wheel, furnishes an example of combined translation and rotation.

The characteristic feature of pure motion of translation is that all points of the moving object travel in the same direction and with the same velocity. All straight lines rigidly connected with the body will remain parallel to their original direction during translation. Suppose a right cylinder to have a number of parallel straight lines ruled on its convex surface, at right angles to the bases, and suppose a number of other lines to be ruled on the bases. If the cylinder is rolled along on a plane surface, the ruled lines on its convex surface may at all times remain parallel to their original direction, but those on the ends will form varying angles with their original directions. The only way in which the parallelism of every line with its original direction could be maintained, with the cylinder in motion, would be to transport the cylinder without allowing it to roll or its axis to change its direction.

If a body rotates in such a way that some straight line drawn through it does not change its position, this line is called the *axis of rotation*. Frequently what is called the axis of rotation is outside the body itself, being a line so placed that if the body were rigidly connected with it the line would not have to change position in consequence of the motion of the body. Thus, if we regard our earth as being at rest, and the moon as revolving in a circular path about it, turning, as the moon does, always the same face toward the earth, the axis of the moon's rotation is a

* In either of these cases, if we were to take into account the earth's motions, the motion of the object in question through space would describe a complex curve.

straight line passing through the centre of the earth and at right angles always with the straight line connecting the centre of the earth with that of the moon.

A set of forces applied to a body may tend to translate it or to rotate it or to do both. If the set of forces is in equilibrium, it tends to produce neither translation nor rotation.

224. Moment of a Force.—The student already knows that the effectiveness of a force in producing rotation about a given axis depends not merely upon the magnitude of the force, but also upon the distance of the line of the force from the axis of rotation, increasing as this distance increases. In fact, as we shall presently see, the importance, or *moment*, of a force with reference to rotation about a given axis, or the measure of the value of the force for the production or prevention of rotation about the given axis, is obtained by multiplying the number representing the force by that representing the perpendicular distance between its line of direction and the axis of rotation.

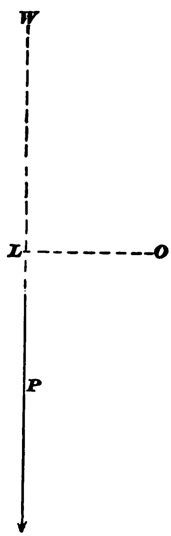

FIG. 154.

Let the line P (Fig. 154) represent a certain force, and let O be the point where the required axis of rotation, perpendicular to the paper, is cut by a perpendicular drawn from the line of P (extended). Then $P \times OL$ is called the *moment* of P with reference to the given axis through O. Moments whose tendency, with respect to a given axis, is to produce rotation in the same direction with the hands of a clock, *clockwise rotation*, we shall call *positive* with respect to this axis, and those of opposite tendency *negative*. In Fig. 154 the moment of P with respect to the axis through O is negative.

225. Illustrations of Moments.—We have, in fact, been calculating moments whenever we have taken *power* × *power-arm* or *weight* × *weight-arm*, in dealing with levers or pulleys, but we have not had the *word* moments. We shall find this word to be of great use in dealing with cases where there are more than three forces, especially when these forces are not all parallel to each other. Indeed, if we discuss a very simple case, like that of a common lever, with the idea of moments in mind, we shall find out some things that have before escaped our notice.

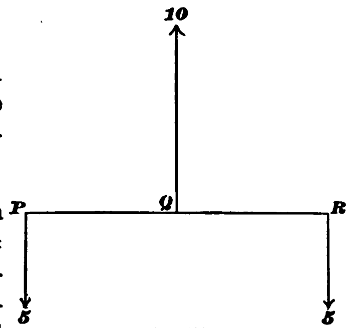

FIG. 155.

Let us take the case shown by Fig. 155, in which $PQ = QR = 2$. Let us first consider the point Q as representing the fulcrum, or axis, and calculate the moments of each force with respect to this axis. We have, taking the forces in their order from the left,

$$\left.\begin{array}{rcr} -(5 \times 2) &=& -10 \\ 10 \times 0 &=& 0 \\ +(5 \times 2) &=& 10 \end{array}\right\} = 0.$$

If we take P for the axis, the moments are

$$\left.\begin{array}{rcr} 5 \times 0 &=& 0 \\ -(10 \times 2) &=& -20 \\ +(\ 5 \times 4) &=& +20 \end{array}\right\} = 0.$$

Take the point R. The moments become

$$\left.\begin{array}{rcr} -(\ 5 \times 4) &=& -20 \\ +(10 \times 2) &=& +20 \\ 5 \times 0 &=& 0 \end{array}\right\} = 0.$$

Take a somewhat less simple case of equilibrium, repre-

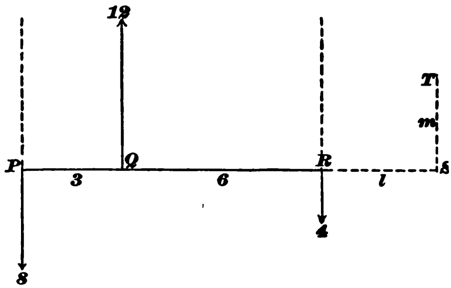

FIG. 156.

sented in Fig. 156. With respect to P we have the moments

$$\left.\begin{array}{rcl} 8 \times 0 &=& 0 \\ -(12 \times 3) &=& -36 \\ +(4 \times 9) &=& +36 \end{array}\right\} = 0.$$

With respect to Q,

$$\left.\begin{array}{rcl} -(8 \times 3) &=& -24 \\ 12 \times 0 &=& 0 \\ +(4 \times 6) &=& +24 \end{array}\right\} = 0.$$

With respect to R,

$$\left.\begin{array}{rcl} -(8 \times 9) &=& -72 \\ +(12 \times 6) &=& +72 \\ 4 \times 0 &=& 0 \end{array}\right\} = 0.$$

But we need not confine the axis to positions indicated by P, Q, or R. Let us imagine an axis perpendicular to the plane of the forces to pass through this plane at a point lying in the extension of PR, at a point S at a distance l to the right from R. The moments with respect to this axis are

$$\left.\begin{array}{rcl} -[8 \times (9+l)] &=& -72 - 8l \\ +[12 \times (6+l)] &=& +72 + 12l \\ -(4 \times l) &=& -4l \end{array}\right\} = 0.$$

Now as l can have any magnitude, and can be either negative or positive, without destroying this equation, it is shown that with the given system of forces the algebraic sum of the moments is zero for all axes, perpendicular to the plane of the forces, that pierce the line PR or its extension.

But imagine an axis to pass in the same direction through a point T at a distance m from S, on a line drawn from S parallel to the forces. The moments with respect to this axis are, as a little consideration of the definition of moments will show, precisely the same as the moments with respect to the axis through S. Now as m, like l, may have any magnitude and be either positive or negative, we see that *the algebraic sum of the moments of the given set of forces is zero with respect to any axis whatever perpendicular to the plane of the forces.*

If we were to try other sets of three or more (see § 226) parallel forces in equilibrium we should find the same statement to be true of all of them.

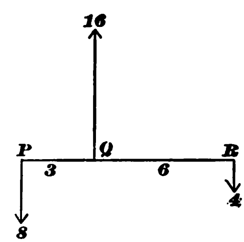

FIG. 157.

Let us now consider the set of forces shown in Fig. 157, which we know is *not* a case of equilibrium.

The moments with respect to Q are

$$\left.\begin{array}{r} -(8 \times 3) = -24 \\ 16 \times 0 = \quad 0 \\ +(4 \times 6) = +24 \end{array}\right\} = 0.$$

But the moments with respect to P are

$$\left.\begin{array}{r} 8 \times 0 = \quad 0 \\ -(16 \times 3) = -48 \\ +(\ 4 \times 9) = +36 \end{array}\right\} = -12,$$

and with respect to R,

$$\left.\begin{array}{r} -(\ 8 \times 9) = -72 \\ +(16 \times 6) = +96 \end{array}\right\} = +24.$$

If we should try other cases of non-equilibrium, we should find a like result always. *The sum of the moments in cases of non-equilibrium may be zero with respect to particular axes, but cannot be so with respect to all axes perpendicular to the plane of the forces.*

226. More than Three Parallel Forces.—A proposition very similar to proposition *c* of § 221 holds for cases of any number of parallel forces, however great that number may be. This enlarged proposition, which is here given without proof, may be put into the following form:

Proposition (*e*).—*In order that any number of parallel forces shall be in equilibrium, as a whole, it is only necessary that*—

1. *The algebraic sum of the forces, those in one direction being called positive and those in the opposite direction being called negative, shall be zero;*
2. *The algebraic sum of the moments of the forces, taken with respect to some axis perpendicular to the plane of the forces, shall be zero.*

To illustrate the use of this proposition, let us find the equilibrant, and so the resultant, of the set of forces described in Fig. 158.

We will call forces acting downward on the page positive and those acting upward negative. Then we must, in order to give the proper signs to the moments, call distance towards the right positive, towards the left negative.

Applying condition (1), we find, x being the unknown magnitude of the equilibrant (§ 222),

$$x + 4 - 6 + 8 - 10 = 0.$$

Hence $x = +4$, a downward force.

It remains to find the point of application of this equilibrant. This we can do by use of condition (2). Calculat-

ing the moments of the forces, including the equilibrant, with respect to the point P, and putting y for the unknown distance of the equilibrant from this point, we have

$$\left.\begin{array}{r} 4 \times 0 = \quad 0 \\ (-\ 6) \times (+2) = -12 \\ (+\ 8) \times (+4) = +32 \\ (-10) \times (+6) = -60 \\ (+\ 4) \times y \quad = \quad 4y \end{array}\right\} = 0.$$

We thus find $4y = 0 + 12 - 32 + 60 = 40$; and $y = 10$, to the right from P.

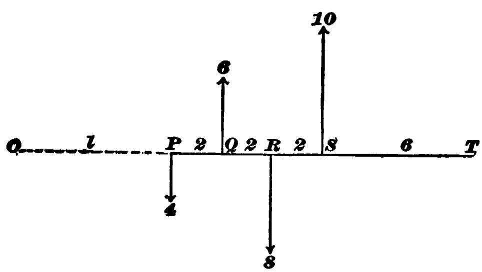

FIG. 158.

Suppose we were to choose R, in Fig. 158, as the point for the imagined axis to pierce. We should have

$$\left.\begin{array}{r} (+\ 4) \times (-4) = -16 \\ (-\ 6) \times (-2) = +12 \\ (+\ 8) \times \quad 0 = \quad 0 \\ (-10) \times (+2) = -20 \\ (+\ 4) \times (y') \quad = \quad 4y' \end{array}\right\} = 0.$$

Then $4y' = +16 - 12 + 0 + 20 = 24$, or $y' = 6$.

But 6 to the right from R is equal to 10 to the right from P, so that the result reached is the same as before.

Let us now take the point T as the intersection of the plane by the imagined axis.

The moments are

$$\left.\begin{array}{l}(+\ 4)\times(-12)=-48\\(-\ 6)\times(-10)=+60\\(+\ 8)\times(-\ 8)=-64\\(-10)\times(-\ 6)=+60\\(+\ 4)\times y''\quad\ \ =+4y''\end{array}\right\}=0.$$

Whence $4y'' = 48 - 60 + 64 - 60 = -8$, and $y'' = -2$, which gives the same point of application for the resultant that has been found before.

In fact, the result will be precisely the same whatever point in the plane is chosen to mark the position of the axis. With the "checker-board" (No. 74) rolling on steel balls the class can test this result, determining whether the equilibrant, as found by calculation, does really balance the original set of forces.

The resultant is at once known as soon as the equilibrant is found, being simply an equal but opposite force applied at the same point.

PROBLEMS UNDER PROPOSITION (e), § 226.

(1) Weights of 20, 50, and 100 gm. respectively are suspended from a horizontal meter-rod at distances of 40, 60, and 80 cm. respectively from one end. The rod * itself weighs .... gm. How far from the end mentioned must a supporting point be placed in order that the whole may balance upon it? (With regard to the influence of the weight of the rod, see § 50.)

(2) The "checker-board" (No. 74) being placed with the cross-bar east and west, forces are applied as follows:

5	cm.	from	the	west	end	7	kgm.	north,
10	"	"	"	"	"	4	"	south,
15	"	"	"	"	"	6	"	north,
20	"	"	"	"	"	3	"	south.

Calculate the magnitude, direction, and point of application of the

* The weight of some actual rod should be given in order that the conclusion may be tested by experiment.

equilibrant of this set of forces, and test the correctness of the conclusion by experiment with the apparatus.

(3) The meter-rod, with its suspended load, as described in Problem 1, is to be supported by two points: one, *A*, 5 cm. distant from the before-mentioned end of the rod; the other, *B*, 95 cm. distant from the same end. Calculate the load borne by each of these points, and test the conclusion by experiment, using two platform balances (No. 71). (Reckon all the moments with respect to the point *A*. The moment of the supporting force *at A* will thus be zero, whatever the force itself. The moment of the other supporting force, at *B*, must balance all the other moments. So the magnitude of the force at *B* is found. Then the magnitude of the force at *A* can be found by (1) of Proposition (*e*).)

227. Anatomical Lever.—Doctors have disagreed as to how the human foot, used to *lift* the body, should be classified among levers. The interest of the discussion lies in the question whether the muscles attached to the heel pull with a force greater or less than the weight of the body when they lift it.

At first sight the foot appears to be a lever of the second class (§ 55), with the "ball" of the foot for the fulcrum, the lifting force, or *power*, being applied at the heel and the weight, or *load*, at the ankle-joint. Taking this view, one might infer that the force applied by the heel muscles is less than the weight of the body.

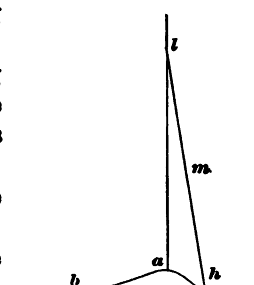

FIG. 159.

But the case is really not quite so simple, as Fig. 159 will show, where *b* is the ball of the foot, *a* the ankle, *h* the heel, and *m* the heel muscles. These muscles are attached at their upper end to the leg, so that in pulling *up* on *h* they must pull *down* on *l*, thus making the downward force at *a* greater than the mere weight of the body.

If we call the weight *w* and the pull of the muscles *p*, we

shall have the downward force at a equal to $w + p$.* So, if we still regard the foot as a lever of the second class with the fulcrum at b, we shall have

$$p \times \overline{bh} = (w + p) \times \overline{ba},$$

or

$$p = w\frac{\overline{ba}}{\overline{bh} - \overline{ba}} = w\frac{\overline{ab}}{\overline{ah}},$$

where $\overline{ab}$ and $\overline{ah}$ stand for the horizontal extents of ab and ah.

But there is another way of looking at the case. A physiologist † points out that the muscular action is the same as if a man, lying on his back with his legs straight in the air, were to lift on the ball of his foot a weight equal to his own. With this view we regard the foot as a lever of the first class (§ 52) with the fulcrum at a, and we have at once

$$p \times \overline{ah} = w \times \overline{ab}, \quad \text{or} \quad p = w \times \frac{\overline{ab}}{\overline{ah}},$$

the same result as before.

228. Moments of Couples.—The student has probably learned already (p. 249) that a *couple* has no single-force equilibrant or resultant, but it is well for him to know something more about couples.

Let Fig. 160 represent a couple applied to a body, and let us consider how the influence of the given couple could be neutralized. If there is a fixed *pivot* at the point Q, we have as the moments, with respect to this pivot, of the two forces making the couple

$$\left.\begin{array}{r} -(8 \times 6) = -48 \\ -(8 \times 6) = -48 \end{array}\right\} = -96.$$

* Not strictly, for w and p are not quite parallel to each other.

† Dr. G. W. Fitz.

This moment of -96 can be neutralized in a great variety of ways. For instance, if we apply at P a force of 16

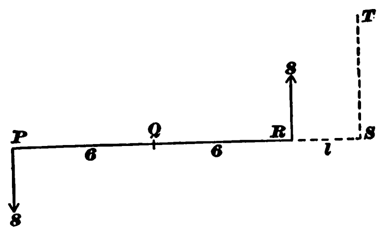

FIG. 160.

upward (on the page), its moment with respect to Q is $+96$, and we have equilibrium.

It may look as if we had here balanced a couple by means of a single force, but we have not. The rule that the algebraic sum of the *forces* must be zero for equilibrium shows that the *pivot* must now exert a force of 16 downward. The fact is that we now have *two* couples, an 8-lb. couple with a distance of 12, and a 16-lb. couple with a distance of 6, the former having a moment of -96, the latter a moment of $+96$. If we had applied a force of 32 upward at a point half-way between P and Q, the pivot would have exerted a force of 32 downward and the moment of this 32-lb. couple would have been $+96$. There is no end to the combinations by means of which we can balance the original couple, but in every case we should find the additional forces equivalent to a couple of moment $+96$.

Another fact is to be noticed. The moment of the 8-lb. couple is, with respect to the point P,

$$\left.\begin{array}{rl} (8 \times 0) = & 0 \\ -(8 \times 12) = & -96 \end{array}\right\} = -96;$$

with respect to R,

$$\left.\begin{array}{rl} -(8 \times 12) = & -96 \\ (8 \times 0) = & 0 \end{array}\right\} = -96;$$

with respect to S, at a distance l from R,

$$\left.\begin{array}{ll} -(8 \times (12 + l)) = & -96 - 8l \\ +(8 \times l) \qquad\quad = & \qquad +8l \end{array}\right\} = -96.$$

With respect to T, on a line drawn from S at right angles with RS, the moments are the same as with respect to S.

In short, *the moment of a couple is the same and in the same direction whatever the position of the axis, provided this axis is perpendicular to the plane of the forces, and this moment is equal to one of the forces multiplied by the perpendicular distance between the lines of the forces.*

The distance just mentioned is called the *arm* of the couple.

Two Sets of Forces at Right Angles.

In the Exercise which follows we shall consider experimentally the case of several forces all in one plane but not all parallel, being grouped in two sets which make right angles with each other. We might work out the laws for such a case from the experiments and discussions that precede, but it is better to reach them by more direct methods. The case is of great practical importance, as we shall see.

EXERCISE 35.

FOUR FORCES AT RIGHT ANGLES IN ONE PLANE.

Apparatus: Four spring-balances (No. 50). No. 74 (without the cross-bar). Four beds (No. 75) for the balances. Hard-twisted cord capable of holding at least 10 kgm., but not over-bulky.

Using the balances carefully, and having the iron pegs set well down into the holes, so that the strings must be near the surface of the board, find by experiment a case of equilibrium, with one of the forces acting north, one south, one east, and one west, no two strings lying in the same line.

Record this case of equilibrium thus:

Hole.	Force.	Direction.	
....	....	....	Equilibrium.
....	....	....	
....	....	....	
....	....	...	

Find and record at least two other cases of equilibrium, varying the arrangements as much as practicable, but observing the conditions above mentioned.

Compare the magnitudes of the forces in each of these cases of equilibrium. Is there any fixed relation between the magnitude of the force north and that of the simultaneous force south? between that of the force east and that of the force west? between that of the force north and that of the force east? etc.

Calculate, in at least one of the cases of equilibrium, the moments of the various forces with respect to some one of the pegs used, and compare the sum of the positive moments with the sum of the negative moments. Do the same with respect to each of the other pegs used in this case of equilibrium, and with respect to one or two other points on the board. State carefully any general conclusion inferred from the Exercise. Has the Exercise anything to do with "couples."

229. Further Applications of Couples.

Example 1. Let us apply the lesson of Exercise 35 to the case of a horizontal bar abP, supported and loaded as in Fig. 161, the weight of the bar itself being disregarded.

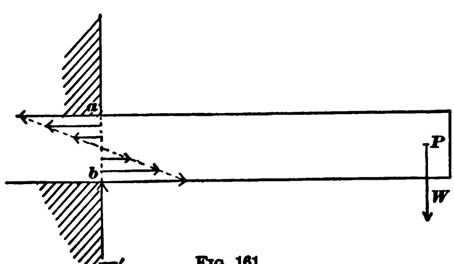

FIG. 161.

We have first the weight W applied downward at the point P. We know from Exercise 35 or otherwise that this

must be offset by an equal force applied to the bar upward. This force must be exerted in the plane of the section *ab*, for that is the only place where the external bar is touched by anything capable of supporting it. The upward force is represented by the arrow W'.

W and W' make a positive couple with an arm from *ab* to P. This must be balanced by a negative couple of equal moment. The only agents that can apply this couple are the parts of the bar itself at the left of the section *ab*. We have already learned (§ 177) that in such a case the particles above the centre of *ab* are stretched and those below the middle compressed, the outer parts being stretched or compressed more than those nearer the centre. In this condition of things the forces exerted upon the external part of the bar *abP* by the imbedded part may be represented, as in the figure, by a number of horizontal arrows. The horizontal forces directed toward the left have a single resultant, those directed toward the right an equal resultant. These two resultants are equivalent to a negative couple, and thus the positive couple WW' is balanced.

Limit of Strength of the Bar.

Influence of length.—The parts of the bar at the section *ab* can bear a certain amount of stretching and compression, and no more. The moment of the couple they can exert has a certain limit. If the moment of WW' becomes greater than this limit the bar breaks. The moment of WW' is $W \times \overline{bP}$. Evidently, the greatest load, W, which the bar can bear without breaking is inversely proportional to the length $\overline{bP}$, the horizontal distance from *ab* to P. (See § 79.)

Influence of thickness.—If the bar is made twice as thick, as in Fig. 162, we may draw twice as many arrows to represent the molecular forces, but the outside arrows have the same limit of length as before; for the limit of stretch-

ing or compression of the substance is the same as in the smaller bar. The average length of the arrows is the same, at the breaking condition, in the thick bar as in the thin one.

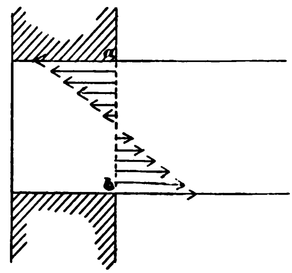

FIG. 162.

The right-hand and the left-hand resultants are twice as great as in the thin bar, and the points of application of these resultants are twice as far apart as in the thin bar. Hence the moment of the molecular couple just before breaking is four times as great in the thick bar as in the other. Doubling the thickness multiplies the strength by *four* (see § 179).

Influence of width.—Doubling the width of the bar

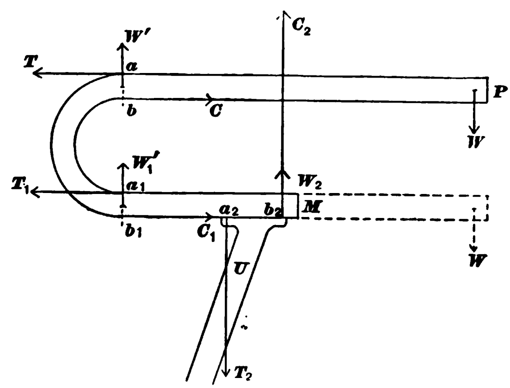

FIG. 163.

would double the resultants of the molecular forces, but would leave the lever-arm of the couple unchanged in length. Hence it would merely double the strength of the bar.

Example 2. Let Fig. 163 represent the saddle-post and bar of a bicycle.

The weight of the rider is applied just over P, and is represented by W. Let us consider what forces must be applied to the part abP by the other part of the bar at the cross-section ab.

There must be a force W', upward, equal to W. W and W' make a positive couple, which must be balanced by a negative couple exerted by the molecular forces, the resultants of which may be represented by T and C.* T corresponds to a tension, a *pull* in the upper part of ab, and C to a compression, a *push* in the lower part of ab.

The forces applied to the part abM by the part abP of the bar would be represented by arrows exactly equal and opposite to W, T, and C, respectively.

The forces applied to the part a_1b_1P by the part a_1b_1M at the cross-section a_1b_1 are represented by the arrows T_1, C_1, and W_1'.

For the equilibrium of the bar as a whole we must find a balance between the weight W and the forces applied to the bar by the post U. These may be represented by the arrows W_2, T_2, and C_2.

The forces W_2 and C_2 may be regarded as united, and there is no necessity of considering *couples* in this case. The law of the lever can be applied. The effect on U is the same as if a straight bar, indicated by the dotted lines, were used.

PROBLEM.

If the post in Fig. 163 were cut through at the top, so that its action upon the bar would be confined to the points a_2 and b_2, how great would be the stretching force at a_2 and how great the compressing force at b_2, if the distance a_2b_2 were 1 inch, the horizontal distance of P to the right of b_2, 2 inches, and the weight, W, 150 lbs.?

Example 3. In Fig. 164 the inclined line M represents

*** T and C are drawn on a smaller scale than W and W' in Fig. 163.**

a man supported by a scaling-ladder L, which hangs upon a window-sill S. A horizontal chain reaches from the man's belt to the hook of the ladder.

The forces applied to the man are his weight,* M_1, downward, and an equal force, M_2, upward, the two making a negative couple; M_3 horizontal toward the right and M_4 horizontal toward the left, making a positive couple. These two couples being equal and opposite, the man is in equilibrium.

The forces applied to the ladder are L_1 and L_2, a negative couple; L_3 and L_4, another negative couple; and L_5 and L_6, a positive couple.

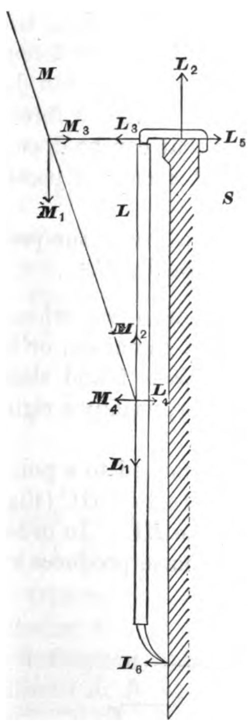

FIG. 164.

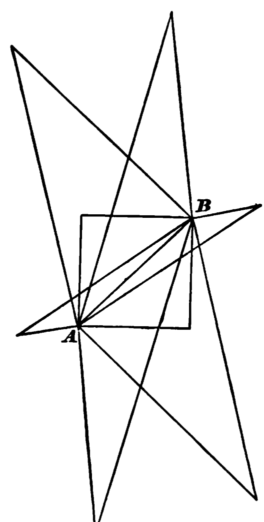

FIG. 165.

* Fig. 164 assumes that the man's centre of gravity is on a level with his belt. This need not be the fact.

Forces whose Lines Meet at One Point.

230. Resolution of Forces.—We have already, in Chapter V, had simple cases of the composition of two crossing forces to find their resultant. The converse operation, namely, finding two or more forces whose effect shall equal that of a single given force, is often necessary.

Since the force to be resolved is to be considered as the diagonal of a parallelogram, of which the required forces form the sides, the number of pairs of forces that can be found, each pair of which is equivalent to the given force, is equal to the number of parallelograms that can be drawn with this force for a diagonal. The number of such parallelograms is unlimited.

The annexed diagram, Fig. 165, shows a few of the possible parallelograms formed upon the diagonal *AB*.

It is often important to resolve a force into two others, one of which shall have a certain definite direction, or to resolve it in such a way that the two forces found shall form a certain definite angle with each other, usually a right angle.

Suppose, for example, that a horse is attached to a point *O* at the front of a car and pulls in the direction *OC* (Fig. 166), while the rails extend in the direction *EF*. In order to find what useful result the pull of the horse produces we

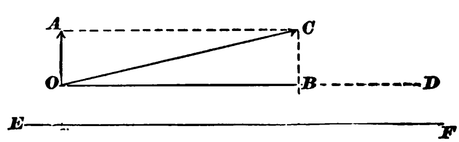

FIG. 166.

must resolve it into two forces, one of which is parallel to the line of the rails and the other perpendicular to this line. Complete a parallelogram with diagonal *OC*, and with sides

OB and *OA*, respectively parallel and perpendicular to the line *EF*. If *OC* represents the whole pull of the horse, *OB* is the useful component, and *OA* is useless, or, because of the friction it causes against the rails, worse than useless.

231. Resultant of Forces in Several Planes.—We are, in proposition of § 221, dealing with three forces only, three forces lying in one plane; but the power we have thereby gained of finding the resultant of any two forces meeting at a point enables us to find the resultant of any number of forces meeting at a point, forces not necessarily confined to one plane, but pointing in any direction. We first find the resultant of any two of the forces and replace the two forces by this resultant, thus reducing by one the number of forces. The same process continued leaves at last a single force, which is the resultant of all those originally given.

For example, let us find the resultant of the five forces *A*, *B*, *C*, *D*, *E* (Fig. 166), all applied at the point *O*.

The resultant of A and B is R_1;
" " " R_1 " C " R_2;
" " " R_2 " D " R_3;
" " " R_3 " E " R_4;

which last is the resultant of all the original forces, *A*, *B*, *C*, *D*, *E*.

Observe that the outline of the resulting figure, from *O* around to *P*, the extremity of R_4, in the direction of its construction, is made up of lines equal and parallel to the arrows *A*, *B*, *C*, *D*, *E*. This leads to a general and very convenient rule for finding the resultant of any number of forces meeting at a point. *Put together the arrows representing the forces, tail to tip, without change of direction, each arrow being used but once. Then draw a straight line from the tail of the first arrow to the tip of the last.* *This*

line represents in magnitude and direction the resultant required.

When this resultant is zero, that is, when the set of forces

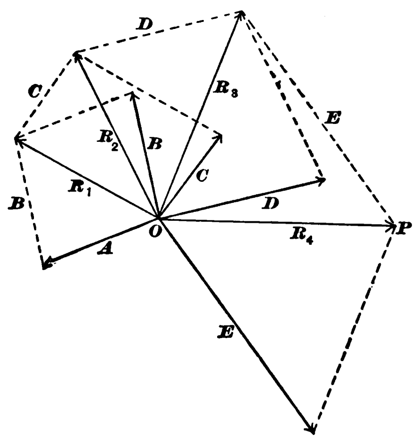

FIG. 167.

is in equilibrium, the arrows when put together in the way just described form the outline of a closed polygon. This

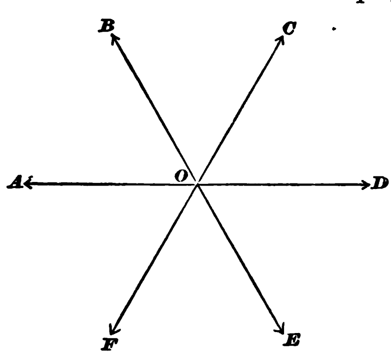

FIG. 168.

is called the *principle of the polygon of forces.* When the

set includes only three forces, we have a *triangle of forces.*

For instance, suppose the six equal forces *A*, *B*, *C*, *D*, *E*, *F*, Fig. 168, to be applied at the common point *O*, so as to make the angles at the centre equal to each other, and then, using the same forces, apply them tail to tip, as above described, so as to form a polygon, Fig. 169. The polygon is seen to be a regular hexagon. Mere inspection of the

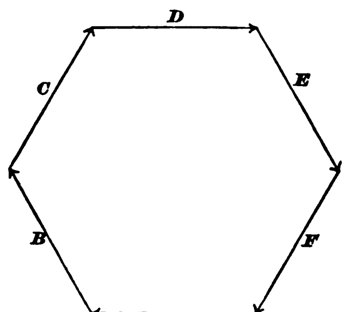

FIG. 169.

six forces in Fig. 168 will suffice to show that they produce equilibrium, and the fact that the polygon in Fig. 169 is a closed one shows the same thing in another way.

232. Forces Acting upon a Sail-boat.—A boat sailing against the wind gives a familiar illustration of the composition and resolution of forces.

Let *KK* (Fig. 170) represent the line of the keel, and *SS* that of the sail. Let *W* represent the direction of the wind.

If we may disregard the *friction* of the wind against the canvas, the direction of the force exerted by the wind against the sail is at right angles with the sail and may be represented by *PN*. This force may be resolved intc two components, one, *PL*, at right angles with the keel, the

other, PF, parallel to the keel. PL tends to drive the boat sideways, and does so to a greater or less extent, in spite of the resistance offered by the broadside, and by the keel or centre-board that extends below the hull. PF tends to drive the boat forward in the direction of the keel, and the resulting motion carries the boat obliquely toward that quarter from which the wind is blowing.

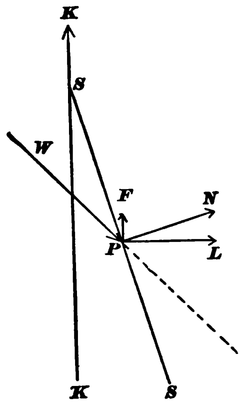

FIG. 170.

QUESTION.

Can a boat sail toward the wind when the sail is parallel to the keel ?

It is said that ice-boats, that is, boats driven by wind across fields of smooth ice, sometimes sail faster than the wind itself is moving. This cannot occur, or, at least, cannot continue, when the boat is moving directly before the wind, as in Fig. 171, for the sail SS, gaining upon the air in front, would compress it, and, outstripping the air behind, would rarefy it, so that it would practically encounter a contrary wind.

But if the direction of the wind is, for instance, at right

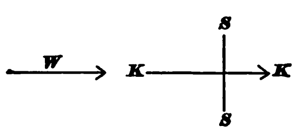

FIG. 171.

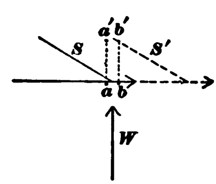

FIG. 172.

angles with the motion of the boat, as in Fig. 172, it is possible, if friction is very slight indeed, for the boat to go faster than the wind. Let S be the sail and let a and b be

two particles of air moving as a part of the wind. Let *a* and *b* be moving with a velocity that will, if they are not obstructed, carry them to *a'* and *b'*, respectively, in one second, and let *S* be moving with a velocity that will carry it to *S'* in one second. A little thought will show that under these circumstances the sail *S* will just dodge the particle *a*, and will pass between *a* and *b* without being hit by either.* In other words, the sail moving the distance *SS'* per second, which is evidently greater than the distance, *aa'*, passed over by the wind in one second, would escape the action of the wind. But if the speed of the sail were made less than it is, while still remaining greater than that of the wind, the sail would be helped on by the wind.

QUESTIONS AND PROBLEMS.

(1) Define the "moment of a force."

(2) A force of 5 lbs. and a force of 10 lbs. are applied in parallel but opposite directions to a straight rigid bar, the distance between the points of application being 8 feet. What is the magnitude, direction, and point of application of a third force that would neutralize the effect of these two?

(3) If a carriage-wheel is resting upright upon the ground and is prevented from slipping at the bottom, how great a force applied horizontally at the top will just neutralize a force of 50 lbs. applied horizontally in the opposite direction at the centre of the wheel?

(4) Let the checker-board (No. 74) be placed with the lines 1–7, 8–14, etc., east and west.

(*a*) Let a force of 10 lbs. north be applied at point 28, and a force of 15 lbs. south be applied at point 26. Tell the direction, magnitude, and point of application of a third force that would just balance the first two.

(*b*) Let a force of 10 lbs. north be applied at point 28, and a force of 10 lbs. south be applied at point 26. Tell the direction, magnitude,

* We shall see later that such particles as *a* and *b*, while moving as a part of the wind, would generally have independent motions of their own which might make them strike the sail, but the blows due to such motions would neutralize each other, merely producing the ordinary atmospheric pressure equal in all directions.

and point of application of a third force, if there is one, that would just balance the two.

(*c*) Let a force of 10 lbs. act north from point 1, a force of 10 lbs. south from point 7, and a force of 15 lbs. west from point 8. Find the magnitude, direction, and point of application of a single force that will make equilibrium with all the others.

(5) Where is the axis of rotation (§ 223) of the wheel of a moving carriage?

(6) The forces P and Q, extending in the same direction, balance each other when applied at perpendicular distances M and N from the axis of rotation. Express the ratio of P and Q in terms of M and N.

(7) Suppose in Fig. 158 all the forces to be in the same direction. Find (*a*) the amount of the resultant; (*b*) its point of application.

(8) Suppose in the same figure everything to remain as there shown, save that the force applied at S must have such a magnitude as to make the resultant 20.

(*a*) What must be the magnitude of the force applied at S?

(*b*) Where must the resultant be applied?

(9) Construct a diagram of four forces, acting in four different directions, not producing equilibrium, and prove that equilibrium is impossible with the forces as given.

(10) Make a diagram of four forces, one north, one south, one east, and one west, which will neutralize each other, marking the magnitudes of the forces and the positions of their lines of action, no two of the forces lying in the same straight line.

(11) A force of 10 lbs. acting south and an equal force acting north are applied to a body, the lines of action being 5 ft. apart. Show fully by means of diagrams two cases of equilibrium in each of which this north and south pair of forces is balanced by an east and west pair.

(12) Find the resultant of a force A of 5 kgm. and a force B of 12 kgm. acting at right angles to each other.

(13) Construct a diagram and thus find the resultant of two forces, A of 5 kgm. and B of 8 kgm., acting at an angle of 60° with each other.

(14) Four forces act on the same point O, and with the following directions: A north, B east, C south, D west. $A > C$; $B > D$. Express in letters the value of the resultant. What is its general direction?

(15) Resolve a force of 100 kgm. into two components acting at right angles to each other, one being twice as large as the other.

Ans. 44.72 + kgm. and 89.44 + kgm.

(16) If the angle *COD* in Fig. 166 is 30°, find, by drawing and measuring, the magnitude of the useful component of a pull of 400 kgm. in the direction *OC*. *Ans.* 346.4 kgm., nearly.

(17) A loaded freight-car weighs 25 tons. It is held at rest on a grade which rises 1 ft. for every 50 ft. of track. What force parallel to the track is needed to hold the car in place, the effect of friction being neglected?

(18) Show by constructing a *triangle of forces* (p. 271) whether the three forces

A (= 5 kgms.), B (= 6 kgms.), C (= 12 kgms.),

can balance each other.

(19) A boat is moored to a pile driven into the bed of the river. The boat is pressed due southward by the river-current with a force of 200 lbs. and due southeastward by the wind with a force of 80 lbs. Construct a diagram to show the amount and direction of the resultant pressure. What is the amount of this pressure?

(20) Resolve a force of 90 lbs. into two components acting at right angles with each other, one being three times as large as the other.

Ans. 28.46 + and 85.38, nearly.

(21) A rope is attached to two hooks which are on the same level and 20 ft. apart. At the middle of the rope, which hangs 10 ft. below the level of the hooks, is suspended a weight of 100 lbs. How great is the pull upon each hook? The weight of the rope is neglected?

Ans. 70.7 + lbs.

(22) A holds one end of a rope, and B holds the other end. At a certain point between them a weight hangs from the rope, and the rope bends at a right angle at this point. A pulls with a force of 12 pounds, B with a force of 16 pounds. How great is the weight which hangs from the rope? (The weight of the rope itself is neglected.)

(23) Construct a diagram to show how equilibrium is secured in the case of a heavy sinker, attached to a fishing-line, dropped into a swift-flowing stream and allowed to move down-stream until the line stops it. (Neglect the action of the water on the line, but consider the buoyant action and the friction of the water on the sinker.)

(24) A child is seated in a swing which is drawn forward from its position of rest and held motionless by a cord. Make a diagram to show the number and direction of the forces engaged, then assume

that the child's weight is 50 lbs., and find by means of your diagram the magnitude of the other forces.

(*The following involve friction. See Chapter VI.*)

(25) Demonstrate by aid of a diagram that the steepness of an incline that will just allow a block to slide down it is independent of the weight of the block.

(26) If a block weighing 50 gm. is dragged at a uniform rate along a horizontal surface by a pull of 20 gm., what is the coefficient of friction?

(27) A block slides with uniform velocity down a board 2 m. long, when one end of the board is raised 50 cm. above the table-top. What is the coefficient of friction?

(28) If the coefficient of friction is 1, what angle must the inclined plane make with a horizontal surface to cause a block to slide at a uniform rate?

(29) A body weighing 5 lbs. rests upon an inclined plane 10 ft. long, whose height is 6 ft. and base 8 ft.

(*a*) How great is the pressure which the body exerts normal to (perpendicular to) the inclined plane?

(*b*) How great a force parallel to the incline is required to prevent the body from sliding down when there is no friction? when the coefficient of friction is $\frac{1}{4}$?

(30) (*a*) If friction of the cars were zero, how steep an incline could a train weighing 400,000 lbs. ascend, the engine exerting a force of 4000 lbs.?

(*b*) The coefficient of friction being $\frac{1}{300}$, how steep an incline could the train ascend with the same pull of the engine? (Express steepness of incline as a certain number of feet rise for a certain number of feet of track.)

CHAPTER XVI.

GRAVITY AND THE CENTRE OF GRAVITY: STABILITY.

233. Direction in which Gravity Acts.—Frequent allusion has been made in the preceding pages to the force of gravity, and its familiar tendency to draw all objects towards the earth has been assumed to be known by the student. The direction of gravity may be found by means of a plumb-line (Fig. 173), and such a line will always be found to point nearly towards the centre of the earth. The line *OP*, or *OC*, passes through the centre of the plumb-bob *P*, and shows the line of direction in which gravity acts upon the bob taken as a whole.

Since the earth is nearly spherical, no two plumb-lines on the same hemisphere could be precisely parallel, except in cases where local disturbances of the direction of gravity are caused by the neighborhood of mountains, etc. Plumb-lines far apart are in general very different in direction, as is shown in Fig. 174. On account of the magnitude of the earth, however, the lines of direction of gravity acting upon the different parts of all ordinary objects form with each other angles too small to be meas-

FIG. 173.

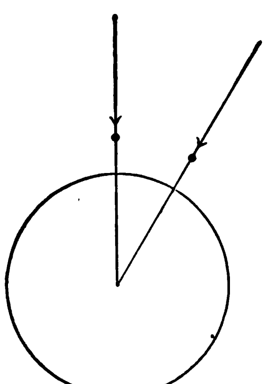
FIG. 174.

ured; that is, they are practically parallel. The action of gravity upon bodies, therefore, gives rise to a large and important class of problems upon parallel forces.

234. Centre of Gravity.—Since gravity must be supposed to act upon every particle of a body, it is clear that the total effect of gravity upon any object is the resultant of a very great number of parallel components. The point of application for this resultant, or the centre about which, as a point of support, a body will balance in any and all positions, is called the *centre of gravity.*

It is, in many cases, an easy matter to find approximately by experiment the position of the centre of gravity. If a slender cylinder, such as a knitting-needle, is, after a series of trials, made to balance across a knife-edge, it is plain that the centre of gravity must lie in the centre of that cross-section of the needle which is vertically above the knife-edge. If a flat piece of card-board can be balanced on the point of a needle or a pin, the centre of gravity lies half-way between the two surfaces of the card-board, vertically above the point of the needle. In a body which is of uniform density throughout, the centre of gravity and centre of figure coincide; that is, the centre of gravity of a sphere is at its centre, that of a right cylinder is at the centre of its axis, that of a cube is at the intersection of its diagonals, and so on.

Such an object as a sheet of writing-paper, of card-board, or of ordinary rolled zinc may safely be assumed to have a nearly uniform thickness throughout. If the shape of such an object is a regular figure, as a regular polygon, it is a simple matter to find its centre by geometry. If the shape is irregular, the centre of gravity may still be easily found by means of the plumb-line, used as in § 49.

The expressions "*centre of gravity of a line*," "*centre of gravity of a surface*," etc., when they are met with in

books on mechanics, may be interpreted to mean centre of gravity of an extremely slender cylinder, of an extremely thin sheet, etc.

235. Stability.—When we speak of the stability of an object resting upon a supporting surface, we usually have in mind the angle through which it must be tilted to overturn it. Stability, in the case of rigid bodies, depends upon two factors, the area of the base upon which the body rests and the height of the centre of gravity above the base. In estimating the size of the base of an object it must be noticed that, for the purpose of stability, the base consists of the whole area included by straight lines which connect the outermost supporting points all around. For instance, the base of a three-legged stool, or a tripod-support for a photographer's camera, is a triangle, of which the foot of each leg forms a vertex.

In illustration of the influence of the area of the base,

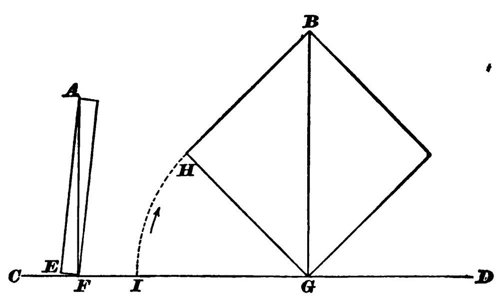

Fig. 175.

compare the angle traversed by the centre of gravity of a square prism, *A* (Fig. 175), 1 cm. square and 10 cm. long, placed on end and then overturned, with the angle traversed under similar conditions by the centre of gravity of a decimeter cube, *B*. The block *A* will not of itself overturn

until after its diagonal AF has passed the vertical position which it has in the figure. In the same way B will of itself overturn only after BG has passed the vertical position. To be upset, therefore, the block A has to be tilted only a bit more than the very small angle EFC, while the block B has to be tilted a bit more than the angle HGI, of 45°.

In order to show the influence of the height of the centre of gravity on stability, suppose a heavy iron nut, with a hole 1 cm. or more in diameter, to have a cylindrical piece of wood inserted in it. When the nut is near the support-

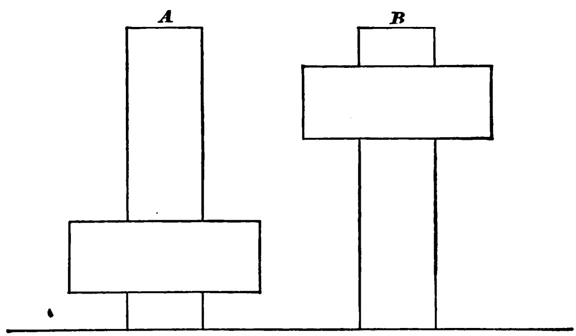

FIG. 176.

ing surface, as shown in A (Fig. 176), the stability will be considerable; when the nut is near the upper end of the stick, as in B, the stability will be very slight.

The common term *top-heavy*, applied to objects like B, well expresses the actual reason of their instability.

236. Kinds of Equilibrium.—Bodies are said to be in *stable*, in *unstable*, or in *neutral* equilibrium, according to their behavior upon being tilted, or displaced through an extremely small angle. If they tend to right themselves, that is, to return to their first position, their equilibrium is stable. If they tend to become overturned, their equilib-

rium is unstable. If they remain indifferently wherever placed, their equilibrium is neutral. A single solid, the cone, in different positions illustrates all three kinds of equilibrium: resting on its base, on a level surface, it is stable; on its apex, unstable; on its convex surface, neutral.

It is evident that the displacement, or tilting through a small angle, alluded to in the preceding paragraph, leaves, in the case of stable equilibrium, the line of direction of the earth-pull, from the centre of gravity downward, well inside the boundary of the base. In the case of the unstable equilibrium it throws the line of direction outside the boundary of the base, and in the case of neutral equilibrium leaves the line of direction unchanged in its relation to the base.

If the plane surface on which a body rests is inclined, the condition of the object as regards stability may be changed. A sphere, for instance, on an inclined plane is no longer in equilibrium, because the line of direction falls outside of the base.

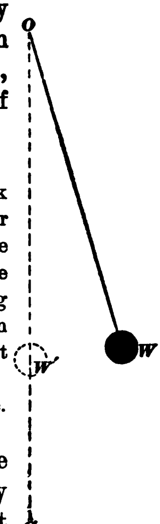

FIG. 177.

EXPERIMENTS.

(1) Place a sphere or a double cone on a track made of two rails placed somewhat higher and farther apart at one end than at the other (No. LIII). The double roller moves toward the higher end of the rails, but its centre of gravity is really descending slightly all the time, for the greater distance between the rails near the upper end lets the rolling object farther down between them.

(2) Set No. LIV rolling along a horizontal surface.

237. Stability of Suspended Bodies.—Since the centre of gravity has a constant tendency towards the earth's centre, it is evident that an object W hung from a point O (Fig. 177), about which it may swing freely, will be in stable equilib-

rium when the centre of gravity of W falls in the line OG which represents the direction of one of the earth's radii. W', then, is the position of rest for the weight when left free to settle. The plumb-line and the pendulum illustrate this principle.

Fig. 178 represents two steel forks stuck into a cork, the whole balancing upon the head of a pin. In this case, as in many others, the centre of gravity, C, is not inside the substance of the balancing body. Other similar toys are familiar.

238. Theory of a Simple Balance.—Fig. 179 represents a balance of a very simple kind, supported at the point O. The pans P_1 and P_2, hanging freely from the points K_1 and K_2, have the same effect as if their weights were concen-

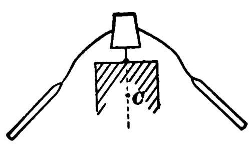

FIG. 178.

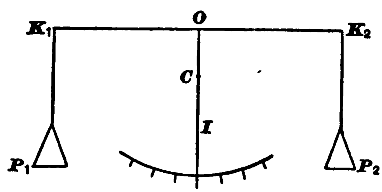

FIG. 179.

trated at these points. The centre of gravity of the whole, consisting of the beam B, the pans, regarded as at K_1 and K_2, and the pointer I, is, we will suppose, at C, a little below the point of support. When the balance is disturbed the pointer swings easily, like a pendulum, through its position of equilibrium in either direction.

If a small weight, p, is placed in P_2 the equilibrium is for the moment disturbed. The pointer swings and presently comes to rest in a new position. The condition of things is then illustrated by Fig. 180.

The point C, at which we may regard the weight of the

beam, etc., concentrated, now hangs to the left of the vertical line through the point of support *O*. We have equilibrium when the moments calculated with respect to *O* balance each other. The moment of the weight which

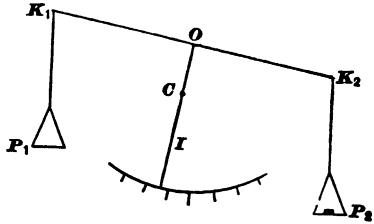

FIG. 180.

we regard as applied at *C* is negative. The moment of *p* with respect to *O* is positive. The moment of *C* increases when the index swings farther from its first position; the moment of *p* decreases at the same time. So, at a certain position of the index the two become equal and there is equilibrium.

The nearer *C* is to *O*, other things being equal, the farther the index must move when the weight *p* is added. By attaching to the index a small rider that can be moved up and down upon it, the sensitiveness of the balance can be changed at will. Frequently an adjustable rider for this purpose is placed on a stem reaching upward from the middle of the beam.

239. Centre of Buoyancy: Stability of Floating Bodies. —In a body of liquid at rest any portion not at the bottom may be regarded as *floating* in the remainder.

The liquid pressure against any body wholly or partly submerged depends upon the shape and position of its submerged surface, but does not depend upon its kind of material. The resultant of the liquid pressure against the submerged part, whatever its density, is a force upward

through what would be the centre of gravity of this submerged part if it were of the same density as the liquid. This point is called the *centre of gravity of the displaced liquid*, or the *centre of buoyancy*.

When a floating body, for instance the hull of a vessel, is in equilibrium, under the influence of gravity and buoyant pressure only, the centre of buoyancy lies in the same vertical line with the centre of gravity of the body. When the vessel rolls, the upward push of the water and the pull of gravity constitute a *couple* which tends to right the vessel.

It is an interesting fact, which the student can verify for himself by floating a light board, that a floating body may be in very stable equilibrium with the centre of gravity above the centre of buoyancy, and even above the surface of the liquid. The truth is in such cases that when the body is tipped, the centre of buoyancy, owing to the change of figure of the displaced body of water, moves in the direction of the tipping more rapidly than the centre of gravity

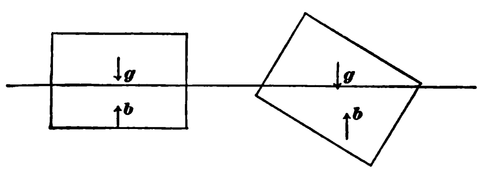

FIG. 181.

does. Fig. 181 shows how this may occur, g being the centre of gravity and b the centre of buoyancy.

QUESTIONS AND PROBLEMS.

(1) Show that a brick laid on a table-top may have three positions, one of greatest, one of least, and one of medium stability. (Disregard the scarcely possible case of balancing it upon an edge or corner.)

(2) What kind of equilibrium has a straight bar balanced across a fulcrum beneath it and having a weight suspended by a string from each end?

(3) How would the equilibrium of the apparatus of Problem 2 be affected by replacing the strings by rigid wires of the same weight?

(4) A telegraph-pole is made of three hollow iron cylinders joined end to end. Each cylinder is 8 m. long. The lowest weighs 200 kgms., the middle one 100 kgms., and the uppermost 50 kgms. Find the centre of gravity of the pole. (Imagine the pole in a horizontal position, calculate the *moment* of the weight of each section with respect to one end of the pole, find the *force* which must be exerted by a single supporting point, then find the *position* of this point.)

(5) A cube of wood, 10 cm. on each edge and of specific gravity 0.5, is covered on one side by a plate of metal 10 cm. square and 1 cm. thick, of specific gravity 5.

(*a*) How far from the outer surface of the metal plate is the centre of gravity of the whole?

(*b*) If this object is floated in water, how far below the surface will the centre of buoyancy be?

(6) A uniform lever, 6 ft. long and weighing 20 lbs., lies horizontally across a fulcrum 2 ft. from one end. A mass of 100 lbs. is suspended from the end of the short arm of the lever. How great must be the force applied at the end of the other arm in order that there may be equilibrium?

(7) A lever weighing 12 lbs. is placed horizontal, and carries at one end a weight of 8 lbs. The centre of gravity of the lever is 4 ft. from the same end. At what distance from this end must a supporting point be placed in order that the whole may be in equilibrium?

(8) A bar 6 ft. long has its centre of gravity 1 ft. from one end and balances upon a point 4 ft. from this end when a load of 10 lbs. is suspended from the other end. What is the weight of the bar?

(9) In the preceding problem suppose the bar to be uniform in cross-section and density and to weigh 4 lbs. How great will be the load supported and how great the pressure on the fulcrum in this case?

(10) A uniform bar, 10 ft. long and weighing 5 lbs., bears at a distance of 2 ft. from the end A a slider weighing 4 lbs., and at a distance of 6 ft. from the same end another slider weighing 7 lbs. How far from the end A must a supporting point be placed in order that the rod with its loads may balance upon it in a horizontal position?

(11) Show what forces are at work to maintain equilibrium in the

case of a vessel heeling (tilting) over somewhat under the action of a wind which blows at right-angles to her course.

(12) A door weighing 40 lbs. has its centre of gravity $1\frac{1}{2}$ ft. distant from a vertical line passing through the pivots of the two supporting hinges. The hinges are of the simplest character, and 6 ft. apart from centre to centre. The load is divided equally between the two hinges, but the upper one is supposed to take all the horizontal pull, and the lower one all the horizontal push, caused by the weight of the door.

(*a*) How great is the horizontal pull upon the upper hinge?

(*b*) How great is the horizontal push upon the lower hinge?

(*c*) Find by the graphical method the direction and magnitude of the total force which the upper hinge *applies to the door*.

(*d*) Find by the same method the total force which the lower hinge applies to the door.

(*e*) Find last by the same method the magnitude, direction, and point of application of the resultant of all the forces applied to the door by both hinges.

(13) A ladder 20 ft. long and weighing 50 lbs., the centre of gravity being at the centre of its length, stands upon level ground and leans against a vertical wall, which is so smooth that the force between it and the ladder is wholly horizontal. The distance from the foot of the ladder to the wall is 8 ft. A boy weighing 100 lbs. is on the ladder. The ladder does not bend under his weight.

Find the magnitude of the horizontal push H and of the vertical push V exerted by the ladder against the ground—

(*a*) When the boy is midway between the foot and the centre of the ladder;

(*b*) When the boy is midway between the centre and the top of the ladder.

Suggestion: Find first the magnitude and line of action of the resultant of the two given vertical forces, the weight of the ladder and the weight of the boy. Then proceed in either of two ways:

(1) Treat the problem as a case of two equal and opposite couples, one horizontal and the other vertical.

(2) Solve the problem by a graphical use of the parallelogram of forces, thus: Draw a horizontal line from the top of the ladder. Draw the line of action of the resultant of the weight of ladder and boy. The point of crossing of these two lines may be regarded as the point of application of the forces represented in direction by

these lines; that is, these forces will have the same effect upon the ladder as if they were applied at the end of an arm reaching out from the ladder to the point described. The only other force which we consider applied to the ladder is the push of the ground. This push is not wholly vertical, for its line of action must pass through the point of application of the other forces; otherwise it could not neutralize them. We thus know the direction of all the forces crossing at the given point, and the magnitude of one of them. Everything else is easily found.

The answers for Case (*a*) are $V = 150$ lbs., and $H = 21.82 +$ lbs.

CHAPTER XVII.

MOTION.

240. Matter, Force, and Motion.—In the preceding chapters it has been necessary to speak of various forms of matter as things familiar to every one, and to consider some of the effects which forces produce on matter. It is well known that the production or change of motion is a very common consequence of the application of any force to any portion of matter. We shall now begin a more careful study of the relation between matter, force, and motion than the student has yet made. This study will require us to look sharply to the meaning of the words *motion* and *velocity*.

241. All Motion Relative.—It may seem at first sight that it would be a simple affair to define the *absolute motion* of a body; that is, to state just how it travels with reference to some fixed point. But there is no point which we know to be fixed. The earth and all the other members of the solar sytem, not excepting the sun itself, have very complicated motions of their own; and since also the stars are probably drifting through space, it is quite impossible to determine the absolute motion of any object. The most that we can do is to observe its departure along a given line from some chosen starting-point which is, for the purpose we have in view, to be regarded as fixed. Thus we might describe the motion of a man moving down the aisle of a car, neglecting in our account the rocking movement of the car from side to side, its pitching, and its general motion of translation along the track; neglecting, in fact, everything

except the *difference* between the motion of the man and the motion of the point from which he started. This difference of motion is called *relative motion*, and all motion that we can describe is relative motion. When nothing is said to the contrary, it will be understood that motions spoken of in this book will be relative motions with respect to the earth regarded as fixed.

242. Composition of Motions.—The student may have gathered from what precedes the inference that motions may be compounded or resolved as forces can be. This is true; and nearly all that was stated in Chapter XV in regard to the parallelogram of forces and the poygon of forces may be re-stated, using the word *motion* instead of force, and the words *rest* or *zero motion* instead of *equilibrium*.

An interesting and important class of recording instruments make use of this fact. Such are the self-recording barometer, the chronograph, much used by astronomers, the plethysmograph, used by physiologists for studying the circulation of the blood, and a number of instruments used in the investigation of sounding bodies (see Exercise 49). In all these pieces of apparatus a moving point is, sometimes by the aid of photography, made to record its motion upon a surface which is itself in motion. The manner in which the motion of the recording surface affects the trace left by the moving point is well shown by a simple phenomenon of frequent occurrence in every-day life. When

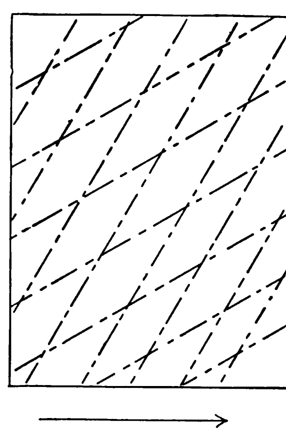

FIG. 182.

rain falls on a comparatively windless day against the window of a railroad-car at rest, the drops trickle vertically down the surface of the pane. But when the car is in motion the line traversed becomes more and more oblique as the speed of the car increases. Fig. 182 contains a graphic record of the course of the drops for two different velocities of the car. The arrow shows the direction of the car's motion.

243. Velocity.—*Velocity is rate of motion.*

When velocity is unchanging it is measured by the number of units of space traversed in a unit of time. When velocity is changing it may be defined as equal, at any instant, to the distance the body would move in a unit of time if the rate of motion which it has at that instant were maintained unchanged. The choice of units depends somewhat upon the kind of motion and the purpose for which it is to be estimated or measured. The speed of railroad trains, of steamers, of pedestrians, and so on, is usually reckoned, in this country, in miles per hour, the speed of bullets or cannon-balls in feet per second, and the rate of transmission of electrical impulses along wires or of light through air in miles per second. For strictly scientific purposes, however, *centimeters per second* is a form of statement frequently employed by physicists.

244. Change of Motion: Inertia.—In the next few Exercises we shall be dealing with objects which are neither at rest nor in uniform motion, but are changing from rest to motion, or from one state of motion to a different state. In these cases we shall encounter a very important property of matter which we have almost totally disregarded thus far. Before we give a name to this property let us assure ourselves that it exists.

EXPERIMENTS.

Suspend an iron ball weighing not less than 10 lbs. from a firm support by a long stout cord, so that it may be free to swing at a slight push. Attach a thread to the ball, and with a gentle horizontal pull set it gradually in motion. Stop the ball, and again set it in motion by means of the thread, this time more suddenly. Repeat, starting more and more suddenly each time, until the thread breaks.

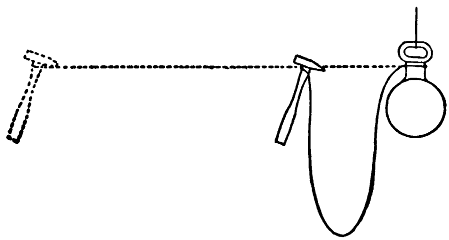

FIG. 183.

Then take a string that will bear the weight of several pounds, attach it to the ball, and break it as the thread was broken.

Take finally a string that will bear considerably more than the weight of the ball, and attach it to the latter. Tie the free end of the string securely to the handle of a good-sized hammer (see Fig. 183) held in the hand, and then, the string being long enough to give the arm free play, attempt to set the ball in horizontal motion with the greatest possible suddenness* by swinging the hammer. The string will probably be broken, while the ball will move but little.

The *hanging back* of the ball, which is so extremely obvious in these trials, cannot be accounted for by its weight; for weight, in its strict sense (§§ 24 and 25), is merely the earth's attraction for the ball, and this downward force cannot, save in some indirect way, oppose horizontal motion.† Nor is the behavior of the ball to be accounted

* Make sure that the handle of the hammer is strong and the head firmly fastened to it.

† It is true that the ball begins to rise a little when it swings from its position of rest, but this rise is very slight at first, and that component of the earth's attraction which is parallel to the path of the ball is therefore very small.

for by the resistance of the air, nor by the action of any other opposing force applied to the ball by known outside agencies. We must conclude from the experiments described, and other similar ones in which the motion of bodies is arrested or changed in direction, that IT IS THE NATURE OF MATTER TO REQUIRE FORCE TO SET IT IN MOTION OR TO CHANGE ITS MOTION IN MAGNITUDE OR DIRECTION,* *little force if the change of motion is very gradual, greater and greater force as the suddenness of the change is increased.*

This is one of the most important facts that we know about matter, and it is convenient to have a single word which will call it to our minds. We therefore say that this behavior of matter manifests a certain property which we will call *inertia.* To say this, to give the name inertia to a supposed property, does not *explain* the facts. It is merely a convenience, for it enables us to use the one chosen word, inertia, in place of phrases or sentences which would otherwise be required.

If we now suspend a much larger or much smaller iron weight than the one just used and repeat some of the experiments with it, we shall find that to start the larger weight with equal suddenness requires a greater force. This being the case, we naturally inquire whether we cannot use the property of inertia in comparing quantities of matter, in determining *how much* is the quantity of matter in one body compared with that of a second body, which process we call *measuring* the quantity of matter in the first body.

245. Quantity of Matter; Mass.—" There are several characteristics belonging, so far as we know, to all kinds

* The words in small capitals are substantially equivalent to what is called Newton's *First Law of Motion.*

of matter,* which force themselves habitually upon our attention and are of such a nature that we turn to them, when we attempt to estimate the quantity of any aggregation of matter. These characteristics are:

"*Matter occupies space;*

"*Matter attracts other matter;*

"*Matter requires the application of force to change its motion.*

"These facts are here put in this order, because it is the chronological order in which they are first recognized by every student. . . .

"We shall call these three tests [suggested by the properties just mentioned] the *volume-test*, the *weight-test*, and the *inertia-test*, respectively. Since they do not all agree, which one shall be selected and agreed upon as the best?

"To a child the volume-test is the natural one, perhaps the only one he can apply or imagine. As he grows older, he observes that bodies may change in volume without addition or subtraction of substance, and that in such cases the weight remains constant. He therefore comes to prefer the weight-test to the volume-test. Continuing, he learns that a given body does not weigh the same [on a spring-balance] at all parts of the earth's surface, and that in regions of space far from the earth, where nevertheless science has to deal with matter, the aspect of weight, if we regard it at all, is quite changed. He finds, however, that there is every reason to believe that a given body, in whatever part of space it might be placed, would require the same force to give it the same velocity in the same time. He therefore in the end comes to regard the inertia-test as more widely applicable than the weight-test. He now agrees with other physicists that two bodies which are equal in the inertia-

* [The luminiferous ether (Chapter XXVII) is a possible exception. There is no evidence that it is subject to gravitation.]

test shall be said to contain, or consist of, equal quantities of matter.

"As an equivalent for the phrase *quantity of matter* the word *mass* is commonly used. *Equal masses*, then, are, *by definition*, quantities of matter which, *whatever their inequality in other respects*, are alike in this, *that they require equal forces to give them equal velocities in equal times*." *

An example of the inertia-test, or acceleration-test, is given in the next Exercise; but as experiment shows that bodies which are equal by the inertia-test are equal by the weight-test also, and as the latter is very much easier to apply, mass, though *defined* with reference to inertia, is practically measured by the weight-test. There are many advantages in this practice, but there is one disadvantage. The disadvantage is that many people, some of whom write books and magazine articles, fail to distinguish between inertia and gravity, between mass, the quantity of matter in a body, and weight, the pull of the earth upon that body.

It is gravity that we have to do with when a bullet rests in the hand. It is inertia that is manifested when the bullet, fired from a pistol, forces its way *upward* through an obstruction against the force of gravity.

It is gravity that tends to draw the earth into the sun. It is inertia that tends to keep the earth moving in a straight line past the sun. It is a balance between the two that keeps the earth moving around the sun, neither falling into it nor receding from it.

EXERCISE 36.

(For two or three students working together.)

COMPARISON OF MASSES BY ACCELERATION-TEST.

Apparatus: Nos. 76, 77, and 78. Lead scraps *ad libitum* for one of the carriages. A pinch-cock.

* From *Elementary Ideas, Definitions, and Laws in Dynamics*, E. H. Hall.

Test of the Rubber Tubes.—The Exercise requires these tubes to exert the same contractive force when equally stretched. After making sure that they are of equal length when unstretched, which can best be done by measuring them while they are hanging straight downward from the carriages, fasten them together with a pinchcock at the free ends, as in Fig. 184, and then pull the two carriages

FIG. 184.

away from each other until each tube is about twice as long as before stretching. They are now subject to the same force, and if the point of junction is midway between the carriages, that is, if the tubes have stretched equally, they require no further adjustment. If one tube has stretched less than the other, slit it open for a distance of fifteen or twenty centimeters and trim off a narrow piece along the edge of the slit, until this tube stretches just as much as the other. The tubes may now be separated. Hereafter they will, until their properties suffer some change, exert equal pulls when equally stretched.

Balancing Friction.—The resistance due to friction is balanced, as nearly as may be, by making each carriage roll upon one of the planks, which are placed side by side, so inclined that the carriage, once started very slowly down the incline, will barely continue in motion to the bottom.*

Friction being thus balanced by the slight pull of gravity down the incline, the elastic tubes when used will have to deal with inertia alone, so that the test may now proceed as if with frictionless carriages on a level surface.

THE ACCELERATION TEST.—Load one carriage with the iron, as in Fig. 185, and the other with an unknown amount of lead scraps. Let one student hold each carriage at the top of its incline, the hind

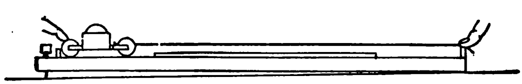

FIG. 185.

* It is quite likely that the two carriages will require different inclinations, but with a good carriage on a good plank the inclination need not be greater than 1 in 50. The bearings of the carriages should be oiled; but if too much oil is used, it is likely to come into contact with the rubber tubes and spoil them.

wheels against the upper cleat. Let another stretch the rubber tubes, each over the middle of its own plank, till the tips rest upon the cleats at the bottom. Let a third place himself by the side of the incline, to watch the progress of the carriages when released, and to stop them before they reach the bottom.

Let the experimenter No. 1 release both carriages at the same instant, for which purpose he will do well to hold them both with one hand, and let No. 3 watch sharply to see which carriage is ahead at that part of the incline where the tubes cease to pull. (It is well to have a pencil-mark drawn straight across both boards at this place.)

Then increase or decrease the load of lead as the case seems to require, and repeat the test and readjustment of load many times, very carefully at the last, until no difference can be detected in the time of arrival of the two carriages at the pencil-mark just mentioned.

THE WEIGHT TEST.—Weigh the carriages with their loads, as finally adjusted in the acceleration test, and thus see how accurate the work of that test has been.

246. Unit of Mass.—There is in charge of an English government officer a certain piece of platinum marked "P. S. 1844, 1 lb." The letters P. S. mean Parliamentary Standard, and this particular piece of platinum is by act of Parliament declared to be " the legal and genuine standard measure of weight [mass] . . . from which all other weights and other measures having reference to weight shall be derived," etc. Very exact copies of this standard are kept in various places in England, so that, if the original were lost or injured, one of the copies might be substituted for it without harm.

It is by reference to this standard that goods are bought and sold by weight in England, and the exceeding care bestowed upon the construction of the standard and its copies shows how much regard the English have for stability and accuracy in commercial and scientific transactions.

With perhaps equal care the French have constructed and established a standard of mass, which they call the "Kilogramme des Archives." This, too, is of platinum and was

"intended to represent the mass of a cubic décimètre of distilled water at the temperature 4° C." The piece of platinum is the standard now, though it may not correspond exactly to the original intention, with reference to water.

The relation of the English standard to the French standard is very accurately known, so that when the English make up their minds to abandon pounds, ounces, grains, etc., and adopt kilograms, decigrams, grams, etc., they will be able to make the change without serious disturbance to commerce.

"The standard avoirdupois pound of the United States is equivalent to the weight of 27.7015 cubic inches of distilled water at 62° Fahrenheit, the barometer being at 30 inches, and the water weighed in the air with brass weights." *

* From Webster's International Dictionary.

CHAPTER XVIII.

UNITS FOR PHYSICAL MEASUREMENTS, MOMENTUM, ETC.

247. Fundamental Units.—We have in physics a great many kinds of quantities to be measured, distance, time, velocity, mass, force, work, energy, heat, electrical resistance, electromotive force, etc., and we need a well-defined unit for each.

Such quantities are not entirely independent of each other, but are so related that if we choose units in which to measure any three of them, we can from these units *make up* units for all the others. We commonly start with the units of *length*, *mass*, and *time*, and call these the *fundamental*, or *primary*, units.

There is a wide range of choice as to the *size* of the fundamental units. We shall often take

for the unit of length the *centimeter*,
for the unit of mass the *gram*,
for the unit of time the *second*,

which gives us what is called the centimeter-gram-second, or C.G.S., system of units, the one most commonly used by physicists the world over. But evidently we might use a *foot-pound-second*, or a *meter-kilogram-second*, or even a *foot-kilogram-hour*, system.

248. Derived, or Secondary, Units.—Units defined by use of the fundamental units are called *derived* units. For

example, the unit of velocity is defined as a velocity of one *centimeter* per *second*, or one *foot* per *second*, etc. Rate of delivery of water through pipes is defined as so many *cubic centimeters* per *second*, or so many *cubic feet* per *minute*, etc.

Some of the derived units will be discussed in following articles.

249. Units of Force.—We have repeatedly measured forces in the Exercises of this book, using spring-balances and reading in pounds or ounces, kilograms or grams. But these are precisely the names we use for our various units of *mass*. How does it happen that units of mass and units of force have the same names?

It is because the pound (force) is the pull of gravity upon a pound (mass), the gram (force) the pull of gravity upon a gram (mass), etc. These force-units are *derived* units, of a kind. They are defined by reference to units of mass and to gravitation, and are therefore called *gravitation-units* of force.

Such units are variable, for the pull of gravity upon a given mass is not equally great at all parts of the earth's surface. For this reason and certain others gravitation-units of force are not satisfactory for all scientific purposes. Engineers find them convenient, using the larger units, pounds, kilograms, etc., and physicists are not likely to discard them altogether; but much use is made of other force-units, defined, without reference to gravitation, from the three fundamental units of length, mass, and time.

Thus we have the

DYNE, *which is the force that, acting for one second upon a mass of one gram, gives it a velocity* * *of one centimeter*

* This does not mean that the gram will *move* one centimeter during the one second for which the force is acting. It means that, if the gram is at rest when the force begins to act, it will gain such a velocity that, if the force ceases to act at the end of one second, the

per second. This is the C. G. S. (§ 247) unit of force, the favorite unit among physicists.

So we have the

POUNDAL, *which is the force that, acting for one second upon a mass of one pound, gives it a velocity of one foot per second.*

Such units as the dyne and the poundal, which are defined without reference to gravitation, are called *absolute* units of force. *Inertia*-units, or *acceleration*-units, would be a more expressive name for them.

The general definition of an absolute unit of force is this: *The unit force is that which, acting upon the unit mass for the unit time, gives to it the unit velocity.*

250. Momentum: Newton's Second Law of Motion.— Experiment shows that the following equation holds true:

$$f = \frac{m \times v}{t},$$

where f is a force, expressed in dynes;
m " mass, " " grams;
t " time, " " seconds;
v " velocity, " " centimeters per second,
which is given to the mass m by the force f in the time t.

The same equation would hold true with force expressed in poundals, distance in feet, mass in pounds, etc.; but if force were expressed in *gravitation*-units, the equation would not be so simple. It would then be

$$f = \frac{m \times v}{g \times t},$$

g having the value shown in § 253.

gram will move one centimeter during the next second, if nothing disturbs it.

Definition.—The product $m \times v$ is called the *momentum* of the mass m with the velocity v.

The equation given above is equivalent to Newton's *Second Law of Motion*, as stated and explained by Maxwell in his little book on *Matter and Motion.*

PROBLEMS.

(1) A force of 10 dynes acts on a mass of 5 grams starting it from rest. How fast is the mass moving at the end of the first second? at the end of the second second? third second? eighth second?

(2) A force of 12 dynes, acting 5 seconds, imparts to a certain mass a final velocity of 3 cm. per second. How great is the mass?

(3) A force of 200 dynes acting upon a mass of 50 gm. gives it a final velocity of 100 cm. per second. How many seconds does the force act?

(4) A certain force acts upon a mass of 20 gm. for 10 seconds and gives it a final velocity of 50 cm. per second. How great is the force?

(5) A force of 50 poundals acts upon a mass of 50 lbs. for 1 second. What velocity in feet per second does it give?

(6) What force in poundals would give to a 10-lb. mass in 5 seconds a final velocity of 20 ft. per second?

(7) The momentum of a certain mass, reckoned in the C. G. S. system, is 400. If this was given by a force of 80 dynes, how long a time was required?

251. Resistance: Loss of Momentum.—A resistance, friction for instance, is merely an opposing force; it tends to destroy motion. The equation

$$f = \frac{m \times v}{t}$$

holds as well for opposing forces as for accelerating forces, but when the force is an opposing one $m \times v$ stands for the momentum *lost* by the moving body in the time t.

Gravitation acts as an accelerating force upon a falling body, and as an opposing, or retarding, force upon a rising

body. A stone, if the resistance of the air may be neglected, gains as much momentum per second in falling as it loses per second in rising.

PROBLEMS.

[*In these and all similar problems in this book the resistance of the air is to be neglected unless expressly mentioned.*]

(1) The pull of gravity upon a certain stone, the mass of which is 1 gm., is 980 dynes. How fast will this stone be moving at the end of its first second of falling? at the end of its second second? its third? (Gravitation pulls steadily upon a body with the same force whether the body be falling, rising, or at rest.)

(2) The same stone is thrown upward with a velocity, at the start, of 2000 cm. per second.

(*a*) How much momentum will it lose in the first second of its rise?

(*b*) What will be its velocity upward at the end of this second?

(*c*) How many seconds will it continue to rise?

(3) The pull of gravity upon a mass of 1 lb. is about 32.2 poundals. If this be taken as the exact force, what velocity in feet per second will the body have at the end of the first second of its fall? at the end of the second second? the third second? the tenth?

(4) The pull of gravity upon a 5 gm. mass being 4900 dynes, what velocity will this mass acquire in the first second of its fall? in the first two seconds? the first three seconds?

(5) How many dynes equal a force of 1 gm.?

(6) How many poundals equal a force of 1 pound?

The Laws of Falling Bodies.

252. Remarks.—We have already had several problems dealing with falling or rising bodies. It is customary in elementary text-books of physics to give much attention to such problems. The most important reason for doing this is that in falling or rising bodies, resistance of air being neglected, we have very simple cases of accelerated or retarded motion, the force of gravity being practically uniform over any point of the earth's surface within any

reasonable height. Experiments* with actually falling bodies are frequently made.

253. Acceleration and Velocity.—The force of gravity acting upon the rising or falling body being supposed unchangeable, or "constant," the law $f = \frac{mv}{t}$, or $\frac{v}{t} = \frac{f}{m}$ (§ 250), shows that the rate of change of velocity $v \div t$, which is called the *acceleration*, is unchangeable, or constant. This fact has been illustrated by some of the problems given.

Moreover, since the pull of gravity upon any given body is proportional to its mass (§ 245), we see that $f \div m$ will be the same for different masses, and so $v \div t$, the acceleration, will be the same for different masses.

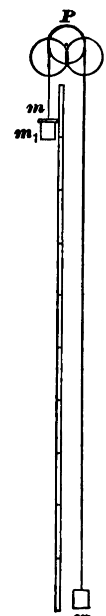

FIG. 186.

* Direct experiments upon the laws of freely falling bodies are rather difficult to make with simple apparatus, on account of the very short time that is required for falling ordinary experimental distances, but certain other experiments to illustrate the action of a uniform force in causing change of motion can be made with the familiar Attwood's machine, the construction of which is described in most of the larger treatises upon physics. This apparatus consists essentially of two equal weights m_1 and m_2 (Fig. 186), with a small additional weight, m, on one, fastened to the ends of a thread passing over a pulley, P, together with an attachment for beating seconds and for determining accurately the time of the beginning and the end of motion. The rate of increase of velocity with this machine is uniform, but is much less than in the case of a freely falling body.

Another device consists of an inclined plane down which some body is permitted to roll or slide. If the body rolls, however, the case is seriously complicated by the very fact of the rotational motion acquired, and if the body slides friction is likely to be troublesome.

In view of all the difficulties in the way of successful experiments upon the effects of a uniform force and successful interpretation of these experiments, we shall not attempt in this book an experimental treatment of the matter.

This acceleration is a very important quantity in physics, so important that a certain letter, g, is set apart to indicate it. "The value of g," "the determination of g," etc., are phrases understood by physicists the world over as referring to this quantity.

As the problems already given indicate, g is about 980* cm., or about 32.2 ft., per second.

If v stands for the velocity *acquired* in t seconds by a body falling freely, we have

$$v = g \times t. \qquad \qquad (A)$$

If at the beginning of the time t the body has already a vertical velocity v_1, the change of velocity will be just as great as if it started from rest, so that the velocity at the end of t seconds will be $v = v_1 + (g \times t)$.

The case in which the body is started with an upward velocity requires the formula $v = - v_1 + (g \times t)$, velocities upward being called negative velocities.

254. Distance.—To represent the vertical distance traveled in a given time by a falling body we shall make use of a graphical method; but let us first consider how we can represent graphically the distance traversed in t seconds by a body moving with a *uniform* velocity v.

Draw a horizontal line OX (Fig. 187) the length of which shall be t units. Draw a vertical line OY, the length of which shall be v units. The area of the parallelogram $OYPX$ is equal to the distance, $(v \times t)$, that is traveled by the body in the time t.

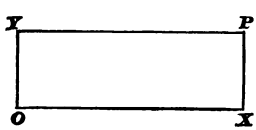

FIG. 187.

If a body travels t_1 seconds with a velocity v_1, t_2 seconds with a velocity v_2, somewhat greater than v_1, etc., the total

* It varies from about 978 cm. per second at the earth's equator to about 983 cm. per second at the poles.

distance traveled is represented by the area of the figure made by joining all the rectangles v_1t_1, v_2t_2, v_3t_3, etc., as in Fig. 188.

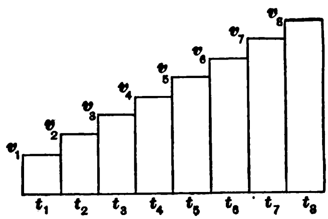

FIG. 188.

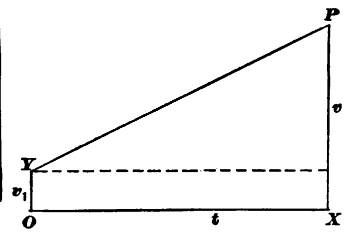

FIG. 189.

If the body starts with a velocity v_1, represented by OY in Fig. 189, and gains velocity steadily and uniformly during t seconds, it will have at the end of that time a velocity v, represented by XP, and the distance traveled in the time will be represented by the area $OYPX$. This area is evidently equal to $t \times \frac{1}{2}(v_1 + v)$. It is equal to the product of the time, t, by the *average*, or *mean*, velocity $\frac{1}{2}(v_1 + v)$.

If the case is that of a falling body having a downward velocity v_1 at the beginning of the t seconds, we have (see § 253) at the end of t seconds $v = v_1 + gt$, and the mean velocity for the t seconds is $\frac{1}{2}(v_1 + v_1 + gt) = v_1 + \frac{1}{2}gt$, so that the distance traveled is

$$s = t \times (v_1 + \tfrac{1}{2}gt) = v_1t + \tfrac{1}{2}gt^2 \quad . \quad . \quad . \quad . \quad \text{(B)}$$

If v_1 is 0, that is, if the body falls from rest at the beginning of the t seconds, the mean velocity is $\frac{1}{2}gt$ and the distance is

$$s = \tfrac{1}{2}gt^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(C)}$$

Observe that the distance s in equation (B) is made up of two terms—v_1t, which is evidently the distance the body would go in t seconds if it kept its initial velocity, gravity

not acting, and $\frac{1}{2}gt^2$, the distance it would go in the t seconds if it had no initial velocity.

If the body were started with an *upward* velocity, v_1 would be called negative, and the distance traveled *downward* would be

$$s = -v_1t + \tfrac{1}{2}gt^2, \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(D)}$$

255. Equations (A), (C), and (E).—The following equation, obtained from (A) and (C) by eliminating t, is often useful:

$$v^2 = 2gs. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(E)}$$

It is well for the student of physics to commit equations (A), (C), and (E) to memory so thoroughly that the tongue can say them mechanically, even when the brain has lost its hold upon them.

New Brit p262-3

PROBLEMS.

(In the following problems call $g = 980$ cm. or 32.2 ft. per second. When the velocity of a falling or rising body is very great, the resistance of the air is an important matter, especially if the body is a small one of little density. All consideration of friction is omitted from these problems, and therefore the results obtained by calculation are in some cases considerably different from those which experiment would give.)

(1) A body is started downward with a velocity of 500 cm. per second and falls freely. How fast will it be moving at the end of 1 second? 2 seconds? 5 seconds?

(2) If the initial velocity were 2000 cm. per second upward, what would be the velocity and direction at the end of 1 second? 2 seconds? 5 seconds?

(3) If the initial velocity were 50 ft. per second downward, what would be the velocity, in feet per second, at the end of 1 second? 2 seconds? 5 seconds?

(4) If the initial velocity were 100 ft. per second upward, what would be the velocity and direction at the end of 1 second? 2 seconds? 5 seconds?

(5) A falling body has acquired a velocity of 3000 cm. per second.

(*a*) How long has it been falling?

(*b*) If it were started upwards with the same velocity, how long would it continue to rise?

(6) How long must a body that starts downward with a velocity of 60 ft. per second fall in order to have a velocity of 100 ft. per second?

(7) A body starts upward with a velocity of 100 ft. per second.

(*a*) How long after this start will it have a velocity of 50 ft. per second upward?

(*b*) How long before it will have a velocity of 50 ft. per second downward?

(8) In Problem (1) what would be the mean velocity during the 1st second? during the first 2 seconds? during the first 5 seconds?

(9) In Problem 3 what would be the mean velocity during the 1st second? during the 2d second? during the 5th second?

(10) A body falls freely from rest. How many cm. does it fall during the 1st second? during the first 2 seconds? the first 3 seconds? the first 4 seconds?

(11) How many cm. does the body fall during the 2d second? the 3d second? the 4th second?

(12) How many feet does the body fall during the 1st second? the first 2 seconds? the first 3 seconds? the first 4 seconds?

(13) How many feet does the body fall during the 2d second? the 3d second? the 4th second?

(14) A bullet is shot upward with a velocity of 300 m. per second. How far will it rise in the 1st second? in the first 2 seconds? How far will it rise in all?

(15) (*a*) How far above the starting-point will the bullet in Problem 14 be after 40 seconds?

(*b*) How many seconds after starting upward will it reach the starting-point again?

(16) A falling apple has acquired a velocity of 15 m. per second. From what height has it fallen?

(17) A boy throws a stone straight upward 100 ft. With what velocity does the stone leave his hand?

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256. Other Cases of Uniform Acceleration.—Gravitation, as we have just dealt with it, is merely a study of uniform

acceleration, with a particular value, g, for that acceleration. If we were dealing with any other case, with an acceleration α, caused by any other force than gravitation, we should have, corresponding to formulas (A), (B), etc.,

$$v = \alpha t; \quad \text{(A')}$$
$$s = v_1 t + \tfrac{1}{2}\alpha t^2; \quad \text{(B')}$$
$$s = \tfrac{1}{2}\alpha t^2; \quad \text{(C')}$$
$$v^2 = 2\alpha s. \quad \text{(E')}$$

PROBLEMS.

(1) A moving mass of 30 gm. is opposed by a force of 150 dynes. How great is the (negative) acceleration?

(2) If the initial velocity of this mass is 200 cm. per second—

(*a*) How long will it move against the given resistance before coming to rest?

(*b*) How far will it move before coming to rest?

257. Force Required to Change the Direction of Motion: "Centrifugal Force."—In many cases change of motion affects, not its magnitude, but its direction. Even this change requires the application of force. If a person swings a weight, fastened to the end of a string, with uniform velocity in a horizontal circle about his head, he is conscious of exerting a continual pull in order to prevent the ball from getting away. The tendency of the ball to escape from its circular path, which tendency makes necessary the retaining pull, is usually called *centrifugal force.*

It must not be supposed that centrifugal force means a tendency to fly straight away from the centre. Experiment and observation will show that if the weight is at any time released it starts off in a straight line, which is a tangent to the circular path at the point where the release occurs. It is this tendency of bodies in motion to move on in a straight line that prevents all the planets of our solar system from falling into the sun. The sun's attraction for

them is like the pull between the hand and the revolving weight.*

It was no mere accident that made equilibrium between gravitation and the centrifugal tendency in our solar system. If, for instance, the sun's attraction for the earth were suddenly doubled, the earth would not in consequence be drawn into the sun. It would approach nearer to the sun, and in so doing would be subject to a stronger and stronger attraction, but on the other hand it would revolve faster and faster around the sun, and in a smaller circle, which would make the centrifugal tendency greater. The centrifugal tendency would increase more rapidly than the attraction, and at last there would be equilibrium again.

If the sun's attraction were somewhat too small, the opposite effect would take place. The earth would move away somewhat farther till equilibrium was restored.

In fact, this process of self-adjustment is continually going on in the solar system. The earth, for example, is slightly nearer to the sun at one time in the year than at other times.

EXPERIMENTS.

(1) Half fill the small brass bucket (No. 6) with water and then, holding it by the wire handle, whirl it in a vertical circle at arm's length.

* We say this now with great confidence, although two hundred and fifty years ago very different theories in regard to the motions of the planets were held. Sir Isaac Newton proved about 1680 that the earth's attraction, supposed to decrease in proportion as the square of the distance between its centre and that of the attracted body increases, is just sufficient to hold the moon in its orbit, if the moon acts like ordinary matter with which we are familiar. He showed, too, that the same natural supposition, a mutual attraction universally proportional to the square of the distance between the attracting bodies, would account for the retention of the various planets in their orbits about the sun, provided these planets act like ordinary bodies upon our earth. We accept Newton's theories because of their simplicity—they do not call into action among the stars properties of matter unknown to us upon the earth,—and because of their completeness—they account for the magnitude of the effects observed as well as for their general nature.

(2) Fill an ordinary set wash-basin, having a stopper at the bottom, with water, give this water a vigorous rotary motion and then pull out the stopper. Note the behavior of the water over the vent, and the number of seconds required to empty the basin.

Repeat the experiment, filling the basin as before, but allowing the water to come to rest, as shown by the stillness of particles floating upon it, before pulling out the stopper.

258. Combination of Horizontal and Vertical Motion.— The student may have some doubt as to the exact action of gravity upon a body having horizontal motion. It is probably a common belief that initial horizontal motion makes a body slower to fall than if it were dropped from rest.

EXPERIMENT.

Fix apparatus No. LV (Fig. 190) in a horizontal position several feet above a level floor. Put the two marbles in place in their notches,

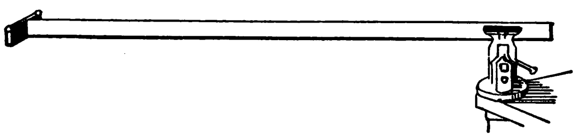

Fig. 190.

then pull the free end of the long bar quickly to one side. One of the marbles is shot out horizontally with considerable velocity. The other is merely deprived of its support and falls nearly straight down. Which strikes the floor first?

259. Relation of Pendulum-Motion to g.—The motion of the pendulum is, of course, due primarily to the earth's attraction. If the earth's attraction for a given mass were greater than it is, the time of vibration of a given pendulum would be less than it now is. In fact, the time of vibration of a given pendulum is different at different parts of our earth, the earth's attraction for the matter of the pendulum being different in different regions, while the mass (§ 245) of the pendulum as determined in the strict way, by the magnitude of the force required to give it a certain velocity in a certain time, is everywhere the same.

The relation between the intensity of gravity,* the time of vibration of a pendulum, and the length of the pendulum is so well known that if any two of these quantities are known the third can be calculated, and from these the intensity of gravity, which is usually expressed by stating the value of g (§ 253), can be found much more accurately than by any other method in common use.

The determination of g at many different parts of the earth's surface is of much interest on account of the information which the variation of g from place to place gives in regard to the shape and interior condition of the earth. Accordingly, various governments have employed skilled observers to make pendulum-observations, and these observers have traveled far and wide over the earth in order that the same pendulum might be studied by the same person in widely different regions. The value of g at the poles is about one part in two hundred greater than its value at the equator, a fact which is accounted for partly by the shortness of the polar diameter of the earth as compared with the equatorial diameter, and partly by the slight tendency of bodies to fly off at a tangent, parting company with the rotating earth (see § 257), which tendency is greater at the equator.

260. All Force a Mutual Action.—We have discussed at some length the effect of a force applied to a body in changing the direction or magnitude of the body's motion. But a force always requires the interaction of two bodies, and the two bodies are always affected in opposite directions by this interaction. Illustrations of such mutual action are common in the student's experience. Two sticks struck together will indent each other. A bat stops a ball and

* By this phrase, *intensity of gravity*, is meant the force exerted by the earth upon unit mass. The *force of gravity* upon any body is equal to the number of units of mass in the body multiplied by the number expressing the intensity of gravity.

turns it back, but the ball stops, or at least checks, the bat. A weight pushes down upon a hand which pushes it up.

It is true that in many cases the effect is visible in one direction only. A pebble thrown against a massive stone-wall may appear to make no impression upon it. But careful observations may detect in the wall a slight jar at the instant of collision; and even if this is not the case, we are so familiar with the fact that mutual shocks are caused by such collisions, and with the further fact that many effects too small to be easily perceived really exist, that we have no difficulty in believing that the case is only an apparent exception to a universal rule.

261. Object of Exercises 37 and 38.—While it is usually evident that the large mass suffers less change of velocity than the small mass in any collision or other interaction between them, the definite law of such cases is not at first evident.

Is the change of velocity of each mass inversely proportional to the mass itself? If so, the change of *momentum* (§§ 250 and 251) of one mass will be just as great as that of the other. The object of Exercises 37 and 38 is to see whether, in a few simple cases of collision, the changes of momentum of the two bodies used are equally great.

In making this test we shall find it convenient to call momentum in one direction positive, and momentum in the opposite direction negative, just as we have already sometimes called velocity in one direction positive and in the other direction negative.

If we have this understanding about signs, it is evident that one body will suffer a + change of momentum and the other a − change of momentum in the collision. If these two changes are *numerically* equal, the total momentum, that is, the algebraic sum of the two momenta, will be just as great after collision as before. It will be like this:

$$\begin{array}{ccc} \text{Momentum Before.} & & \text{Momentum After.} \\ M_1 + M_2 & = & (M_1 + ch.\,M) + (M_2 - ch.\,M), \end{array}$$

where M_1 is the momentum of one body, and M_2 that of the other body, before collision, and *ch. M* is the numerical value of the change of momentum of each.

For example, if a mass of 20 gm., moving north at the rate of 50 cm. per second, which we will call a + velocity, should strike a mass of 15 gm. moving south at the rate of 40 cm. per second, which we will call a − velocity, and if the change of momentum suffered by each were 500, we should have

$$\begin{array}{ccc} \text{Momentum Before.} & & \text{Momentum After.} \\ (20\times 50)-(15\times 40) & = & [(20\times 50)-500]-[(15\times 40)+500]. \end{array}$$

In the two Exercises which follow we cannot find the actual velocities in centimeters or feet per second, or the actual momenta. But we can find, approximately, the *relative* velocities, that is, quantities which will bear the same ratios to each other that the actual velocities bear, and so we can get the *relative* momenta, which will be all we need for the purpose of our test.

For instance, suppose that in a certain case we are told not the real momenta, M_1, M_2, M_1', and M_2', but the relative momenta, M_1k, M_2k, $M_1'k$, and $M_2'k$, k being the same quantity throughout, we can tell whether

$$\begin{array}{ccc} \text{Moment Before.} & & \text{Moment After.} \\ M_1 + M_2 & = & M_1' + M_2', \end{array}$$

even when we do not know the value of k. For if

$$M_1k + M_2k = M_1'k + M_2'k,$$

it must be that

$$M_1 + M_2 = M_1' + M_2'.$$

EXPERIMENT.

(Preliminary to Exercise 37.)

Apparatus: Two pendulums like those used in Chapter VII, with suspensions of equal length, the longer the better. Wooden blocks to be struck by the pendulum-balls at any chosen points of their paths. A meter-rod.

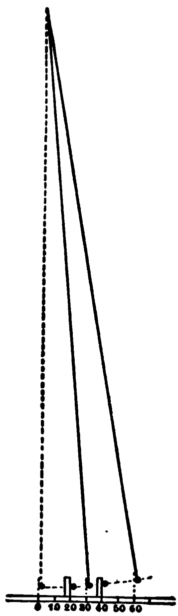

Fig. 191.

Suspend the pendulum so that there shall be about 4 cm. of clear space between their balls. Place the meter-rod between the balls, with its length at right angles with the line connecting the centres of the balls. Draw one of the balls to one side 60 cm., as measured on the rod, and the other ball in the same direction 30 cm., as in Fig. 191. Place the blocks so that each ball will strike after traveling one third of the horizontal distance to its lowest position. Release the balls at the same instant, and listen very attentively to decide whether they strike at the same instant.*

Let the balls be stopped in the same way after having traversed one half, two thirds, then the whole, of the distances to the lowest point. If time permits, the examination may be carried beyond the lowest points of the arcs.

If in all cases here suggested, and any others that may be tried, the blows given by the two balls come so nearly together that no interval between them can be detected, it may fairly be concluded that the ratio of the velocities of the balls at any instant—when each

* The method just described is, owing to the limited power of the ear to note small intervals of time, not very accurate. It serves, however, to indicate methods which would be accurate, and it is therefore given to relieve the teacher from the necessity of stating to his class entirely without proof the fact which he wishes to use. Another method, devised in its essentials by Mr. Schobinger, of the Harvard School, Chicago, will now be described. It is suited to individual experimentation rather than to lecture-room use. Suspend the two pendulums, made as nearly isochronous as practicable, some little distance apart—for instance, 60 cm Then stick a tall pin or a pocket-knife upright in the table just in line with the two balls and about 60 cm. distant from the nearer ball, so that when the eye of the observer is

is passing through its lowest point, for instance—is nearly, if not quite, equal to the ratio of the horizontal components of the paths in which the balls travel. Conclusions for the cases tried must not be extended to cases in which the arcs are much larger parts of circles.

EXERCISE 37.

ACTION AND REACTION: ELASTIC COLLISION.

(For two or three students working together.)

Apparatus: All the articles of No. 79, the beveled suspension-board being attached to the highest * convenient support, the wires in slots 1 and 2 (Fig. 192), so that the suspended balls just graze each other, with their centres about 4 cm. above the graduated rod.

Place the detents about 50 cm. apart, and so adjust the base-board that both of the suspension-wires shall lie in the line of sight through the middle of the slots of the uprights (see Fig. 193), the point of contact of the balls being exactly over the middle point, the 50-cm. mark, of the base-board.†

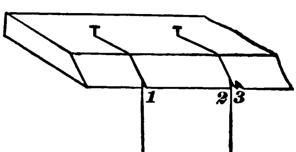

FIG. 192.

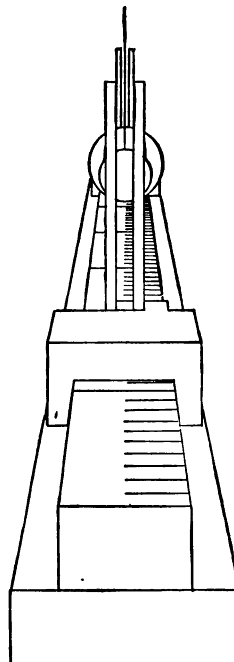

FIG. 193.

placed at the top of the pin or knife the nearer ball hides the other. Then let the two balls be pulled to one side, with such displacements that the nearer ball still hides the other from the observer placed as before. Let the balls be released at the same instant. If the nearer continues to hide the other during one whole swing, it is evident that the velocities must have nearly the same ratio throughout the swing.

* The distances mentioned in this Exercise are appropriate to a suspension 2 m. in length.

† The adjustment of the base-board is very important, and there should be some means of fastening it in place after the adjustment is made.

The larger ball will be called A, the smaller B.

The mass of the larger ball will be called m_a; the mass of the smal'er ball, m_b; the distance moved by A before collision will be called D_a; the distance moved by B before collision, D_b; the distance moved by A after collision, d_a; the distance moved by B after collision, d_b; movements from small to large on the scale will be called $+$; movements from large to small on the scale, $-$.

The movements of each ball are to be determined by watching that side of each which is turned toward the other ball.

FIRST CASE.—Make $D_a = 0$; that is, leave A in its position of rest. Make $D_b = 30$ cm.; that is, place the detent in such a position that when B is released it will have to move 30 cm., measured horizontally, before it can strike A.

After making sure that A is at rest, as nearly as may be, release B suddenly but quietly, and let it swing against A. Then note d_a, the distance A swings as a result of the collision, and d_b, the distance B rebounds after the collision.

Repeat this experiment several times, and finally, having decided upon the values of d_A and d_B, see whether this equation holds true.*

$$\underbrace{(m_a \times D_a) + (m_b \times D_b)}_{\text{Representing momentum before collision.}} = \underbrace{(m_a \times d_a) + (m_b \times d_b)}_{\text{Representing momentum after collision.}}$$

SECOND CASE.—Make $D_a = 15$ cm. and $Db = 0$. Determine d_a (the onward movement of A after the collision) and d_b by several trials, and then compare the total momentum before collision with the total momentum after collision, as in the preceding case.

In considering the result of this Exercise it should be borne in mind that a pendulum does suffer some slight loss of motion from the resistance of the air. As the momentum just before collision is estimated from the length of swing preceding contact, and the momentum just after collision from the swing following contact, it is evident that the former estimate will be a little larger, and the latter a little smaller, than it should be. A rough estimate of the error caused in this way may be obtained by studying the rate of decrease of the pendulum-swings.

* No account is here taken of the mass of the suspending wires. If it seems worth while to consider them, one half the mass of its own suspending wire may be added to the mass of each ball. One half, because the upper end of the wire is at rest, while the lower end moves as fast as the ball.

EXERCISE 38.

ELASTIC COLLISION CONTINUED; INELASTIC COLLISION.

This Exercise is a continuation of Exercise 37. It requires the same apparatus, with the addition of a small quantity of putty. The Cases of the two Exercises are numbered consecutively.

THIRD CASE.—Make $D_a = 15$ cm. and $D_b = 15$ cm. Release both the balls *at the same instant*. (This must be done by one person. It is well to attach a string to the upright of each detent, as in Fig. 194, pass the middle of the string through a hole or screw-eye near the centre of the base-board, and when all is ready pull suddenly upon this part of the string, taking care not to derange the base-board.)

Determine d_a and d_b by several trials, and then compare the total momentum before collision with the total momentum after collision, as in the preceding Cases.

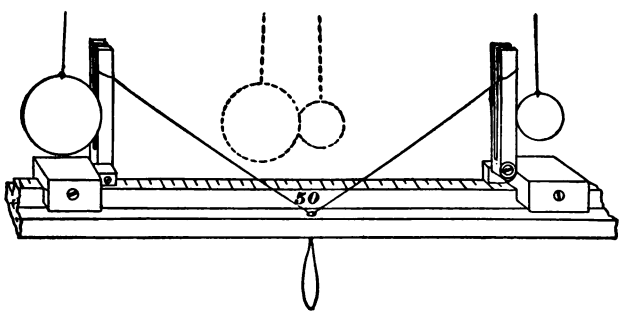

FIG. 194.

FOURTH CASE.—Let the suspending wires hang in slots 1 and 3 (Fig. 192). Put around the smaller ball a belt of putty about 1 cm. wide and 0.3 cm. thick, so that this belt will just touch the large ball when both are hanging at rest (Fig. 195). (The object of the putty is to destroy the elasticity of the collision, in order that the conditions of the momentum-test in this Case may be very different from those of the other cases. It will do little harm if the balls stick together at collision, though this may be prevented by covering the putty with thin paper at the point of contact.

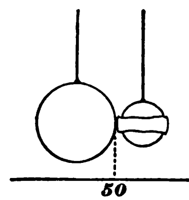

FIG. 195.

Make $D_a = 30$ cm. and $D_b = 0$ cm. Deter-

mine d_a and d_b by several trials, and then apply the momentum-test as before, taking for m_b the mass of the ball B and its belt of putty.

262. Newton's Third Law of Motion.—It is intimated in the last paragraph of Exercise 37 that, on the whole, the total momentum before collision, as estimated in the experiments of that Exercise, may appear to be somewhat greater than the total momentum after collision, as estimated. But the more carefully the experiments are performed, and the more intelligently and accurately the errors of experiment are allowed for, the less the difference becomes.

This is just as true for inelastic collision as for elastic collision. It is a general law of very great importance that: *The algebraic total momentum of any two bodies is not changed by any interaction that may take place between them.* Under the form, *Action is equal to Reaction and opposite in direction*, this is called Newton's *Third Law of Motion.*

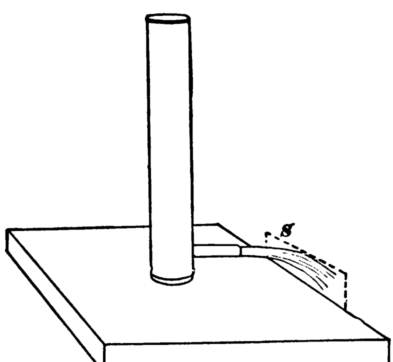

FIG. 196.

263. Illustrations of Action and Reaction.—Good illustrations of action and reaction are familiar, but are not always recognized as such.

EXPERIMENT.

Remove the plate *S* (Fig. 196) from No. LVI, close the tube at the bottom, fill the cylinder with water, float the whole in a tank of water, and then open the tube. The horizontal pressure within the water forces a horizontal stream from the tube, and the cylinder itself is driven in the opposite direction.

Repeat, with *S* in place.

This experiment is a type of many others that might be shown. The revolving lawn-sprinkler is a case in point. A bent tube through which water is flowing, as in Fig. 197, tends to straighten. The water entering at *N*, with a northerly velocity let us say, is, by the pressure *P′* exerted upon it by the tube at the angle, made to emerge with a westerly velocity, but at the same time the water reacts upon the tube with a force represented by *P*, tending to straighten it.

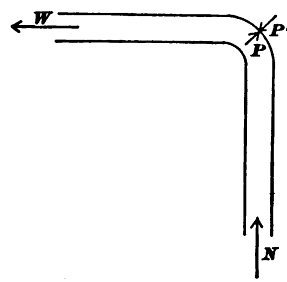

FIG. 197.

A swimmer pushes himself upward and forward by pushing the water downward and backward, and what water is to the swimmer air is to a flying bird. An air-tight box in which a bird is sustaining itself on its wings would press as heavily upon its support as if the bird were at rest upon the bottom; for a stream of air driven downward by the wings would press upon the floor.

An athlete can cover about a foot more in the "standing broad jump" by holding in each hand a stone of suitable size, which he flings swiftly behind him when in mid-air.

A child soon learns that he cannot move a sled, even upon the smoothest ice, by sitting upon the sled and tugging at his own rope. It is true that by standing upon the sled, drawing his body back upon one foot and projecting it forward upon the other foot, he can move the sled forward.

The explanation of this latter case is, that there is some friction between the runners and the ice. This friction is enough to prevent the sled from moving backward while the body is slowly acquiring its forward momentum, but is not sufficient to resist the shock when the body is suddenly arrested by the forward foot.

When a charge of dynamite explodes in the open it leaves a great hole in the ground beneath the spot where it lay, and some people, seeing this permanent effect, after the tremendous temporary effect has vanished from the atmosphere, exclaim, "See what a queer thing the action of dynamite is! It spends its force downward." But dynamite has no such eccentric habit. It presses, like every other explosive, in all directions. The hole in the ground testifies merely to the suddenness and violence of the explosion, which are so great that the repelled air cannot give way swiftly enough to save the earth below from injury.

PROBLEMS.

(1) A bullet weighing 10 gm. and moving 200 m. per second, strikes horizontally into a suspended block of wood weighing 500 gm. and lodges there.

(*a*) How great is the momentum of the bullet before striking?

(*b*) How great is the momentum of bullet and block together just after the collision?

(*c*) How great is the velocity of the block just after the collision?

(The velocity of bullets and cannon-balls was formerly estimated by means of experiments and calculations like those here indicated.)

(2) If a bullet weighing 5 gm. is shot from a 4000-gm. rifle with a velocity of 300 m. per second, how great is the backward velocity acquired by the rifle itself during the discharge, if it hangs free.

(3) (*a*) If the discharge in the preceding problem lasted .005 second, how many dynes was the average force exerted upon the bullet during that time?

(*b*) How many grams weight would this force equal?

(4) A repulsion equal to 50 poundals acts for 3 seconds between two bodies, one of which weighs 100 lbs. and the other 20 lbs. How great is the velocity in feet per second that each body acquires?

(5) If the force in the preceding problem were 50 pounds-weight, how great would the velocity be?

(6) A ball weighing 50 gm., moving north at the rate of 90 cm. per second, strikes squarely a ball weighing 100 gm. at rest, and rebounds with a velocity of 10 cm. per second south. What velocity does the other ball gain by the collision?

(7) A ball weighing 30 gm., moving north with a velocity of 200 cm. per second, strikes squarely a ball weighing 70 gm. moving south with a velocity of 100 cm. per second. After the collision the first ball is moving south with a velocity of 100 cm. per second. What is the direction of motion and the velocity of the other ball?

CHAPTER XIX.

WORK AND ENERGY.

264. Work: Units of Work.—The next physical quantity to be considered is *work*. This word is used by the physicists to signify *the doing of something against opposition.** If a man pushes a saw through a board against friction and cohesion of the particles of wood, if he raises a weight against the force of gravity, he does work. If the saw sticks, so that he pushes in vain, if after lifting the weight a certain distance he encounters some obstacle and ceases to raise it, he is no longer doing work, although he may be making more effort than before.

A statement like this frequently arouses a feeling akin to indignation in the mind of the student, who is apt to feel that scant justice is done to the supposed toiler. But the science of physics does not concern itself with emotions and purposes. Looking to the result accomplished, it says that a man who is merely sustaining a weight is doing no more for that weight than a post could do. He is not prevailing over the opposition of gravitation. He is not doing work in the scientific sense of the word.

The nature of a work as a definite mechanical *process* being thus defined, the next point to be considered is the measurement of work. For an English-speaking person the most natural unit of work is the amount necessary to raise one pound one foot. This is called the *foot-pound*,

* Consult Clerk Maxwell: *Treatise on Heat*, chapter IV.

and is in almost universal use among English-speaking engineers. Evidently, to lift 2 lbs. 1 ft., or 1 lb. 2 ft., requires 2 ft.-lbs.; to lift 4 lbs. 3 ft., or 3 lbs. 4 ft., requires 12 ft.-lbs. In short, to lift m lbs. h ft. requires mh ft.-lbs.

Although the foot-pound is *defined* by reference to lifting a certain mass a certain distance, any kind of work can be expressed in foot-pounds. Thus if a man pushing a carpenter's plane horizontally along a board exerts a horizontal force as great as the vertical force that he would exert in holding up a weight of 10 lbs., we are in the habit of saying that he exerts a force of 10 lbs., and if, exerting this horizontal force all the time, he pushes the plane 5 ft., we say that he does 10×5 ft.-lbs. of work.

French engineers, and others who have the kilogram as the unit of mass and the meter as the unit of distance, take as their unit of work the *kilogram-meter*, that is, the amount of work required to lift one kilogram one meter. Various other units of work which are in common use among physicists will be defined later in this book (§§ 269 to).

Measurement of work, then, always involves both force and distance. When the force is parallel to the motion maintained or assisted by it, the *work done by the agent exerting the force is reckoned by multiplying the number expressing the force by the number expressing the distance which the point acted on by the force moves during the action of the force.*

PROBLEM.

How many ft.-lbs. of work must be done in dragging a 100-lb. wt. a distance of 50 ft. along a horizontal surface upon which its coefficient of friction is $\frac{1}{8}$?

265. Work Done by Forces Oblique to the Line of Motion.—When the force applied to any point is not parallel

to the motion of that point, the calculation of the work done is not quite so simple.

For instance, suppose that a street-car moves 10 ft. along its track while a man pushes against the car, exactly at right angles with the track, with a force of 50 lbs. His effort does not help or hinder the motion of the car, save, possibly, by changing a little the friction against the sides of the rails. He does no *work* upon the car.

If the line of the force exerted by the man makes an acute angle with the direction of motion of the car, the case is like that discussed in § 230. To find the work done by the man we multiply the useful component of the force, OB (Fig. 166), by the distance the car moves while the force acts, disregarding the useless component OA.

Observe that in such a case the product of the whole distance by that component of the force which is parallel to the motion is equal to the product of the whole force by that component of the distance which is parallel to the force. Thus in Fig. 198, where the whole force is repre-

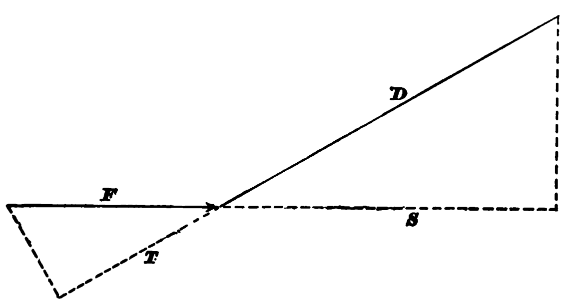

FIG. 198.

sented by F and the whole distance by D, we see from the similarity of the two triangles that $T : F :: S : D$, whence $T \times D = F \times S$.

266. Rate of Work: Horse-power, etc.—To express the *rate* at which work is done by a machine or an animal we have to tell the amount of work done in a certain time; for example, the number of foot-pounds done per second, or per minute.

The term *one horse-power*, used to indicate a rate of work, is equivalent, in English-speaking countries, to 550 ft.-lbs. per second, or 33,000 ft.-lbs. per minute. This rate of work is somewhat greater than that of an ordinary draught-horse moving at his average pace and exerting his average pull.

The following table, taken from one given by the Scotch engineering authority Rankine, compares the rate of work of a man and of a horse under various conditions. R stands for the resistance, expressed in pounds-force, overcome in each case, and V stands for the velocity of motion, in feet per second, against this resistance.

Man.	R lbs.	V ft. per sec.	$R \times V$ ft.-lbs. per sec.
1. Raising his own weight up stair or ladder	143	0.5	71.5
2. Hauling up weight with rope	40	0.75	30
3. Lifting weights by hand	44	0.55	24.2
4. Shoveling up earth to a height of 5 ft. 3 in.	6	1.3	7.8
5. Pushing or pulling horizontally	26.5	2.0	53
6. Working a pump	13.2	2.5	33
Horse.			
7. Thoroughbred, cantering and trotting, drawing a light railway carriage	30.5	$14\frac{2}{3}$	447.3
8. Draught-horse, walking, drawing cart or boat	120	3.6	432

PROBLEMS.

(1) If a horse drawing a cart exerts a force of 110 lbs. in the direction of motion, how many ft.-lbs. of work does he do in moving the cart 5 ft.?

(2) If a machinist exerts upon a file a force equivalent to 15 lbs. downward and 20 lbs. forward, how many ft.-lbs. of work does he do in 50 horizontal strokes of the file, each stroke being 8 in. long?

(3) A railroad train weighing 1,000,000 lbs. is drawn at a uniform speed along a level track, the coefficient of friction being .005. How many ft.-lbs. of work are done in drawing it 1000 ft.?

(4) A mass of 30 lbs. is placed on an inclined plane which is 10 ft. long, the base being 8 ft. and the height 6 ft. Friction is to be called zero.

(*a*) How large a force, parallel to the incline, is needed to keep the mass moving up the incline?

(*b*) How many ft.-lbs. of work will be done by this force in drawing the mass the whole length, 10 ft.?

(*c*) How large a force, parallel to the base, would be needed to keep the mass moving up the incline?

(*d*) How many ft.-lbs. of work would be done by this force in drawing the mass the whole length of the incline?

(*e*) How many ft.-lbs. of work would be done in lifting the mass straight up 6 ft., the height of the incline?

(5) The meter being, approximately, equivalent to 3.28 ft., and the kilogram to 2.2 lbs., how many kilogram-meters are equivalent to 100 ft.-lbs.?

267. Energy.—Work and energy are so closely connected that they can hardly be discussed separately. In fact, by definition, *energy is the power of doing work.*

As work involves both force and motion, so energy involves not only the power of exerting force upon a body, but also the power of following up the body, for a greater or less distance, if it yields to the force. A compressed spring, a charge of gunpowder, a flying cannon-ball, each of these is a body possessing energy. A weight raised above the surface of the earth is ordinarily said to possess energy, bacause it can do work during its descent, but strictly the energy in this case belongs not to the weight alone but to

the system made up of the earth below and the weight above, which by their mutual attraction tend to approach each other and to overcome any resistance which opposes their approach.

268. Kinetic Energy and Potential Energy.—Two classes of energy may be distinguished.

The moving cannon-ball, quite apart from any attraction or repulsion, has energy by virtue of its inertia (see § 244) and its motion. This is called *kinetic energy*.

On the other hand, in the case of the raised weight just described, in a bended bow, in an explosive mixture of hydrogen and oxygen, in a Leyden jar charged with electricity, etc., we have energy due to some attraction or repulsion between different parts of the same body or different bodies of a system. This is called *potential energy*.

Each of these forms of energy is the power of doing work, and each is measured in the same units as work, that is, in foot-pounds, kilogram-meters, etc.

For example, let us find the number of foot-pounds of energy possessed by a body whose mass is m lbs., and which is moving with a velocity of v ft. per second. Since experience shows that kinetic energy does not depend upon the direction of the motion, we shall for convenience suppose that the body is moving straight upward and that its kinetic energy is to be entirely expended in raising the body itself. If the body rises h ft. from its present position before stopping, its kinetic energy will have done $m \times h$ ft.-lbs. of work. But how great is h? From § 255 we know that h (there called s) $= v^2 \div 2g$, g being equal to 32.2 nearly. Putting for h this value, we have as a general expression, good for every case in which a body of m lbs. is moving with a velocity of v ft. per second,

$$\textit{kinetic energy} = \frac{mv^2}{2g}\ \textit{ft.-lbs.}$$

Everywhere in the example just discussed we may, without impairing the reasoning in the least, write *kilograms* for pounds and *meters* for feet, g becoming in this case 9.81 nearly (see § 253). So we may write *grams* for pounds and *centimeters* for feet without altering the reasoning or the general formula obtained; but g in this case will have the value 981 nearly.

269. Statement of Work in "Absolute Units."—Sometimes work is expressed in foot-*poundals* instead of in foot-pounds. As the earth's attraction for a one-pound mass is g poundals (§§ 249–253), it is evident that one foot-pound equals g foot-poundals. Hence, if we use this latter unit in expressing kinetic energy, we have, in the case of m lbs. moving with a velocity of v ft. per second,

$$\textit{kinetic energy} = \left(\frac{mv^2}{2g} \times g\right) \textit{ft.-pls.} = \frac{mv^2}{2} \textit{ ft.-pls.}$$

Physicists, as distinguished from engineers, are the world over in the habit of reckoning in grams and centimeters, rather than in pounds and feet or kilograms and meters. They generally take as their unit of force the *dyne* (see § 249), and as their unit of work the work done by one dyne acting a distance of one centimeter. This unit may be called a *dyne-centimeter*, but it is used so much that a special short name is given it. It is called the *erg*. If in the measurement of kinetic energy we use grams instead of pounds, centimeters instead of feet, and ergs instead of foot-poundals, we shall arrive at the general formula

$$\textit{kinetic energy} = \frac{mv^2}{2} \textit{ergs.}$$

In short, when we use a *gravitation*-unit of work, that is, a unit of work based upon a gravitation-unit of *force*, we must write

$$\textit{kinetic energy} = \frac{mv}{2g};$$

but when we use an *absolute* unit of work, that is, a unit of work based upon one of the so-called absolute units of force, we must write

$$\textit{kinetic energy} = \frac{mv^2}{2}.$$

270. Estimation of Potential Energy.—It would be difficult to give here any useful formula for the amount of potential energy contained in a body or system of bodies, in any case except that of bodies raised above the earth's surface. In such a case if m is the mass of the body, and if h is its height above the earth's surface, we see that the body in descending to the earth may be made to exert m *gravitation* units of force throughout a distance h. Hence its potential energy, reckoned in gravitation-units of work, is $m \times h$. In "absolute" units of work it is $m \times h \times g$.

PROBLEMS.

(1) How many ft.-lbs. is the kinetic energy of a 500-lb. cannon-ball moving with a velocity of 2000 ft. per second? How many ft.-pdls.?

(2) How many kgm.-m. is the kinetic energy of 200 kgm. moving with a velocity of 600 m. per second?

(3) How many gm.-cm. is the kinetic energy of 200 gm. moving with a velocity of 600 cm. per second? How many ergs?

(4) How many ft.-lbs. is the potential energy of 75 lbs. 12 ft. above the earth's surface?

(5) A certain body has 800 ergs of kinetic energy.

(*a*) How *far* will this supply of energy carry the body against a constant resistance of 40 dynes?

(*b*) Can you tell from the information given how *long* the body would move against this resistance?

271. Conservation of Energy.—A body thrown upward spends its kinetic energy in doing work against gravitation, in pulling itself away from the earth, but the work thus

done goes to store up potential energy. While the body falls this potential energy is turned back into kinetic energy. This is a familiar example of the change from one form of energy to the other which is continually going on about us, sometimes visibly, frequently unseen. Energy takes extremely subtle shapes—in heat, in light, in electric currents, in chemical conditions—but the two classes *kinetic* and *potential* include all forms of which we have knowledge. All such forms are so related that energy in any one of them may be turned into any or all of the others in succession, and back finally to its first shape.

"In actually performing such a circle, or *cycle*, of transformations we observe an apparent loss. We bring back to the original form a smaller amount of energy than we started with. Thus a ball thrown upward returns to the hand with diminished velocity. We must not, however, conclude from such trials that there is any real destruction of energy in the process of transformation.

"If we were to take a quantity of water filling a certain vessel and pour it into a dozen vessels in succession until we reach the first vessel again, this vessel would not now be filled, but we should not infer from this any real destruction of the substance of the water. We should account for the apparent loss by the amount left clinging to the sides of the vessels, or absorbed by them, or spilled, or evaporated, or possibly changed by chemical action into some different form of matter; and we should have not the slightest doubt that, if all these small amounts could be collected, they would just fill the space now empty in the vessel. This confidence is the result of long experience, which has taught us to believe that matter cannot be destroyed.

"Equally long experience has been teaching the somewhat more subtle truth that energy cannot be destroyed; that its apparent annihilation in its transformation, or wasting away without transformation, is to be explained, like the dis-

appearance of water in the process described above, merely as an escape in various ways from the immediate scope of our observation. But so numerous, and in many cases obscure, are these ways of escape, that their full extent and significance have been perceived only within quite recent years.

"The belief that energy cannot be destroyed or diminished, and the converse belief that energy cannot be created or increased,—which latter belief is practically rejected by those who attempt to 'invent perpetual motion,'—constitute the doctrine of the Conservation of Energy. All the familiar devices for the advantageous application of mechanical power, such as the lever, the inclined plane, and the hydraulic press, are simple examples of the truth of this doctrine, and their laws may be at once deduced from it."*

272. Comparison of Momentum and Kinetic Energy.—Momentum and kinetic energy are often confounded or mistaken for each other. The following comparison or contrast may therefore be of use:

Momentum is $m \times v$. There is no one-word name in common use for the unit of momentum.

Kinetic energy, in "absolute units," is $\frac{1}{2}mv^2$. It is reckoned in ft.-pdls., ergs, etc., the same units in which work is measured.

Momentum is equal to the product of *force* and *time.* If we know the momentum only of a body, we can tell how long it would move against a given resistance, but not how far.

Kinetic energy is equal to the product of *force* and *distance.* If we know the kinetic energy only of a body we can tell how far it would move against a given resistance, but not how long.

* *Elementary Ideas, Definitions, and Laws in Dynamics.*

We have already seen that, by agreeing to call momentum in one direction + and momentum in the opposite direction −, we can frame the law, that the (algebraic) total momentum of two bodies is unchanged by any interaction between them. Thus, if two masses, each of m gm., one having velocity v cm. per second and the other being at rest, collide and then move on as one, the velocity, v', after collision will be one half as great as v.

The total kinetic energy is not usually the same after a collision as before. Thus, in the case just described, the kinetic energy before collision is $\frac{1}{2}mv^2$, and that after collision is $\frac{1}{2}(2m)v'^2 = \frac{1}{4}mv^2$.

273. Internal Energy.—In the last paragraph preceding we considered merely the kinetic energy of the masses moving as wholes. We left out of account the energy, partly kinetic, of the vibratory or rotary motion of the particles within each mass. If this internal energy were considered, we should find the total energy to be unaffected by the collision, as the law of conservation of energy requires, but we should not necessarily find the total *kinetic* energy the same after collision as before.

One form of internal energy is *heat*, which will be considered in Chapter XXI.

CHAPTER XX.

FLUIDS IN MOTION.

274. Torricelli's Law.—Let Fig. 199 represent a tank containing water, S being the surface exposed to atmospheric pressure, and N a small nozzle from which the water can flow. The distance h, from the level of the surface to the level of the middle of the nozzle, is called the *head* of the water.

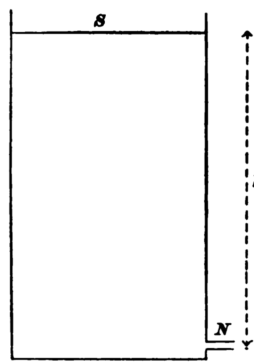

FIG. 199.

Let the nozzle be opened and let water flow in at the top as fast as it flows out at N, so that the level in the tank remains unchanged. Under these conditions the total energy of the water in the tank remains unchanged. Therefore, every gram or pound of water escaping through N must carry away just as much energy as the same amount of water brings in at the top.

It is evident that the *potential* energy of the unit-mass of water flowing from N is h gravitation-units less than the potential energy of the unit-mass at the top. Consequently the *kinetic* energy of the unit-mass at N must be h gravitation-units greater than the kinetic energy of the unit-mass at the top. If we suppose the kinetic energy of the latter to be zero, the kinetic energy of the unit-mass escaping from N must be *h gravitation-units.*

Accordingly, if v is the velocity of the escaping jet, we have

$$h = \frac{v^2}{2g}, \quad \text{or} \quad v = \sqrt{2gh}.$$

According to this formula, which is obtained by disregarding friction, the velocity of a water-jet escaping into the atmosphere at a distance h below the water-surface in a tank open to the atmosphere is the same as the velocity acquired by a body in falling the distance h (see § 255).

This law, which was discovered by Torricelli, is called *Torricelli's law.*

The law holds for other liquids as well as for water, provided friction may be neglected; but in some liquids, molasses, for example, friction cannot safely be neglected.

275. Water in Tubes.—Torricelli's law, which neglects friction, gives very inaccurate results, even for water, if the tube through which the water escapes is long in comparison with its diameter.

The rule for finding the velocity in such a tube, or pipe, is rather complicated, and no proof of its correctness will here be attempted, but it is worth while to state it and illustrate its use.

Let v = the velocity of the stream in a pipe;
h = the *head*, that is, the height of the water in the open tank, from which the pipe leads, above the point where the pipe discharges into the air;
l = the length of the pipe;
d = the diameter of the pipe;
g = the acceleration of gravity, as usual.

Then, for a pipe of fairly smooth internal surface, the formula is

$$v = \sqrt{2g\left(h - 0.03 \times \frac{l}{d} \times \frac{v^2}{2g}\right)}.$$

This formula holds when all the distances are reckoned in feet as well as when they are all reckoned in centimeters. If the inside of the pipe is very rough, the quantity 0.03 is replaced by a larger one, but otherwise the formula is the same for rough pipes as for smooth ones.

276. Loss of Head.—By comparing the formula of § 275 with that of § 274, we see that the only change made by considering friction is to subtract $\left(0.03 \times \frac{l}{d} \times \frac{v^2}{2g}\right)$ from h, the head. This subtracted quantity is called the "loss of head due to friction," the velocity being the same as if the flow took place through a frictionless pipe with a head $\left(h - 0.03\frac{l}{d}\frac{v^2}{2g}\right)$.

EXAMPLE.

To show the use of the formula of § 275, let us write it in the form

$$v^2 = 2gh - 0.03 \times \frac{l}{d} \times v^2,$$

whence $$v = \sqrt{2gh \div (1 + 0.03 \times \frac{l}{d})},$$

and then apply it to find the velocity of water flowing through a 2-in. pipe 1 mile long, from a spring in which the surface is 16 ft. above the outlet of the pipe. We have, calling g 32,

$$v = \sqrt{2 \times 32 \times 16 \div (1 + 0.03 \times \frac{5280}{\frac{1}{6}})} = 1.04 \text{ ft.}$$

If there were no friction, we should have in this case, by Torricelli's formula, $v = 32$ ft.

277. Flow of Gases.—Formulas similar to those of §§ 274 and 275 can be applied to gases, but the definition of "head" in the case of a gas is somewhat more complicated than in the case of a liquid, and the reader is referred to more advanced books for the discussion. That friction of a gas may be very great in a long narrow tube is made plain by experiments in Chapter VI.

QUESTIONS AND PROBLEMS.

(1) A bar 20 ft. long is placed horizontally. A force of 100 lbs. due north is applied to the bar at a point 5 ft. from one end, and a force of 250 lbs. due south is applied at a point 10 ft. from the same end. How great a force would just neutralize these two forces, and at what point of the bar should it be applied?

(2) A uniform bar 12 ft. long and weighing 10 lbs. bears at one end a load of 8 lbs. and at the other end a load of 16 lbs. The bar so loaded is placed horizontal upon a supporting point. How great is the pressure upon the support, and how far must the support be from the end bearing the 8-lbs. load?

(3) A beam 12 ft. long and weighing 8 lbs. has

at 9 ft. from one end a force of 7 lbs. acting downward,
" 4 " " same " " " " 5 " " upward,
" *b* " " " " " " " *a* " " " ,

and is in equilibrium. Find the magnitude of *a* and of *b*.

(4) The deck of a steamer is 40 ft. wide. A ball is rolled across the deck at right angles to the steamer's course. If the ball crosses in 6 seconds, and the boat's rate of motion is 10 miles per hour, how far does the ball travel in the 6 seconds?

(5) A car is moving at the rate of 20 miles per hour; how far will a bullet, let fall from the car at a point 7 feet above the ground, move along the track before striking the earth?

(6) State what kind of motion is caused by a single constant force. Illustrate your answer.

(7) Two bodies initially at rest move towards each other in obedience to mutual attraction. Their masses are respectively 20 gms. and 100 gms. If the force of attraction be $\frac{1}{10}$ of a dyne, find the velocity acquired by each mass in 30 seconds.

(8) The momentum of a certain body is 600. It encounters a constant resistance of 50 absolute units.

(*a*) How long will it continue to move?

(*b*) Can you tell how far it will move?

(9) If a body whose mass is 100 gms., originally moving due north with a velocity of 20 cm. a second on a perfectly smooth horizontal surface, be acted upon by a force of 500 gms. due northeast and an equal force due northwest, what velocity will the body have at the end of one second, and how far will it have moved during that second?

(10) An engine pulls with a force of 4000 lbs. upon a train of cars

weighing 400,000 lbs. upon a horizontal track. If the friction of the train were zero, how great a velocity would it acquire in one second, starting from rest? If the coefficient of friction were $\frac{1}{300}$, how great a velocity would be acquired in one second?

(11) A ball weighing 50 gms., moving horizontally at the rate of 80 cm. per second, strikes a ball weighing 200 gms., which is at rest. If the smaller ball rebounds with a velocity of 20 cm. per second, how great is the velocity imparted to the large ball by the collision?

(12) A ball weighing 50 lbs. and moving with a velocity of 10 ft. a second strikes squarely a ball at rest weighing 150 lbs., and imparts to it a velocity of 3 ft. per second. What velocity has the smaller ball after the collision?

(13) A bullet weighing 10 gms. is shot with a velocity of 300 m. a second from a rifle weighing 6 kgms. How great a backward velocity is given to the rifle by the discharge?

(14) Suppose that Exercise 38 is performed with balls of soft putty, *C* and *D*. *C* weighs 150 gms., and swings south 25 cm., when it collides with *D*, weighing 20 gms., at rest. How far, approximately, will *D* now swing?

(15) If a mass of 1 mgm. is acted on by a force of 1 dyne for 10 seconds, no other forces being applied to it, how much energy will it require in that time if it starts from rest?

(16) In a place where the average barometric height is 75 cm., what is the value in dynes of the atmospheric pressure per square centimeter?

(17) Define carefully the dyne and erg, the poundal and foot-poundal. Define work.

(18) A street-car is moving northeast along a track. The team is pulling exactly north with a force of 800 lbs., and continues to do so while the car moves far enough along the track to go 12 ft. north. How much work does the team do during this movement? (State the unit in which the work is reckoned.)

(19) A block weighing 10 lbs. rests upon an incline such that the block must move 5 ft. in order to rise 3 ft. The pressure of the block against the incline is in this case 8 lbs. Let the coefficient of friction be $\frac{1}{4}$. How much work must be done against gravity by a force parallel to the incline in drawing the block 5 ft.? How much work against friction?

(20) A body weighing 100 lbs. rests upon an incline. How much

work must be done in drawing the body 10 ft. up this incline, the vertical ascent being 6 ft. and the coefficient of friction being $\frac{1}{3}$?

(21) A hammer hangs vertically, head downward. The centre of gravity of the head, which weighs 16 lbs., is 3 ft. below the point of support. The centre of gravity of the handle, which weighs 1 lb., is 1 ft. below the point of support. How much work is necessary to lift the hammer into a horizontal position at the level of the point of support?

(22) The kinetic energy of a certain body is 600 ergs. It encounters a constant resistance of 50 dynes. How far will it go before coming to rest? Can you tell how long it will continue to move?

(23) If the mass of a body is 50 lbs. and it is moving with a velocity of 30 ft. per second, how great is its kinetic energy as reckoned in absolute units? reckoned in gravitation-units? Name the units of energy in both cases.

(24) The mass of a steamer is 5000 tons. It is drifted against a wharf by the tide with a velocity of 2 ft. per minute. What is its kinetic energy?

(25) A mass of 10 lbs. is thrown upward with a velocity of 20 ft. per second. (*a*) How many foot-pounds of kinetic energy has it? (*b*) How many foot-poundals? (*c*) How much work was done in giving this velocity?

(26) (*a*) How long will the mass mentioned in (25) rise? (*b*) How far will it rise? (*c*) What can you say of its energy when the mass reaches the greatest height?

(27) A mass of 10 lbs. is 20 ft. above the earth's surface and is moving with a velocity of 5 ft. per second. (*a*) How many foot-pounds is the potential energy? (*b*) How many foot-pounds is the kinetic energy?

(28) A bullet which travels at the rate of 1200 ft. per second is found to penetrate a wooden target to the depth of 6 in. What velocity would be required to enable a bullet of the same size and shape to penetrate 8 in. into the same target, if the resistance encountered is the same at all depths?

CHAPTER XXI.

HEAT: TEMPERATURE, CONVEYANCE OF HEAT.

278. Sensation of Warmth.—The bodies which we touch and handle in every-day life might be roughly classified as warm, cold, or in a neutral condition, that is, giving us neither the sensation of warmth nor of cold. This classification is an exceedingly variable one at best, depending largely upon the condition of the observer at the time. To the hands of any one coming indoors on a cold winter's day, a basin of water at the temperature of an ordinary living-room feels warm, while the same water at the same time would feel cool to any one just risen from a warm bed.

But all of us agree in a general way as to the meaning of the words hot and cold.

279. Historical.—We have now to consider the nature of the agent, heat, which is added to bodies in order that their temperature may rise, and is taken away from them, in some measure, in order that their temperature may fall.

During the first half of the nineteenth century, heat was generally supposed to be a fluid, invisible and without weight, capable of working its way into bodies somewhat as water enters a sponge, thereby causing expansion, and capable, too, of being forced out of bodies by friction or blows. Experiments which should have been fatal to this belief were made about a century ago by Count Rumford, who in boring cannon for the Bavarian government showed that there appeared to be no limit to the amount of heat which

continued grinding would bring out of a body, and by Sir Humphry Davy, who showed that two pieces of ice could be melted by rubbing them together.* The theory survived these attacks for a long time, but at length, between 1840 and 1850, the careful experiments of Joule, who measured the amount of heat developed in many different processes, gave it the death-blow, and it steadily, though still gradually, lost its hold upon students' minds.

280. The Present Theory.—In order to understand the present theory we must think of the very little particles of which all bodies, whether solids, liquids, or gases, are made up. These particles, or molecules, which are too small to be seen, one by one, even with the most powerful microscope, are believed to be all the time in a state of more or less rapid motion. In solids, this motion may be compared to the movement of people seated close together in a large company, each individual having a certain freedom and a certain activity of his own, although confined to a particular seat and kept in close contact with his neighbors. In liquids, we may think of the particles as moving about more or less from one place to another, the crowd in which they are packed being less rigid, though perhaps not less dense, than that of the previous case. In gases, we consider the particles as moving about with very great freedom, like skaters on a wide field of ice, occasionally bumping against each other or against the boundary of the space in which they move, but most of the time free from such disturbances.

We do not mean that a gas-particle goes any long time without hitting something. We mean that the time between two hits is usually long compared with the time a hit lasts. In fact, the particles are believed to move so fast that any one of them is likely to hit thousands and even

* See Tyndall's *Heat as a Mode of Motion.*

millions of times a second, and as each cubic centimeter of an ordinary gas contains millions upon millions of such particles, the continual pelting of this multitude against the walls of the containing vessel makes what seems to be a perfectly steady pressure, tending to burst the vessel or make it larger.

The following experiment gives evidence of the incessant activity of gas-particles, the phenomena observed being due to the fact that the particles of the illuminating-gas move, under like conditions of pressure and temperature, with a greater average velocity than those of the heavier air.

EXPERIMENT.

In Fig. 200 *b* is a bottle, *c* is a porous cup, such as is used to separate the two liquids of a galvanic cell, *s* is a glass stem leading from *c* to the colored water in a tumbler *t*. At first the cup and bottle contain air at the ordinary pressure. When gas is allowed to flow through the rubber tube *r*, it soon drives most of the air out of the space between the cup and the wall of the bottle. Then bubbles of air come out from the stem *s* and rise through the water in the tumbler. This is because the particles of gas are working their way into the cup through its porous wall faster than the air-particles can work their way out by the same passages.

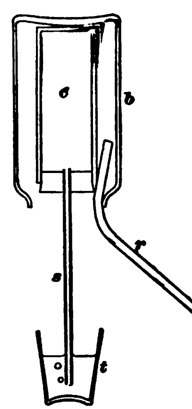

FIG. 200.

When the bubbling has continued for a little while, stop the flow of gas through the tube *r*, and then lift the bottle, exposing the porous cup to the outside air, but leaving the lower end of *s* still in the water. Now the gas which has entered the cup will begin to work its way out faster than air can work its way in, and the water will begin to rise in the stem.

By pushing the bottle, which should be held mouth downward all the time, down over the cup again, the water can be made to descend promptly in the stem.

281. Velocity of Gas-particles.—The aggregate weight of all the particles in a given volume of gas being known, it is possible to make an estimate of the mean, or average, velocity with which they must be moving in order to account for the expansive pressure exerted by the gas. In hydrogen at 0° C., the average velocity of the particles is supposed to be something like 2000 m., more than a mile, per second.

282. Relation of Heat and Temperature.—Two bodies are said to have the same *temperature* if there is no flow of heat from one to the other when they are brought into contact with each other, but two bodies in this condition would not necessarily contain the same amount of *heat*.

The case is somewhat like that of two communicating vessels containing water. If the water surface is at the same height in both vessels, there will be no flow from one to the other, though the first may contain a thousand times as much water as the second.

We shall in the next chapter begin to study the effects of change of temperature and shall undertake to measure, after a fashion, differences of temperature, but the *measurement* of heat will be taken up later, in Chapter XXIII.

Conveyance of Heat.

Before attempting any strict measurement of either heat or temperature, it will be well to consider the modes by which heat is conveyed from one place or body to another. These are three—*conduction*, *convection*, and *radiation*.

283. Conduction.—The following experiment will give some rude idea of the relative freedom with which heat travels through the three solids examined.

EXPERIMENT I.

Take a rod of copper, a rod of iron, and a rod of glass, each about 5 mm. in diameter and 30 cm. long (No. LVII). Support all three

on bits of brick or pieces of asbestos laid on a piece of sheet-iron on the ring of a retort-stand. Insert one end of each rod a very little way into the flame of a Bunsen burner, and keep the outer ends of the rods apart. After 15 or 20 minutes test the temperature of each rod by putting on it at successive short intervals, beginning with the outer end, a pinch of powdered sulphur. In the case of each rod stop putting on the sulphur at the point where the latter is seen to melt. The melting-point of sulphur is 111° C. Finally, measure the distance from the 111° point on each rod to the flame.

Transmission of heat in this way is called *conduction.* It is a flow of heat *as such*, that is, without change into any other form of energy, and without the help of any movement, except, of course, molecular heat-movements, in the substance through which the heat is passing.

Most ordinary substances have been examined in regard to their power of conducting heat, and it is found that solids, and particularly metals, are the best conductors, while liquids, except mercury, and gases are extremely poor conductors.

EXPERIMENT 2.

Wind a piece of soft copper wire or a slender strip of sheet-lead about a bit of ice of the size of a hazelnut, and drop it into a test-tube three-quarters full of ice-cold water. Heat the water at the top of the tube, the ice being at the bottom, by holding the tube inclined in the flame of a Bunsen burner, and notice whether the ice melts rapidly as the upper portion of the water is made to boil. See Fig. 201.

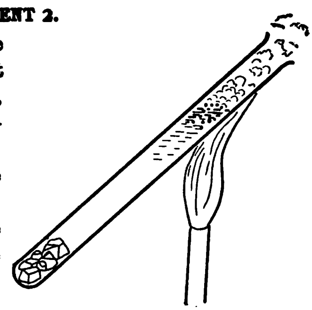

FIG. 201.

284. Illustrations of Good and Poor Conduction.—Good and bad conductors of heat can frequently be recognized as such by merely touching them. If a number of bodies all of the same temperature are touched in succession, the good conductors will feel warmer than the bad conductors if all

are warmer than the hand, and will feel cooler than the bad conductors if all are, as is generally the case, cooler than the hand. For instance, the coated and uncoated sides of a photographic dry-plate can often be distinguished in the dark by the different degrees of rapidity with which they cool the palm of the hand or the tip of a finger.

The explanation is that when one touches a bad conductor the spot in contact with the hand, not being freely assisted by the other parts of the conductor, soon rises or falls to a temperature approaching that of the hand at the point of contact. In a good conductor the part touched is not so readily brought to the temperature of the hand. In this test, however, as in the experiment of § 283, the effect depends upon the specific heat (Chap. XXIII) and density of each substance as well as upon its conducting power.

The following familiar experiment gives striking proof of the great effectiveness of brass, as compared with wood, in carrying off heat from a body, paper, in contact with the two substances.

EXPERIMENT.

The junction of the compound rod (No. LIX), wrapped with paper secured by rubber bands (Fig. 202), is held over a small flame and moved back and forth at such a height that the paper begins to char.

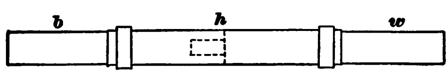

FIG. 202.

It will presently be noticed that the charred area has a pretty definite boundary on one side, and a knife-blade cutting through the paper at this line will enter the niche between the brass and the wood.

The density and conducting power of gases being small, it is possible to hold one's hand for some little time without injury in gases much hotter than the hottest solid or liquid that one would care to handle. It is a well-known fact that dry steam coming in contact with the skin does less harm than steam mixed with water, although the dry steam may

be hotter. Very bad conductors, like cork, felt, and non-metallic powders, are used to impede the flow of heat into or out of bodies; that is, to keep hot bodies hot and cold bodies cold.

285. Davy's Safety-lamp.—This is an instrument which makes use of the fact that the flame of a burning gas will, under certain conditions, not pass through a sheet of wire-gauze, the wire carrying off the heat so rapidly that the gas within the meshes will not burn.

EXPERIMENT.

Light a Bunsen burner and press down upon the flame a sheet of fine wire gauze. Observe that the *flame* does not, until the wire becomes red-hot, pass through the gauze. To show that the gas passes through unburned, hold a lighted match above the gauze.

The safety-lamp is used by miners in regions where they are likely to meet with inflammable gases, which an ordinary lamp or candle would ignite. It has (see Fig. 203) a cylinder and cover of wire-gauze over the flame. The inflammable "fire-damp" of the mines works its way into the lamp through the meshes of the gauze and causes the flame to grow larger and change color; and the miner, seeing this, leaves the dangerous region before the flame can set fire to the gas outside the gauze. This lamp was the invention of Sir Humphry Davy, a famous English chemist, who lived from 1778 to 1829.

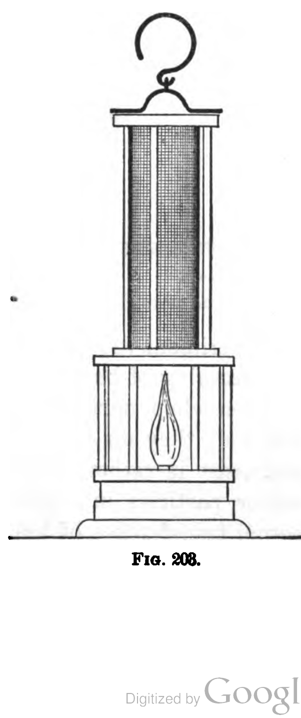

FIG. 203.

286. Convection.—This is the conveyance of heat by conveyance of the body containing the heat. Such a movement occurs naturally in a liquid or a gas which is unsymmetrically heated; for, as we shall see later, the hotter parts are, in general, less dense than the colder parts, and so the former tend to rise while the latter tend to fall. The movements thus produced are called convection-currents. It is evident that they should be especially strong when heat is applied at the bottom. It would be a difficult matter, as the experiment of § 283 has shown, to boil water by heating it at the top.

EXPERIMENTS.

(1) Light a small piece of touch-paper (No. LX) and place it under a bell-jar held mouth downward. Currents, made evident by the smoke, will be seen moving up and down within the jar.

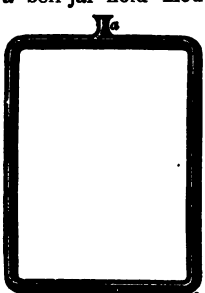

FIG. 204.

(2) To show convection currents in water, apparatus LXI, shown in Fig. 204, may be used. Fill the apparatus completely with water, hold it as shown in the figure, and drop in through the tube *a* a very small quantity of any of the aniline dyes soluble in water—for instance, Hoffman's violet rubbed with a drop of water into a thick paste. Now gradually heat one angle, *b*, of the apparatus by cautiously applying the flame of the Bunsen burner, and note the evidence of currents in the tube.

Experiment (2) illustrates the method of circulation in the warming of buildings by means of hot water.

Even when currents are set up in a fluid that is being warmed, they do not work independently of conduction, but assist it rather. This they do by constantly bringing colder particles into contact with warmer ones. But the actual transference of heat from one portion of the fluid to another portion is effected mainly by conduction.

287. Radiation.—One body may be heated at the expense of another without either conduction or convection between them. The earth is warmed by the sun, but if we were to start from the earth's surface towards the sun with a thermometer, we should very soon find that we were not following an ascending scale of temperature in the medium between us and the sun, as we should be if heat came to us through this medium by conduction. On the contrary, we should get into colder and colder regions above the general level of the earth. If, on the other hand, we look for streams of matter coming from the sun to the earth, and bringing us heat by convection, we do not find them.

Physicists are convinced, by means of experiments not to be described here, that the heat we get from the sun is transmitted to us by means of a wave-motion of the intervening medium, whatever that medium may be, that fills the space between the heavenly bodies. As waves in deep water travel straight forward, although the single water-particles go but short distances and then return, so waves travel swiftly from the sun to the earth, while the particles of the substance that transmits them merely vibrate to and fro.

Now wave-motions, in which every particle repeats the same movements again and again, not jarring against its neighbors irregularly, but moving in system and harmony with them all, are not what physicists call heat-vibrations. We shall therefore speak of the medium between us and the sun as containing, not "radiant heat," as books formerly said, but *radiant energy*, which, coming from the heat of the sun, may be turned back into heat when it strikes the earth.

It is not from the sun alone that radiant energy is transmitted. On a small scale the same kind of action is noticeable at moderate distances on the earth's surface, from fires, masses of heated metal, and so on; it is the principal means

of heating in rooms warmed by open fires. *Radiant energy may traverse a medium without appreciably warming that medium. Its velocity between us and the sun is about* 30,000,000,000, *or* 3×10^{10}, *cm. per second.*

EXPERIMENT.

Hold a convex lens 6 or 8 cm. in diameter in the sunlight above a sheet of paper, at such a distance as to throw upon the paper as small and bright a spot of light as possible. The radiations passing through the lens set the paper on fire. Is the lens itself much heated?

The discussion of radiation is continued in Chapter XXVII.

CHAPTER XXII.

EXPANSION WITH RISE OF TEMPERATURE: THERMOMETRY.

288. Expansion Caused by Rise of Temperature.—The hotter a body is the more active its particles are. The hotter a gas, for instance, becomes the more it tends to expand; for if it is not allowed to expand, its particles will hit harder and oftener against the walls of the containing vessel than they did before the heating.

The expansion of gases is so great, even with moderate changes of temperature, that it would be a familiar fact to every observing person, if gases were not, as a rule, invisible. The following simple experiment makes this expansion evident to the eye.

EXPERIMENT 1.

(1) Take an "air-thermometer bulb" (No. LXII) 4 or 5 cm. in diameter. Invert this bulb, and immerse the open end of its tube in a glass of water colored red. Heat the bulb by applying the hand, or cautiously by means of a Bunsen flame, and note the effect. Then allow the bulb to cool and observe what occurs, the end of the tube remaining all the time in the water.

So, too, in solids and liquids a rise of temperature generally, though not always,* makes the substance tend to expand.

EXPERIMENT 2.

(2) Take apparatus No. LXIII, which consists of a metal sphere, and a metal ring just large enough to encircle it easily at ordinary temperatures (Fig. 205). Heat the sphere in the Bunsen flame and try to drop it through the ring.

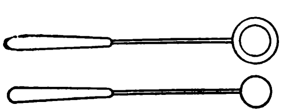

FIG. 205.

While the sphere is still hot, heat the ring and see whether the sphere will now drop through.

* See foot-note to p. 358.

(3) One flask of apparatus No. LXIV, shown in Fig. 206, is filled with colored water to a point which is marked by a rubber ring on the tube, all large bubbles of air being excluded. When the flask thus filled is plunged into warm water, the first visible effect is a slight fall of the water-surface; but presently the liquid becomes warmed and expands enough to raise the surface much higher than it was at first.

What is the cause of the fall at first?

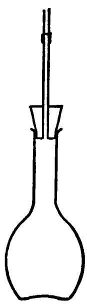

FIG. 206.

This last experiment illustrates the action of a mercury thermometer. What we observe in the thermometer is not the total expansion of the mercury. It is the excess of the expansion of the mercury over the expansion of the capacity of the glass.

289. Different Rates of Expansion.—It is easily seen that air expands more under such conditions as those of the preceding experiments than water or the solids. But different solids and different liquids may have different rates of expansion.

EXPERIMENTS.

(1) Move the compound metal bar (No. LXV, Fig. 207) back and forth in the flame of a Bunsen lamp until it becomes hot throughout,

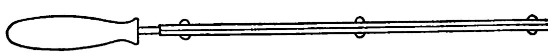

FIG. 207.

taking care not to heat it to redness at any point. Observe the bending caused by inequality of expansion, and note which of the metals expands the more.

(2) Take the two flasks of No. LXIV and fill each to a height of 1 or 2 cm. of its stem, one with water and the other with kerosene. Now, after marking the height of the liquid surface in each stem, immerse both bulbs in a vessel nearly full of warm water, and note the height of each liquid surface in its tube after the bulbs have been immersed some minutes.

290. Thermometry.—Further experiments like those preceding would show that, with few exceptions, solids, liquids, and gases expand with rise of temperature, but that each solid or liquid substance has a rate of expansion peculiar to itself. Moreover, these rates of expansion do not bear a constant ratio to each other; that is, if liquid A expands just twice as fast as liquid B between certain limits of temperature, it would probably not expand just twice as fast as B between other limits of temperature.

Accordingly, although the expansion of almost any substance can be used to indicate a rise of temperature, it is by no means a matter of indifference what substance is selected and used commonly for this purpose. The general experience of mankind has fixed upon mercury, contained in a glass bulb and graduated stem, as the best *thermometer*, or gauge of temperature.

In adopting the mercury-thermometer as a standard instrument we practically say this: *Every increase of temperature which produces a certain apparent* expansion of the mercury, shall be called one degree, and all such degrees shall be considered equal, though they might not produce equal expansions of any other substance.*

Evidently this is a very arbitrary standard, and the student need not be surprised to find that scientific men do not consider it a final standard for their work. See § 305.

291. The Mercury-thermometer.—One method of making this instrument will be here described.

First a piece of thick-walled small-bore tubing, known as thermometer-tubing, is taken, and the uniformity of the bore is tested by means of a short column, or "thread," of mercury, which is placed in various positions in the bore

* That is, the real expansion of the mercury minus the expansion of the glass envelope. See Exp. 3 under § 288.

and measured as to length in each position. A bulb, *R* (Fig. 208), is then blown upon one end of the tube, and at the other end a reservoir for mercury, *B*, with a slender prolongation and a minute opening. *B* is heated until the air within it is somewhat rarefied and then plunged into mercury. The cooling effect produced by the latter causes the inclosed air to contract, and considerable mercury enters the reservoir *B*. The end *B* is now raised, *R* and the stem are somewhat heated to rarefy the contained air and then cooled to allow the mercury to sink into *R*. The mercury in *R* is boiled so that its vapor shall drive out the air in the stem and *B*. This and similar operations are continued until nothing is left in the bulb and tube but mercury and mercury-vapor. Then, finally, the tube is strongly heated just below *B*, drawn out to a point, and sealed.

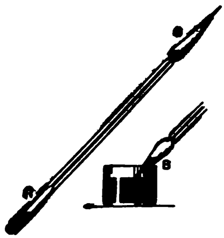

Fig. 208.

Inexperienced students should not try to boil mercury, as its vapor is poisonous.

292. Graduation of a Mercury-thermometer.—It has been ascertained by an immense number of observations that the temperature at which ice melts and that at which water boils are substantially invariable, provided certain conditions are maintained.* These conditions are chemical and mechanical purity and a standard, unvarying, pressure. Slight chemical or mechanical impurity affects these temperatures but little. Ordinary changes of atmospheric pressure affect the melting temperature extremely little, but they affect the boiling temperature of water very per-

* A like statement can be made concerning other solids and liquors, but ice and water are the most convenient for use in graduating thermometers. Some solids gradually soften before melting, so that they have no definite melting temperature.

ceptibly. These changes are easily observed by means of the barometer, and allowance for their effects is, as we shall see, easily made.

Advantage is taken of the peculiarly constant behavior of water to establish "fixed points" on the thermometer-tube. The instrument is first placed upright, with the bulb and the stem buried in a mixture of ice and water to such a depth that the top only of the mercury-column is visible. The position of the top of the mercury-column, when it has ceased to descend, is marked by a fine transverse scratch on the tube. The thermometer is then removed from the ice and placed upright in a vessel in which the bulb and the greater part of the stem can be perfectly surrounded by an abundant supply of freely escaping steam, the operation being performed, if convenient, at a time when the barometer reads very nearly 76 cm. After the mercury has ceased to rise in the stem, a fine scratch across the latter is made at the top of the mercury-column.

If the thermometer is to be graduated by the Centigrade system (§ 293), the portion of the tube between the melting-point and the boiling-point marks is divided into a hundred equal * parts, numbered from 0 at the bottom to 100 at the top; and these divisions are engraved on the stem of the thermometer, or ruled on a slip of paper fastened to the thermometer-stem and protected by a larger inclosing tube, or engraved on the metal case to which the thermometer-bulb and tube are fastened, or in some other

* If the thermometer is intended for any accurate work, the tube before or after filling with mercury should be "calibrated," a short column of mercury being forced through it a few millimeters at a time, and the length of this column measured in successive portions of the tube. By this means, even if the tube is not of equal diameter throughout, it may be divided into successive portions, all of equal *capacity*, and the graduation afterwards made in accordance with these; or, if the scale is made in parts of equal *length*, the calibration helps toward the making of necessary corrections in the ordinary use of the thermometer.

way fixed in their proper places. The same graduation may be continued above the melting-point and below the boiling-point.

All thermometers, however carefully made, are likely to be more or less in error, for the glass changes somewhat after manufacture, thus altering the capacity of the bulb and stem. Cheap thermometers — and inexperienced workers should use no others—frequently have errors of half a degree or more at various points. The easiest points to test are the zero- and the boiling-points, and these are the only ones that will be attempted in Exercise 39.

A number of interesting effects will be noticed incidentally in connection with the tests. Thus, if the thermometer has been kept at the ordinary temperature of the room for some weeks preceding this Exercise, it is quite likely that the zero-point found at the beginning of the Exercise will be notably different from that found at the end, the glass not recovering at once, upon cooling from the boiling temperature, the same volume that it had before heating.

EXERCISE 39.

TESTING A MERCURY-THERMOMETER.

Apparatus: Nos. 68, 80, 81, 82, and 83. Snow or ice. A short rubber tube and a pinch-cock. A rubber band to close the joint at the base of the cover of the boiler.

Examine the thermometer, holding it all the time bulb downward, to see whether any mercury has separated from the column and lodged at the top of the stem. If any is so lodged, it can probably be freed by grasping the thermometer near the middle of the stem (bulb outward) and swinging it rapidly back and forth, the arm being held at full length downward.

To test the freezing-point: Break the ice into fine pieces, the finer the better, and fill the dipper with it. Add only enough water to fill the space between the lumps of ice. Thrust the bulb of the thermometer down into the middle of the cup until the point marked 0° is only 1 or 2 mm. above the surface of the water. Give the mer-

cury-column time to descend as far as it will, and then record the reading. (If its shortest divisions are a millimeter or more in length, try to read to tenths of these divisions. Place the eye in such a position that a straight line drawn from its centre would strike the thermometer at right angles at the top of the mercury-column.)

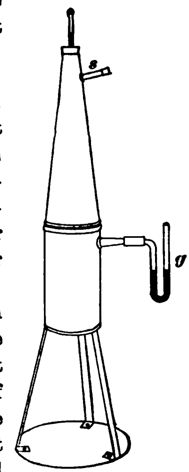

FIG. 209.

To test the boiling-point: Fill the boiler (Fig. 209) with water to the depth of 3 or 4 cm., put on the conical top, pressing it carefully down into place, attach the small mercury pressure-gauge, and thrust the bulb and stem of the thermometer down through the perforated stopper until the point marked 100° is not more than 2 or 3 mm. above the top.

Boil * the water and keep it boiling, the steam escaping from the side tube near the top of the cone, until the mercury-column rises as far as it will, and then record the reading. (The flame of the burner beneath the boiler should not be allowed to flare out much beyond the bottom, lest the current of hot gases rising from it should overheat the mercury-gauge and the upper part of the vessel.)

Record the reading of the barometer.

To test the effect of increased pressure † *upon the boiling temperature:* Close the steam-outlet by means of a short rubber tube and a pinch-cock, or a tight stopper, the flame still burning, and watch the thermometer and the pressure-gauge. If the apparatus is properly closed, the gauge will very soon indicate a steam-pressure 4 or 5 cm. of mercury greater than the atmospheric pressure. The cover must be pushed in *hard* to prevent this pressure from lifting it. If there is too much leakage around the base of the cover, close the joint

* To save time, it is well to begin heating the water before making the freezing-point test.

† The introduction of the pressure-gauge into this exercise is due to a suggestion from Mr. Rollins, formerly of the Newton High School. He had devised and used a more elaborate apparatus than that here described.

there by means of a rubber band. When everything is satisfactory, record both the rise of temperature and the rise of pressure.

This experiment, if successful, will be of service in estimating the error of the boiling-point. The true boiling-point error of the thermometer examined is obtained by finding what its reading would be in steam at a pressure of 76 cm. and subtracting 100° from this reading.

To test the effect of leaving the stem of the thermometer exposed to the air while the bulb is heated: Give the steam free escape once more, and then, after noting the reading again, draw the thermometer up through the stopper until only the bulb and a few degrees of the stem are exposed to the steam. Watch the effect upon the reading as the stem cools.

Finally, if time permits, test the freezing-point again, taking care, as before, to have the snow or ice to reach the bottom of the dipper. It is not sufficient to have snow or ice *floating* in the water.

The operation of determining the freezing-point and boiling-point is profitable for the student on account of the light it throws on the theory of the thermometer, but the inaccuracies detected in the operation will not, unless they are greater than 1°, be serious in the heat-experiments of this course. It will be important, however, not to exchange thermometers during such experiments as require careful observation of small changes of temperature.

293. Comparison of Thermometer-scales.—Unfortunately there are three thermometer-scales in somewhat general use, the Centigrade, already described, the Réaumur, and the Fahrenheit. The Réaumur scale differs from the Centigrade only in the fact that the space between the two standard points, of freezing and boiling water, is divided into 80 instead of 100 equal parts, so that 1 Réaumur degree equals 1.25 Centigrade degrees. The Fahrenheit scale, which is the one commonly used in this country except in laboratories, bears a less simple relation to the Centigrade. Fahrenheit, its inventor, divided the space between the

temperature found in melting ice and that found in boiling water into 180 equal parts, and placed the zero * of his scale at a point 32 like divisions below that of the melting-point of ice.

The annexed diagram (Fig. 210) shows the relation between the two scales more simply than any verbal statement. In order to find the equivalent of a Centigrade degree in Fahrenheit degrees we need only remember that 100 Centigrade degrees = 180 Fahrenheit degrees. Therefore 1 Centigrade degree is $\frac{180}{100}$ (= $\frac{9}{5}$) of a Fahrenheit degree, or 1 Fahrenheit degree = $\frac{5}{9}$ of a Centigrade degree. In order to reduce *temperatures* from one scale to another we must take account not only of the different values of the degrees of the two scales, but also of their different starting-points.

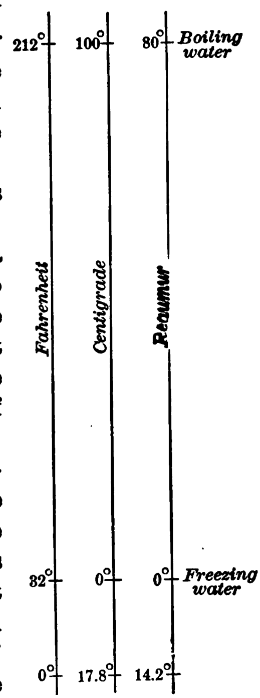

Fig. 210.

Suppose that it is required to reduce a temperature of 15° C. to the corresponding temperature on the Fahrenheit scale: 15 Cent. degrees = 15 × $\frac{9}{5}$ (= 27) Fahr. degrees, but the Cent. temperature was measured from a point coinciding with 32° Fahr., so the required temperature is 27° + 32° (= 59°) Fahr.

Suppose that it is required to reduce the temperature 68° Fahr. to the corresponding Cent. temperature: 68° Fahr. is 68° − 32° (= 36°) Fahr. degrees above the Cent. zero, so that the required temperature is 36 × $\frac{5}{9}$ (= 20°) C.

* The lowest temperature he reached with the freezing-mixture which he used.

Stated as formulæ, the rules for reduction of these two scales of temperature are as follows:

$$\text{F.}^\circ = (\text{C.}^\circ \times \tfrac{9}{5}) + 32^\circ;$$
$$\text{C.}^\circ = (\text{F.}^\circ - 32^\circ) \times \tfrac{5}{9}.$$

PROBLEMS.

(1) If a change of 2.7 cm. in the barometer makes a difference of 1 cent. degree in the boiling-point of water, at what temperature will water boil when the barometer reads 78.0 cm.?

(2) If the barometer column falls 1 cm. for a rise of 10° m. from the sea-level, at what height will water boil at a temperature of 99° C.?

(3) What temperature on the Cent. scale corresponds to 20° Fahr.? to − 20° Fahr.

(4) What temperature on the Fahr. scale corresponds to 40° C.? to − 40° C.?

294. Linear Expansion of Solids.—Nearly all of the solid substances which have been tested are found to expand in all directions as their temperature rises.* If the increase of size is observed or estimated only in one direction, as in the lengthening of a bar of metal, the increase is called the *linear expansion.* In all ordinary cases this lengthening is so slight as to be nearly imperceptible. Although spaces are left between the ends of the rails in laying a railroad-track, to allow of the expansion which may be produced by hot weather, yet to the unobservant eye the spaces appear unchanged through summer and winter. It is only when long pieces or structures of metal or of other solid substances are examined that the lengthening is very manifest. In the spans of long iron bridges the motion of the ends under the influence of changes of temperature may become

* Exceptions are iodide of silver, the compound iodide of lead and silver, Rose's "fusible alloy," garnets, and india-rubber. The last-named substance, if heated when unstretched, expands in all directions, but if it is stretched rise of temperature increases its already existing tendency to contract.

considerable, as in the case of the iron and steel railroad bridge across the Forth in Scotland. The total length of the bridge, including the approach-viaducts, is 8098 feet, and the allowance for expansion of ironwork over the whole length is 6 feet.

The amount of expansion of most metals and of various other solids per degree Centigrade has been ascertained by many accurate experiments, and it is found that the expansibility varies greatly with the substance, no two ordinary metals, for example, expanding the same amount for the same increase of temperature.

295. Coefficient of Linear Expansion.—Experience shows that, if we represent the length of an ordinary solid at $t°$ by l, and the length at $t'°$ by l', the relation between these quantities can be shown with sufficient accuracy for ordinary purposes by the equation

$$l' = l + l(t' - t) \times k = l(1 + k(t' - t)),$$

where k is a quantity that does not vary with the temperature. This quantity is called the *coefficient of linear expansion.* It has different values for different substances.

From the equation just given we may get

$$k = (l' - l) \div l(t' - t),$$

which shows that we can find k, for any particular substance, by finding its length, l, at any convenient temperature $t°$, and its increase of length, $(l' - l)$, in rising from $t°$ to another known temperature $t'°$. The most convenient temperatures for our use are the ordinary temperature of the room and the temperature of steam. These temperatures are used in the following Exercise.

The increase of length in this Exercise is so small that it cannot be measured accurately without some magnifying device. The one adopted in the following description is

exactly similar in principle to that used in Exercises 29 and 30. A small movement at the end of the short arm of a pivoted index produces a large movement at the end of the long arm. The short arm is here nearly vertical and the long arm nearly horizontal.

EXERCISE 40.

LINEAR EXPANSION OF A SOLID.

Apparatus: Nos. 80 (without the gauge), 82, 83, 84, 85, 86. A barometer. A meter-rod. A rubber tube 30 cm. long to carry steam from the boiler to the sheet-iron tube; another, 15 cm. long, to carry the steam and water from the same tube (see Fig. 211). A small tumbler to catch the water thus drained off. (It is better not to place this tumbler on the wooden rack. Steam or water coming from it might wet some part of the rack and cause a swelling, in or near the back-post, that would seriously affect the position of the index. A

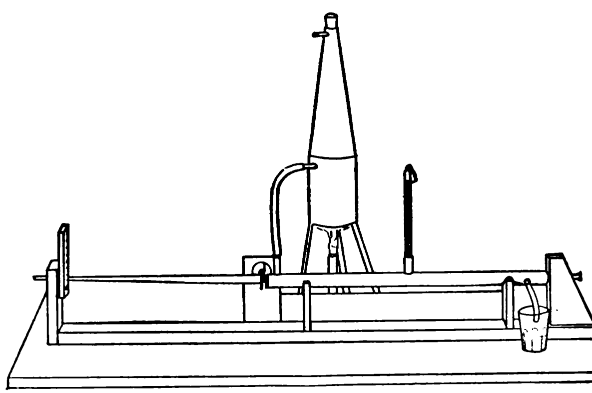

FIG. 211.

similar, though less, danger threatens if much water or steam escapes from the other end of the sheet-iron tube.)

Fill the boiler to a depth of 3 cm. with water and place the lighted Bunsen burner beneath it, all outlets being closed except the lower side-tube.

Raise the long arm of the index and place the sheet-iron tube, the brass rod within it, in position on the rack, the untipped end of the rod resting against the screw back-stop, and then gently lower the index until the short arm bears against the sharp tip fastened to the rod.

Attach the 30-cm. rubber tube to the inlet-tube of the sheet-iron tube, making no connection as yet with the boiler; turn the cylinder in its bed till the inlet-tube rests against a firm support; attach the outlet rubber tube, leading to the tumbler; put the thermometer-bulb through the middle side-opening into the steam-cylinder alongside the rod, as in Fig. 211.

Measure and record the length of the brass rod, not including the sharp tip. This measurement need not be made with great care, as an error of 2 or 3 mm. here would make very little difference with the final result.

Measure, with about the same care, and record the length of the long arm of the index, from the centre of the pivot to the vertical scale.

Measure with much care and accuracy the short arm of the index, from the centre of the pivot to the point of the sharp tip. To do this push up the thin brass sleeve on the vertical arm of the index (see Fig. 212) until its upper edge touches the tip; then measure carefully the distance from the *base-board* of the rack to the top edge of the sleeve. Lift the horizontal arm and turn it back till it lies upon the

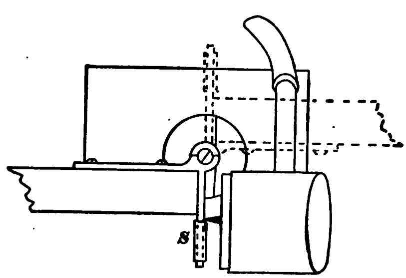

FIG. 212.

sheet-iron cylinder, then measure carefully the distance from the base-board to the same edge of the sleeve, which is now the lower edge, the short arm of the index now extending upward, as indicated by the dotted lines in Fig. 212. One half the difference between these two measurements is the effective length of the short arm of the lever.

Read the thermometer, then remove it, and put a stopper in its place. There is no further use for the thermometer, as the higher temperature, that of the steam, can be found better from the barometric pressure. (See Exercise 39.)

Make sure that the brass rod still rests against the back-stop screw, then read, as accurately as you can, the position of the index, upper or lower edge according to convenience, upon the vertical scale.

By this time a good flow of steam should be coming from the boiler; Slip the end of the inlet rubber-tube over the vent of the boiler and let the steam take its course through the tube on the rack. See that there is a free escape of steam from the other end of this tube. Watch the index until it ceases to rise along the vertical scale, then read its position as carefully as before.

From the difference of the first and last readings upon the vertical scale we can, making use of the ratio of the index-arms, calculate the increase of length which the rise of temperature has produced in the rod.

From the data obtained is to be calculated the *coefficient of expansion of brass for 1 Centigrade degree*, that is, the fraction of itself by which the length of a bar of brass is increased when its temperature is raised 1 Centigrade degree. The calculation will assume that the expansion is equally great for each and every degree of the rise of temperature. This is not quite true. The quantity which the calculation gives is the average, or mean, value of the coefficient between the limits of temperature used.

Alternative Method.

Instead of the lever-index described in this Exercise, a micrometer-screw, similar in principle to that used in Exercise 26 for measuring the diameter of the wire, may be used for measuring the expansion of the rod. The delicacy and accuracy of this device can be increased by using an electric sounder to show when contact between rod and micrometer-screw occurs.

296. Coefficient at Different Temperatures.—For metals between 0° and 100°, the coefficient of expansion is usually nearly uniform, but through wider ranges of temperature this uniformity does not hold good in the case of metals or

of other solids. It is found that as the temperature grows higher expansion usually proceeds at a more rapid rate. In the case of a specimen of glass whose mean coefficient per degree was ascertained, first from 0° to 100° and then from 0° to 300°, the coefficient k was found in the second case to be about 20 per cent larger than in the first.

297. Applications.—The expansion of most substances with rise of temperature and the inequality of expansion of different substances are facts which must be taken account of in many kinds of work.

The need of allowing room for expansion of steel car-rails has already been mentioned. It is said that the iron frameworks within brick or stone walls, used so commonly in the construction of "fire-proof" buildings, have proved dangerous, because in case of fire the expansion of the iron ruins the walls.

The warmth communicated from the hand to a small steel cylinder or the gauge which measures it may cause a misfit in the works of a watch.

The steel tires of carriage-wheels and the steel jackets which strengthen barrels of large guns, are put into place while hot, and their contraction in cooling gives the necessary snugness of fit.

In steam-boilers having steel shells, or bodies, and brass internal tubes allowance must be made for the greater expansion of the brass.

Steam-pipes placed nearly horizontal along the walls of a room are not rigidly fastened to the wall at both ends, but are supported in some way that allows them to expand and contract. The expansion is easily seen in long pipes when the steam is turned on, and part of the noise then heard is due to the slipping of the pipes upon their supports.

The unequal expansion of two metals, already illustrated in § 289, is made useful in a number of ways.

Thermometers are made (No. LXVI) in which an index is caused to move by the bending or unbending of a compound bar consisting of two strips of brass and steel placed side by side and soldered together.

The "gridiron pendulum" (Fig. 213) is so constructed that the expansion of the brass bars (shown light) offsets the expansion of the steel bars (shown dark), so that the effective length of the pendulum is nearly unchanged by change of temperature.

In the "balance-wheel" of a watch or clock (No. LXVII, Fig. 214) the adjustment for changes of temperature is made by setting the change of curvature of the rim, RR, which is open at two places, C and C, to counterbalance the change of length of the diameter, DD.

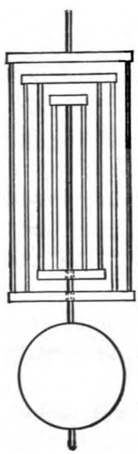

FIG. 213.

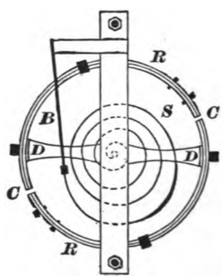

FIG. 214.

If a body to which heat is rapidly applied is thicker in some parts than in others, or if the heat is unsymmetrically applied to it, some parts grow hot faster than others, and the result is an unsymmetrical expansion, which may break the body. Glass vessels are often broken in this way by pouring hot water into them. Like results may come from unsymmetrical cooling of hot bodies.

298. Trevelyan's Rocker.—One of the most interesting effects of heat-expansion is shown in the following experiment, in which the slight impulses given, first to one edge and then to the other of the "rocker" (No. LXVIII, Fig. 215) by the lead expanding at the points of contact, keeps the rocker in rapid oscillatory motion for a considerable time. The result is a more or less musical note, subject to eccentric changes of pitch when the rocker is disturbed by a touch.

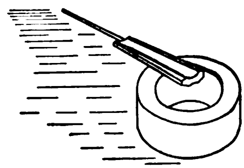

FIG. 215.

EXPERIMENT.

Lay the lead ring of No. LXVIII upon the table. Heat the rocker till it is nearly hot enough to melt lead, scrape its two ridges till they are bright, then lay these ridges upon one edge of the lead ring, the end of the handle resting upon the table. Joggle the rocker until it begins to give out a musical tone.

299. Coefficient of Cubical Expansion.—Since each edge of a cube whose length at 0° is 1, becomes $1 + k$ (§ 295) at 1°, the volume of the cube which at 0° was 1, will at 1° become $(1 + k)^3$, or $1 + 3k + 3k^2 + k^3$.

Now the value of k is always a very small fraction, and consequently its square or its cube is so extremely small a quantity that it may for common purposes be altogether neglected, so that the value of $(1 + k)^3$ is practically equal to $1 + 3k$. Suppose, for instance, that for some particular substance $k = .0001$. Then, if the object experimented upon is a cube 1 cm. on an edge, it will, after being heated 1°, have a volume of $(1.0001)^3 = 1.000300030001$; so that, reckoned to seven places of decimals, the cubical expansion, expressed as a fraction of the original volume, is just three times the linear expansion expressed as a fraction of the original length.

The *coefficient of cubical expansion* is defined as the

ratio which the increase of volume for 1° rise of temperature bears to the original volume.* In the case supposed it is .0003, which is three times as large as the coefficient of linear expansion, .0001. If we had not 1 cu. cm. only, but a body of V cu. cm., having k as its coefficient of *linear* expansion, the volume after a rise of temperature from $t°$ to $t'°$ would be $V' = V(1 + 3k(t' - t))$, the increase being $3k(t' - t)V$. We shall represent the quantity $3k$, the coefficient of *cubical* expansion, by K.

300. Expansion of Liquids.—Liquids, like solids, vary greatly among themselves in the amount of expansion which they undergo for equal increments of temperature. In liquids we generally have to do with cubical expansion, and we may, as appears from § 288, in measuring this take account either of apparent or of real expansion.

The real expansion of a liquid may be determined by using the device of balancing columns described in Exercise 32. This method properly applied gives the ratio between the density of the liquid cold in one branch of the tube and the density of the same liquid hot in the other branch, and from this ratio the rate of expansion is easily found. Mercury is the liquid that has been most carefully tested in this way.

The real expansion of other liquids may be determined by a like test, but the more common method is to measure first their apparent expansion in glass bulbs with slender stems attached, and then add to the apparent expansion the expansion of the bulb. The expansion of the bulb is measured by means of the already determined real expansion of mercury, as follows: Fill the bulb and stem with mercury at 0°; raise the temperature to 100°, and catch the small

* In the case of gases, which are very expansible, it is customary to divide the increase per degree by the volume at 0° C., even when the actual starting-point of the expansion is some other temperature.

quantity of mercury which overflows during the heating. Weigh this amount of mercury, and weigh also the amount which remains in the bulb and stem. Knowing the real expansion of mercury, one can calculate how great the overflow would be if the bulb did not expand. Comparing this with the actual overflow, one can find how much the bulb has expanded. This apparatus is called a *weight-thermometer*.

For equal increments of temperature liquids in general expand much more than solids.

301. Irregular Expansion of Water and other Liquids. —If a thermometer-bulb and tube of suitable proportions were filled with boiling-hot water and then allowed to cool slowly to the temperature of melting ice, the surface of water in the tube would fall slowly until a certain point was reached, after which it would rise until the water became of the same temperature as the melting ice. The turning-point, at which contraction ceases and expansion begins, when the water reaches its temperature of maximum density, is very near 4° C. Water taken at this temperature, then, becomes less dense upon being either cooled or heated. Its density at 0° C. is substantially the same as at 8° C. If the volume of a given portion of water is 1.00000 at 0° C., it would be about 0.99987 at 4° C., 1.0118 at 50° C., and 1.0430 at 100° C.

An important consequence of this curious behavior of water, on cooling below 4°, is that lakes, ponds, and rivers in severely cold weather, when the temperature at the surface approaches 0° C., tend to maintain at the bottom a temperature of 4° C., so that ice does not ordinarily form in their depths. Large bodies of fresh water in which the natural course of things is not disturbed by deep currents or the presence of springs—deep lakes like Lake Tahoe in California, for instance—show a bottom-temperature of about 4° C. at all seasons.

Salt water continues to contract below 4° C., and if it is salt enough, down to and below its ordinary freezing-point, which is itself below 0° C.

EXPERIMENT.

Stir a quantity of ice and water together till the mixture is at 0° C. Then fill the brass bucket (No. 6), already cooled, with the water. Hang up the bucket, place the thermometer in it, and note the temperature of the water at the top and at the bottom after a few minutes of quiet.

Most liquids expand more rapidly at high temperatures than at low temperatures, according to the mercury-thermometer. Accordingly, if a thermometer were made after the fashion of a mercury-thermometer, but with a different liquid,—glycerine, for instance,—and if the expansion of this liquid between the freezing and boiling temperatures of water were divided into 100 equal parts, each called 1°, this thermometer would generally not agree with a mercury-thermometer, if each were true to itself. According to the mercury-thermometer glycerine expands irregularly. According to the glycerine-thermometer mercury expands irregularly. We take as a standard for ordinary purposes that thermometer which we find most convenient, the mercury-thermometer; but for more refined purposes we find a better standard in some form of gas-thermometer. See § 305.

QUESTIONS AND PROBLEMS.

(1) If K, the coefficient of cubical expansion of copper, is .000051, how great is k, the coefficient of linear expansion?

(2) What is the excess of length at 30° C. of a steel meter-rod which is correct at 16° C., K being, we will suppose, .000036?

(3) An iron ring, of 10 cm. internal diameter at 20° C., is to be slipped onto a cylinder which is 10.001 cm. in diameter at 20° C. If k for the ring is .000012, to what temperature must it be heated in order to be placed upon the cylinder?

(4) If the cylinder in the preceding problem is of brass, with $k =$

.000018, at what temperature would the cylinder slip into the ring, if both were cooled together and equally?

(5) If K is .000025 for glass, what is the capacity at 100° C. of a flask that contains 800 cu. cm. at 0° C.? (In such problems the glass wall is supposed to expand exactly as if it were the outside layer of a solid lump of glass, and the internal capacity is supposed to increase just as much as a lump of glass capable of filling the flask would increase in bulk.)

(6) If this flask held 10,848 gm. of mercury at 0° C., how many grams would it hold if both glass and mercury were at 100° C., the mean value of K for mercury between these two temperatures being .00018?

302. Expansion of Gases.—The rapidity with which gases expand when heated was well illustrated by the promptness with which air-bubbles began to escape when the bulb was heated in Exp. 1 of § 288. The accurate measurement of the expansion of gases is, however, complicated by the fact, already well known to the student, that the volume of a gas is greatly dependent upon the pressure it bears. In fact, the pressure upon a gas which is being heated may be so manipulated as to make the rate of expansion anything we please, large or small.

It is customary, however, in studying the effect of rise of temperature in gases to follow, as closely as may be, one of two courses: 1st. The volume may be kept unchanged during the heating, in which case we find the *increase of pressure with volume constant;* 2d. The volume may be kept unchanged during the heating, in which case we get the *increase of pressure with volume constant.* The first method is followed in Exercise 41, the second in Exercise 42.

From the data obtained in these two Exercises two coefficients of expansion are to be calculated. One is the coefficient of expansion of *pressure*, with volume constant; the other is the coefficient of expansion of *volume*, with pressure constant. The first is obtained by dividing the

increase of pressure per degree rise of temperature *by the pressure which would keep the gas at the same volume at* 0° *C.* The second is obtained by dividing the increase of volume per degree rise of temperature *by the volume which the gas would have under the same pressure at* 0° *C.*

Reference to 0° C. was not made in defining the coefficient of expansion of a solid (§ 295). It made practically no difference there whether the lower temperature was 0° or the temperature of the room. But in the case of gases this does make a good deal of difference. So, in accordance with the definitions just given, we shall in Exercises 41 and 42 begin by cooling the gas to 0° C.

It is very important that these Exercises should be performed with *dry* air. If at the lower temperature there is any liquid water in the tube, even an invisible film on its inner wall, the rise of temperature will evaporate part or all of this water, thus increasing the pressure too much, and causing error in the result. Accordingly, much care is taken to fill the tubes with dry air. See Appendix.

EXERCISE 41.

INCREASE OF PRESSURE OF A GAS HEATED AT CONSTANT VOLUME.

Apparatus: Nos. 80 (without the gauge), 83, 85, 86 (with the index put on in such a way that the long arm points in the usual way, while the short arm points upward, the pivot-screw being driven in so hard as to hold the index securely in any position which may be given it; see Fig. 216), 87, and 88. A meter-rod, a barometer, cork stoppers and rubber connecting tubes, a quantity of snow or pounded ice.

In handling the tube that contains the dry air and the mercury keep the sealed branch nearly horizontal, do not lower the other branch enough to let any mercury flow out, and avoid shocks lest the mercury-column be broken so as to include bubbles of air.

Fill the boiler with water to a depth of several centimeters, put on the top, close all the openings except the lowest, and place the

flame beneath. Lift and shake the boiler occasionally during the exercise, to find whether it is in danger of getting dry.

Measure the distance from the rubber tube attached to the dry-air

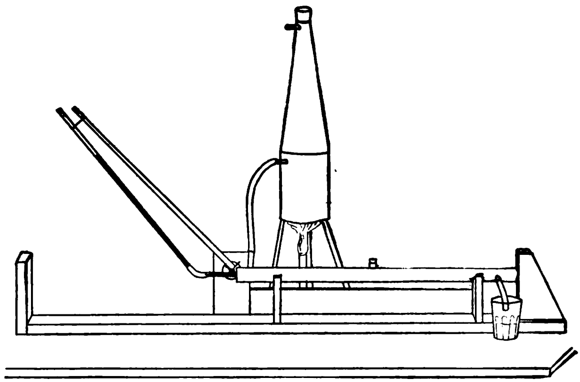

FIG. 216.

tube to the sealed end of the bore of the latter tube. (If the sealed end of the bore *tapers*, do not measure to the extreme tip, but, as well as you can, to the spot where the bore would end if it stopped without tapering but had the same total volume as now.)

Push the sealed end through a short stopper into the cooling tray (Fig. 217), and cover the whole length of the imprisoned air-column with snow, or with water kept at 0° C. by means of pounded ice.

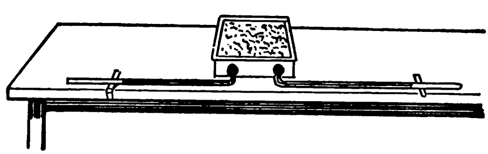

FIG. 217.

Support the outer glass tube in such a way that the two ends of the mercury-column shall be at the same level, pulling the sealed tube out through the stopper till the inner end of the mercury-column is just visible. Leave the tube in this condition for a few minutes, till contraction of the air-column has ceased; then measure the distance from the rubber tube to the inner end of the mercury-column.

The difference between the two measurements now recorded is to be taken as the length of the imprisoned air-column at 0° C. under atmospheric pressure.

The barometer-reading should now be noted.

Close the rear end of the heating-tube on the rack, and fit the other end with a short cork-stopper, perforated to admit the sealed end of the air-tube. Take the latter tube from the cooling tray, and push the sealed end through this stopper till the whole length of the air-column disappears. Attach the outer glass branch loosely but securely to the long arm of the index, which now turns stiffly on its pivot, and set this index in such a position as to raise the outer end of the glass tube some 40 cm. above the horizontal air-tube.

Lead steam from the boiler to the heating-tube on the rack. As soon as a good flow is established, draw the air-tube carefully through the perforated stopper, in which it should fit snugly, till the distance from this stopper to the rubber tube is just equal to the distance that was measured from the inner end of the mercury-column to the rubber tube when the air-tube was in the cooling tray.

Without allowing the distance just measured to change, raise or lower the outer glass branch till the pressure is such as to hold the inner end of the mercury-column just at the outer end of the cork stopper, thus making the length of the imprisoned air-column just* what it was at 0° C. under atmospheric pressure.

After making sure that expansion of the imprisoned air has ceased, measure the difference in height of the two ends of the mercury-column.† This difference represents the *increase* of pressure required to keep the volume of the air the same at the temperature of steam as at the temperature of melting ice.

Calculate α_1, the *coefficient of expansion of pressure with constant volume*, thus:

$$\alpha_1 = (p_t - p_0) \div p_0(t - 0),$$

where p_0 = pressure (atmospheric) upon the air at 0° C.,
p_t = total pressure upon the air at steam temperature,
t = temperature of steam (found from the barometer-reading).

* The expansion of the *glass*, very small compared with that of air, is neglected in Exercises 41 and 42.

† Lack of level in the table-top or the base of the rack may cause considerable error in this measurement.

EXERCISE 42.

INCREASE OF VOLUME IN A GAS HEATED UNDER CONSTANT PRESSURE.

Apparatus: The same as in the preceding Exercise.

Proceed exactly as in the preceding Exercise, until the steam begins to flow through the heating-tube. Then put and keep the two ends of the mercury-column at the same level, the inner end being kept just at the outer surface of the cork-stopper, and, when expansion ceases, measure the distance from this end to the rubber tube.

It is evident that when this measurement is made, the air is under the same pressure as when it was in the cooling tray. The increase of length of the air-column, representing its increase of volume, is found from the measurements now recorded.

Calculate α_2, the *coefficient of expansion of volume with constant pressure*, thus:

$$\alpha_2 = (l_t - l_0) \div l_0(t - 0),$$

where l_0 = the length of air-column at 0° C.,
l_t = " " " " " the steam temperature,
t = " temperature of steam.

303. Discussion of Results.—If the two preceding Exercises, 41 and 42, have been successfully performed, the student will find that the two coefficients obtained are very nearly equal. In other words, the rate of increase of pressure per degree for air maintained at a constant volume is very nearly the same as the rate of increase of volume per degree for air maintained at constant pressure.

An application of Boyle's law (§ 211) might have led us to anticipate this conclusion without the performance of these Exercises. For we might have reasoned as follows: Suppose a volume V of any gas to be heated to such a temperature that it would acquire the volume nV, the pressure P remaining unchanged. By Boyle's law, temperature remaining unchanged, $V \propto \frac{1}{P}$; therefore to bring volume nV of the heated gas back to volume V without cooling it,

a pressure nP must be applied; or, in other words, starting with a gas at volume V and pressure P, we can by a certain rise of temperature get, as we may choose, a volume nV at pressure P, or a volume V at pressure nP.

The fact is, however, that Boyle's law, upon which the reasoning just given is based, is not perfectly obeyed by gases. Accordingly, the two coefficients described in Exercises 41 and 42 are not exactly equal, as careful experiments have shown.

304. Behavior of Different Gases; Law of Charles.—It is found that all gases, at temperatures far from their points of condensation into liquids, expand nearly alike; so that if the student had used coal-gas, carbonic-acid gas, or hydrogen, in Exercises 41 and 42, he would have obtained nearly identical results. The equality of gases in regard to expansion is set forth in a law known from its discoverer as the *law of Charles.* It is variously worded by different authors. Clerk-Maxwell in his *Theory of Heat* states it as follows:

The volume of a gas under constant pressure expands when raised from the freezing to the boiling temperature [of water] by the same fraction of itself, whatever be the nature of the gas.

305. Relative Merits of Gases and Mercury as Thermometric Substances; Air-thermometer.—The uniformity of the behavior of all gases within a wide range of temperature is in striking contrast to the behavior of liquids and solids, no two of which, so far as we know, agree exactly in their rates of expansion, or even have rates that maintain a constant ratio at different temperatures.

In assuming that the apparent expansion (§ 288) of mercury is regular, as we do in making and using mercury-thermometers, we have to assume that all other substances expand irregularly. Yet mercury as a thermometric substance has much in its favor. A very low temperature is

required to freeze it, and a very high temperature to boil it. Its excellent conductivity for heat enables it to become heated or cooled quickly. It does not wet the tube which incloses it and leave an adhering film, when the column descends, as most liquids would. If a liquid is to be used, mercury is the best one for general purposes.

But as all the permanent gases agree very closely in their rates of expansion, it seems better to take a gas as the standard thermometric substance, and to test the mercury-thermometer by comparing it with a gas-thermometer.

As sometimes constructed, the gas-thermometer (No. LXIX) consists of a bulb, *B* (Fig. 218), filled with carefully dried air, connected by means of a slender stem *S* and a short glass tube *G*, with one end of a rubber tube *R* containing mercury, this tube connecting with a glass funnel *F* at its other end. The mercury-column confines the dry air and, the outer end being raised or lowered at will, serves to exert upon it the varying pressure required to keep the volume of the air unchanged when its temperature is raised or lowered. The temperature of the air at any time is calculated from its pressure.

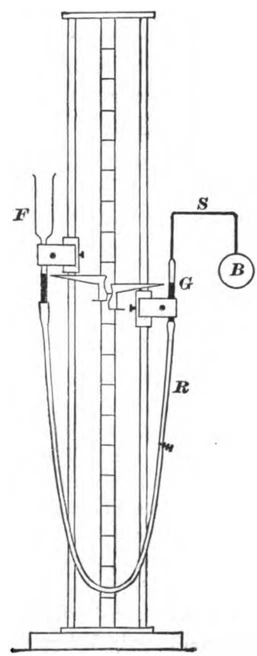

FIG. 218.

The bulb may have a capacity of one or two hundred cubic centimeters and the whole instrument is too cumbrous for common purposes. It can be used, however, for testing and correcting mercury-thermometers intended for accurate work. Far below the temperature at which mercury freezes and

far above the temperature at which it boils, the gas-thermometer can be used, and when we read of — 150° C. or + 1000° C., we may be sure that such temperatures, if determined accurately, have been determined by means of a gas-thermometer or some instrument, usually some electrical device, tested and approved by it.

306. Scale of the Air-thermometer: "Absolute Scale." —The increase of pressure in Exercise 41 and of volume in Exercise 42, during the rise from the freezing-point to the boiling-point of water, is about .3665 times the value of the pressure and volume, respectively, at the lower temperature.

We might, in using the air-thermometer, divide the interval from freezing to boiling of water into 10 or 50 or 500, or any other number of parts, and call each of these one degree. But it is customary to count 100 degrees on the air-thermometer scale between freezing and boiling water. The total relative expansion of pressure or volume, about .3665, divided by 100, gives the value of the coefficients, α_1 and α_2, already described.

We take as the definition of 1 degree on the air-thermometer scale such a change of temperature as will change the pressure or the volume .00366 +, we will say $\frac{1}{273}$, of its value at 0° C.

PROBLEMS.

(1) A certain quantity of air has a volume of 273 cu. cm. at 0° C. under a pressure of 546 cm. of mercury. It is heated under the same pressure till its volume is 293 cu. cm. What is its final temperature?

(2) The same quantity of gas is cooled under the same pressure till its volume is 253 cu. cm. What is the final temperature?

(3) The gas is heated until its pressure is equal to 819 cm. of mercury, its volume being 273 cu. cm. What is its final temperature?

(4) It is cooled with the same volume, 273 cu. cm., till its pressure is equal to only 500 cm. of mercury. What is its final temperature?

(5) If the gas could be cooled, with this same volume, till its pressure were zero, what would the final temperature be, according to the definitions given above?

In the problems just given 0° means the temperature of freezing water, but for some purposes it is more convenient to call the freezing-point of water 273°, in which case the boiling temperature of water is called 373°. The length of the degree remains the same as before, but the scale is started at a different place. This gives us what is called the "absolute scale" of the air-thermometer. We shall use T for temperatures on this scale, t being used for the ordinary scale.

The merit of the absolute scale is that it enables us to say, in dealing with a quantity of gas, *Pressure, with volume constant, is proportional to the temperature; volume, with pressure constant, is proportional to the temperature.*

Thus, if V is the volume at temperature $T°$ and V' the volume of the same quantity of gas at $T'°$, the pressure being the same in both cases, we have

$$V' : V :: T' : T, \quad \text{or} \quad V' = V \times \frac{T'}{T}.$$

Similarly, if volume remains constant,

$$P' : P :: T' : T, \quad \text{or} \quad P' = P \times \frac{T'}{T}.$$

PROBLEMS.

(6) What temperature of the ordinary Centigrade scale corresponds to 400° of the absolute scale? to 250?

(7) Find T when t is 300° C.; when t is − 100° C.

(8) What is the volume at 400° absolute of a quantity of gas which occupied 1000 cu. cm. at 300° absolute, the pressure being unchanged?

(9) What is the pressure at 500° absolute of a quantity of gas which bore a pressure of 76 cm. of mercury at 400° absolute, the volume being unchanged.

(10) What is the volume when $t = 200°$ C. of a quantity of gas that occupied 500 cu. cm. when t was 50° C., the pressure being unchanged?

When both the pressure and the volume change, we have

$$P'V' : PV : T' :: T, \quad \text{or} \quad \frac{P'V'}{T'} = \frac{PV}{T}.$$

(11) If with a certain quantity of gas $P = 100$ and $V = 500$, when $T = 300$, what must T' be in order that P' may be 150 and V' 400?

(12) If a quantity of gas under a pressure of 76 cm. at 500° absolute has a volume of 800 cu. cm., what pressure will make its volume 400 cu. cm. when its temperature is 300° absolute?

(13) If $P = 10$ lbs. to the square inch and $V = 50$ cu. ft. when $t = 300°$ C., what will be the volume V' when $P' = 25$ lbs. per square inch and $t = 600°$ C.?

(14) If a room 20 m. long, 10 m. wide, and 5 m. high contains 1290 kgm. of air at 0° C. when the barometer reads 76 cm., how much air will it contain when the temperature is 30° C. and the barometer reads 74 cm.?

307. Applications of Cubical Expansion.—All tables of density or of specific gravity should be stated with reference to a standard temperature. Specific gravities are often reckoned for the substances tabulated taken at 0° C. compared with water at 4° C. Since the determination could not well be made with the water at one temperature and the object in it at another, a calculation is necessary to obtain the specific gravity under the required conditions from that observed under the actual conditions of the experiment.

The draft in a stove or an ordinary furnace, the circulation of hot air through furnace-heated houses, and of water through those heated by hot-water pipes, all are due to the unequal density of air or water at different temperatures. Winds are due, directly or indirectly, to unequal heating in different portions of the atmosphere, and ocean-currents to the rise of water heated by a tropical sun and by the flow of this water away from the equatorial regions, while its place is taken by the in-rush of cooler waters from regions more distant from the equator. The importance of this

joint circulation of air and water in equalizing the temperature of the earth's surface is incalculable.

Questions and Problems on Chapter XXII.

(*See Appendix for Coefficients of Expansion.*)

(1) How could a mercurial Centigrade thermometer be constructed so as to make a degree very long—for instance, a centimeter or more?

(2) The space above the mercury in a good thermometer is meant to be a vacuum. Can you give any reasons for this?

(3) What is the temperature of boiling water when the barometer reads 77 cm.?

(4) What is the error of the boiling-point in a thermometer that reads, in free steam, 100°.1 when the barometer stands at 77.8 cm.?

(5) The standard platinum meter of France is correct at .0° C. What is its length at 20° C.?

(6) Indicate the calculations by means of which you found the coefficient of expansion of brass from your experimental data.

(7) What force would be required to prevent contraction of a rod of brass of 1 sq. cm. in area of cross-section and 10 m. long, cooling from 30° to 0°? (It would be the same as the force required to stretch the rod the amount that it would naturally contract in cooling 30°. Young's modulus (§ 175) for brass may be called 10^9, reckoned in grams and centimeters.)

(8) A glass rod is graduated in millimeters and is correct at 0°; a rod of steel is graduated in millimeters and is correct at 15°. At what temperature (above 15°) will the lengths of the divisions on the two scales be equal?

(9) Tell what measurements in Exercise 40 must be made with the greatest care, and give reasons for your statement.

(10) A glass bulb is just filled by 100 cu. cm. of mercury at 0° C. If the coefficient of cubical expansion of mercury is .00018 and that of glass .000025, what decimal part of the original mass of mercury will remain in the bulb when it is heated to 100° C.? *Ans.* 0.9847.

(11) The same bulb, refilled at 0°, is heated until it loses 0.8 cu. cm. of unexpanded mercury. To what temperature is it raised?
Ans. 52°.1 C.

(12) The barometer out of doors at 0° stands at 75 cm. What will be the reading after it has been brought into a room in which it

attains a temperature of 18° C., the atmospheric pressure remaining unchanged.* *Ans.* 75.243 cm.

(13) A certain thermometer is filled to a given height with mercury at 0° and the tube is then graduated. Each Centigrade degree is found to measure just 1 mm. The same tube, at 0°, is afterwards filled to the same height with alcohol, and once more graduated. How long are the new degrees? (Take .00106 as the coefficient of cubical expansion of alcohol, .00018 as that of mercury, and .000023 as that of glass.) *Ans.* 0.66 cm. nearly.

(14) At what temperature would a liter of air weigh 1.419 gm., the barometer reading 76 cm.? *Ans.* – 24°.3 C.

(15) A room measures 3 × 3 × 3 m. How many cubic centimeters of air, supposed unexpanded, will escape from it when the remaining air is warmed 1° C.?

(16) Under what circumstances would a liter of air weigh just a gram?

(17) A chimney 20 m. tall and 50 cm. square inside is filled with air at a temperature of 300° C. The outside air is at 0° C. and the barometer reads 76 cm. The top of the chimney is covered by a board. How much does the upward pressure upon this board exceed the downward pressure? (The case is like that of holding a cork under water.)

(18) Pressure remaining unchanged, at what temperature would the density of a quantity of air be one-half as great as at 10 C.?

* Neglect the expansion of the scale.

CHAPTER XXIII.

CALORIMETRY.

308. Heat as Energy.—As we by doing work can set bodies in motion, thus endowing them with *energy* (§ 267), by means of which they in turn can do work, so, it is believed, we can by friction or blows, or other purely mechanical means, set into more violent motion among themselves the invisibly small particles of which bodies are made up, thus adding to their energy, their power of doing work. We now believe, in short, that *heat is energy*, *the energy of individual molecules*, as distinguished from the energy of visible bodies. It is like the energy of a mob, each individual of which may be in motion, though the crowd as a whole does not move, while the energy of visible motion is like that of an army moving as a unit.

Heat-energy, like any other energy, can be measured in foot-pounds. The experiments of Joule, which have already been alluded to, and of which more will be said farther on, showed very exactly the number of foot-pounds of work which must be done in order to heat a pound of water one degree by stirring, and we have similar information concerning the heating of many other substances.

EXPERIMENT.

Lay one end of a rod of lead or solder upon some firm support, pound it briskly with a hammer, and note by touch the rise of temperature produced.

309. Measure of Heat.—Heat is not commonly measured in foot-pounds or other similar units, the *thermal unit*

adopted for convenience being *the amount of heat required to raise the temperature of a certain amount of water one degree.*

The thermal unit used in this book is the amount of heat required to raise the temperature of one gram of water one degree C. This is the unit most used by physicists. It does not have exactly the same magnitude for all degrees, but the difference is small, at ordinary temperatures, and in this book will be disregarded.

Let us now inquire by means of an experiment whether the amount of heat required to raise the temperature of a certain number of grams of another substance is equal to that required to raise equally the temperature of the same number of grams of water.

EXPERIMENT.

Make ready 100 gm. of water 10 degrees C. colder than the air of the room, and 100 gm. of mercury 10 degrees C. warmer than the air of the room. Pour both, the water first, into a thin glass beaker which has the same temperature as the air, stir the two liquids thoroughly with a thermometer for one-half minute, then note the temperature of the mixture. Which liquid appears to have had the greater influence in producing the final temperature?

Make a similar experiment with water warmer and mercury colder than the air. Which liquid has the greater influence in this case?

310. Specific Heat: Thermal Capacity.—*The ratio which the amount of heat required to raise the temperature of a given weight of any substance one degree bears to the amount of heat required to raise the temperature of the same weight of water one degree is called the Specific Heat of the given substance.*

It is evident from this definition that the specific heat of water is 1. It is evident, too, that the specific heat of any substance is equal to the number of thermal units required to heat one gram of the substance one degree. The specific

heat of water is greater than that of most other substances. It is surpassed by that of hydrogen.

The amount of heat required to raise a given body, large or small, one degree is called the *thermal capacity* of that body. If the mass is m grams, and if the specific heat of the material of which the body consists is called h, the thermal capacity of the body is mh.

311. Calorimetry: Measurement of Specific Heats.—The process of measuring heat, as distinguished from temperature, is known as *calorimetry.**

One of the objects of calorimetry is the determination of specific heats. For this determination several methods have been employed. One of the earliest of these was to place a portion of the heated substance in a hole scooped in a cake of ice and cover it with a slab of ice. The amount melted from the ice in this way, the quantity of heat required to melt a given quantity of ice being known, served to determine the specific heat required. This general method has been brought to very great perfection by means of an exquisite piece of apparatus known as Bunsen's ice-calorimeter.

The method most commonly used for the determination of specific heats is called the *method of mixtures.* In this method a known mass of the substance to be tested is plunged at a known temperature into a known mass of some liquid, usually water, at a different known temperature, and the resulting temperature, called the temperature of the mixture, is noted. In the use of this method there are various opportunities for error, even if the balances and thermometers are correct and are correctly read. Some of these will now be considered:

1st. Each substance when its temperature is taken may

* When heat was regarded as a weightless substance this supposed substance was frequently called *caloric.*

not have the same temperature throughout. If the substance is a liquid, or a finely divided solid, it should be thoroughly stirred before its temperature is taken.

2d. A substance may gain or lose considerable heat while it is being poured from one vessel to another. The pouring should be prompt and quick, and through the shortest practicable air-space.

3d. The substance in the mixture may not reach the same temperature before the "temperature of the mixture" is noted. They should be stirred well together, and the thermometer should read the same whether its bulb be near the bottom or near the top of the mixture, before the final temperature is taken.

4th. The vessel in which the mixing takes place will probably be heated or cooled by the substance or substances put into it. Allowance must be made for this in the calculations, and in order that this allowance may be small and readily made the vessel should be thin-walled, not unnecessarily large, and, in general, made of metal.

5th. Heat may be lost or gained by the mixture to or from the surrounding air and other bodies before its temperature is taken. An attempt is usually made to keep this loss or gain small by having the liquid, into which the heated substance is to be plunged, about as much below the temperature of surrounding objects before the mixing as it will be above that temperature at the end of the mixing. As a further precaution, the temperature of the mixture should be taken as soon as it can, by stirring, be made the same throughout.

EXERCISE 43.

SPECIFIC HEAT OF SHOT.

Apparatus: Nos. 71, 80 (without the conical top or the gauge), 81, 82, 83, 89, 90. A piece of pasteboard to cover the top of the dipper.

Half-fill the boiler with water and place the flame beneath it. Adjust the balance and weigh the calorimeter.

Weigh out in the calorimeter about ~~500~~ 400 gm. of shot, pour * them into the copper heating-dipper, place this dipper in the boiler, the bottom in the water, and cover it with the pasteboard. See Fig. 219.

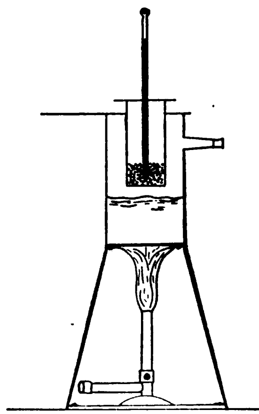

FIG. 219.

Weigh out in the calorimeter about 100 gm. of water, already cooled to a temperature 7 or 8 degrees below the temperature of the room.

Push the bulb of the thermometer through a hole in the pasteboard cover down into the shot, and note the rise of temperature, stirring the shot frequently with the thermometer. When the mercury has finally ceased to rise, withdraw the thermometer.

After the thermometer has cooled 40 or 50 degrees in the air, place the bulb in the water within the calorimeter, stir this water very thoroughly and take its temperature with care, reading to 0.1 of a degree. Meanwhile the shot should be occasionally stirred with a pencil or other convenient instrument.

Remove the thermometer, take the dipper, still covered, from the boiler, pour the shot quickly into the calorimeter, put in the thermometer and stir the water and shot vigorously † and thoroughly for about 10 seconds. Continue the stirring gently, note the temperature of the water frequently, and when it has ceased to rise read and record it.

In calculating the specific heat of shot from the observations of this Exercise it is well to add 0°.5 to the highest reading of the thermometer in the shot, because the greater part of the stem was exposed to the air.

It may be assumed that the whole calorimeter rises in temperature with the water it contains when the shot are poured in. The

* It is well to *practice* pouring the shot from the dipper into the calorimeter in order to acquire the art of doing it quickly and surely when there is need.

† If the common paper-scale thermometer is grasped as near the bulb as possible, it may be used very vigorously as a stirrer, with little danger of breaking.

"thermal capacity" or "water-equivalent" of the calorimeter, if it is of brass, may be found with sufficient accuracy by multiplying its weight by 0.1, that being, approximately, the specific heat of brass.

312. Record and Calculation of Results.—Since the heat lost by the shot in cooling to the final temperature is gained by the water and the calorimeter in rising to the final temperature, it will be possible to state the losses and gains in the form of an equation.

Let m_w = mass of water;
m_s = " " shot;
t_w = temperature of water;
t_s = " " shot;
t_m = " " mixture;
m_c = mass of calorimeter;
h_c = specific heat of material of calorimeter;
x = " " " shot.

The expression for the amount of heat lost by the shot is

$$m_s \times x \times (t_s - t_m).$$

The amount of heat gained by the water is

$$m_w \times 1 \times (t_m - t_w),$$

the specific heat of water being 1.

The amount of heat gained by the calorimeter is $m_c \times h_c \times (t_m - t_w)$, as this is supposed to rise in temperature just as much as the water does.

Stating the equation, *loss of heat by shot = gain of heat by water and calorimeter*, we get

$$m_s \times x \times (t_s - t_m) = m_w \times (t_m - t_w) + m_c \times h_c \times (t_m - t_w).$$

From this we find

$$x = \frac{(m_w + m_c \times h_c) \times (t_m - t_w)}{m_s \times (t_s - t_m)}.$$

The quantity $m_c \times h_c$ is frequently called the "water-equivalent" of the calorimeter, because the calorimeter absorbs as much heat as $m_c \times h_c$ grams of water would absorb during the same increase of temperature.

313. Other Calorimetric Experiments. — Exercise 43 serves as an example of calorimetric work. Other calorimetric experiments will be found in the next Chapter in connection with a study of changes of physical state.

PROBLEMS ON CHAPTER XXIII.

(See Appendix VI for specific heats of iron, glass, etc.)

(1) How many thermal units will be required to raise the temperature of a kilogram-weight of iron from 15° to 30° C.?

(2) Equal weights of water at 0° C. and oil of turpentine at 50° C. are shaken together. The specific heat of oil of turpentine being about 0.47, what is the temperature of the mixture?

(3) If the specific heat of mercury is .0333, what will be the temperature of 100 gm. of water taken at 0° C., into which 1000 gm. of mercury at 100° C. are poured and thoroughly stirred?

(4) Into 110 gm. of water at 15° C., contained in a vessel the thermal capacity of which is equal to that of 10 gm. of water, are put 200 gm. of a certain solid at 100° C., and the resulting temperature of the whole is 25° C. Calculate the specific heat of the solid.

(5) From the following data find the temperature after mixing:

Weight of water used	100 gm.
" " mercury used	1000 "
Original temperature of water	10° C.
" " " mercury	100° "
Specific heat of mercury	.0333
Number of heat-units absorbed by the calorimeter.	80

Ans. 31°.88.

(6) What is the water-equivalent of a thermometer which is made of 20 gm. of glass and contains 10 gm. mercury?

(7) A piece of tinned iron is found to have a specific heat of .09. What is the percentage of iron and of tin present?

(8) If the specific heat of copper is 093 when the Centigrade scale is used, what would it be if the Fahrenheit scale were used? (Be sure of the definition of specific heat before answering.)

CHAPTER XXIV.

CHANGES OF PHYSICAL STATE.

314. Change of Properties in Solids by Addition or Subtraction of Heat.—Besides the expansion discussed in Chapter XXII as a very familiar effect of added heat on most solids, a number of other changes are usually produced. Solids usually have their rigidity and tenacity lessened by heating. Iron pillars and floor-beams which are amply sufficient to support the floors of buildings become so much weakened upon being heated, if the building becomes thoroughly on fire, that they sometimes yield and fall sooner than fireproofed wooden ones (that is, wooden ones coated with plaster or tiles) would have done under the same circumstances. Zinc, which is not very malleable at ordinary temperatures, may be easily rolled into thin sheets between heated rollers at a temperature of 100° to 150° C., while at a temperature of 200° C. it is so brittle as to be readily powdered in an iron mortar. The power of metals to conduct electricity undergoes such diminution with rise of temperature that this diminution is used as a means of estimating very high temperatures. Sir William Thomson (Lord Kelvin) says: "Every known property of a piece of matter, except its gravity and inertia, varies with variation of temperature." *

Fusion and Solidification.

315. Fusion.—Most of the solid *elements*—that is, substances which consist of only one kind of matter—pass at a

* Article *Heat* in Encyc. Brit., 9th Edition.

more or less definite temperature from the solid to the liquid state. So do many chemical compounds and mixtures of compounds, such as common salt, paraffin, beeswax. This change of state is called *fusion*, or *melting*.

A comparatively small number of substances, like oxide of arsenic, iodine, and camphor, may pass directly and freely from the solid into the gaseous condition; although iodine and camphor may also be readily melted and then boiled.

316. Melting-points.—The temperature at which a substance fuses, or melts, is called the *melting-point*. Appendix VI shows that the melting-points of many substances have been pretty definitely determined.

There is a noticeable difference in the abruptness of the transition from solid to liquid in the case of different substances: ordinary glass becomes plastic at temperatures below redness, while it melts only at an orange or straw-yellow temperature. Wrought-iron and mild steel act in the same way; while cast-iron, antimony, and many other substances, notably water, pass abruptly from the solid to the liquid condition.

EXPERIMENT.

Half-fill the copper dipper (No. 81) with fine chips of paraffin. Place this dipper in the cylindrical part of the copper boiler nearly filled with water kept at a temperature of about 60° C., and stir the paraffin about with a thermometer without touching the sides of the vessel, taking readings four or five times a minute, as the paraffin melts. If the melting goes on rapidly, lift the vessel of paraffin out of the hot water for a few seconds at a time. When all the paraffin has melted, let it rise several degrees in temperature, then take it out of the hot water altogether, and continue to note its temperature as it cools and hardens. How much difference is there between the melting-point and the solidifying-point? How does paraffin in this regard compare with water?

317. Variations of the Melting-point.—The melting-point of a substance is affected by the presence in it of impurities and by change of pressure. Increased pressure lowers the melting-point of a substance which contracts upon melting, and raises that of one which expands upon melting (§ 319). The change from this cause is, however, very slight. For instance, one atmosphere additional pressure lowers the melting-point of ice only about .0075 of a degree Centigrade.

The change of melting-point produced by change of pressure, though slight, may produce very curious results; for a change to the extent of .001 degree, or less, will determine, under certain conditions, whether freezing or melting shall take place.

EXPERIMENT.

Freeze two lumps of wet ice together by pressing one hard against the other between the hands.

In this experiment the pressure is borne by a few points of contact, and is therefore rather intense at these points. Melting occurs, the points are flattened down, making somewhat larger surfaces of contact; and the water in the narrow chinks near these surfaces (see Fig. 220), being shielded somewhat from the atmospheric pressure by its own concave surface (§ 195), is in a condition to freeze even if its temperature were a trifle above 0° C. Accordingly, freezing at once occurs.

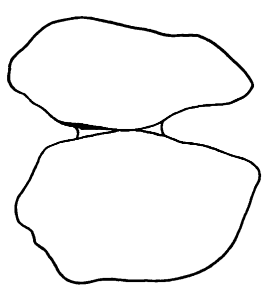

FIG. 220.

The apparently *flowing* movement of glaciers, the great masses of ice in mountain ravines, is explained by melting under great pressure at the points of resistance and freezing again at places of less pressure beyond the obstacles.

BOTTOMLEY'S EXPERIMENT.

Take a block of ice about 15 cm. long and 4 cm. square, support it in a horizontal position at the ends, and suspend from it, by means of a fine wire passing over its middle, a weight of a kilogram or more. Observe the rapidity with which the wire passes through the ice and the condition of the ice after it is cut through.

318. Sub-cooling and Sudden Freezing of Water.—Although pure water under standard atmospheric pressure freezes or melts at 0° C. in a vast majority of cases, it is possible to cool it many degrees lower without allowing it to freeze.

EXPERIMENT.

Fill a common "test-tube" with clear water to a depth of 5 or 6 cm. Place the bulb of a thermometer in the water, the stem, which should be graduated about 10° below the freezing-point, fitting snugly in a perforated cork placed in the mouth of the tube (see Fig. 221). Leave an opening for the escape of steam at one side of the cork, and then cautiously boil the water in the tube for several minutes to remove any air it may contain, taking care to leave water enough to cover the bulb of the thermometer. Allow the whole to cool; and, while it is cooling, pour in oil enough to cover the water to a depth of a few millimeters.

Prepare a freezing-mixture (§ 320) of salt, snow (or ice), and water, having a temperature about − 10° C.—not colder, lest its action be too rapid.

After cooling the tube nearly to 0° C. in ice-water, place it in the freezing-mixture, taking care not to disturb its contents, and watch the thermometer as the cooling progresses.

If all goes well, the mercury will fall steadily below the freezing-point to a distance of several degrees. When a temperature of − 4° C. is indicated, lift the tube gently, and observe that the water is still clear liquid. Shake the tube, or stir the water by means of the thermometer, and note the swift change that occurs after freezing begins. Note, too, the behavior of the mercury-column when this change occurs.

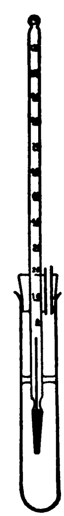

FIG. 221.

If the experiment is to be repeated, make sure that *every particle* of ice is melted, and that *no air-bubbles* have reached the water, before placing the tube again in the freezing-mixture.

The use of the oil in this experiment is twofold. It excludes air from the water, and it prevents the freezing, which may begin prematurely on the wet upper wall of the tube, from extending downward into the body of the liquid.

319. Change of Volume during Fusion or Solidification.—Most solids change their volume during the melting or the solidifying process.

EXPERIMENT.

Fill with ice-water and cork very tightly a bottle of 50 or more cu. cm. capacity, and bury the bottle in a mixture of pounded ice and salt. Allow the bottle to remain there a few minutes, then remove and examine it to see whether the water has expanded or contracted while freezing.

The fact that ice has a lower specific gravity than water, and will therefore float, co-operates with the fact that the maximum density of water is at or near 4° C. (§ 301) to prevent large bodies of water in cold climates from freezing solid.

Most metals and alloys contract in solidifying, but a few, as cast iron and type-metal, expand, and these alone can be readily and successfully cast when it is necessary to obtain a sharp clear impression of the mould in which the cast is made. In casting steel cannon the melted metal has sometimes been submitted to the action of a powerful hydraulic press, which forces the steel into every portion of the mould and at the same time greatly diminishes the size of any contained air-bubbles.

320. Latent Heat of Fusion, or Melting; Freezing-mixtures.—The student may have noticed that a melting body absorbs heat, even when it shows no rise of temperature. A kilogram of crushed ice and a kilogram of water at 0°

put into similar vessels, and exposed to such equal sources of heat as would be furnished by adjacent lids of an ordinary hot cooking-stove, would be found, at the end of the few minutes necessary to melt the ice, to be many degrees apart in temperature, the water in the vessel which contained ice being little above 0°, while that in the other vessel would be hot. The heat which disappears in the melting is said to become *latent*, that is, *hidden*, and the phenomenon is not confined to water, but occurs whenever a solid is liquefied by true melting.

When a solid is liquefied by *dissolving* in another substance, the action is more complicated, and frequently produces a rise of temperature. Acids in which metals are being dissolved may rise from the temperature of the room to a temperature of 100° C. or over, as is readily shown by putting strips of zinc into strong hydrochloric acid.

On the other hand, many cases are known in which two solids or a solid and a liquid have sufficient attraction for each other to form a liquid mixture when brought together, but which do not in uniting furnish sufficient heat to provide for the work of liquefaction without fall of temperature. The result is a greater or less cooling, not merely of the mixture itself, but of the surrounding objects as well. Such combinations of substances are therefore known as *freezing-mixtures*.

The commonest and most convenient freezing-mixture is that of ice and common salt (about two parts by weight of the former to one of the latter), so generally employed in ice-cream freezing. This mixture may easily be made to produce a temperature of — 20° C. Fahrenheit took for the zero of his thermometer-scale the lowest temperature which he obtained by means of it.

With a mixture of properly prepared calcium chloride and snow a temperature of — 48° C. may be reached, and mercury rapidly solidified.

To illustrate the demand for heat which the process of liquefaction involves, the following experiments may be tried, in one of which alcohol is mixed with *liquid* water while in the other it is mixed with *solid* water. (By use of an air-thermometer [§ 305], the bulb of which is placed in the mixture [see Fig. 222], the changes of temperature may be made visible to a whole class at once. The sensitiveness of this instrument may be greatly increased by use of a water-column instead of a mercury-column.)

EXPERIMENTS.

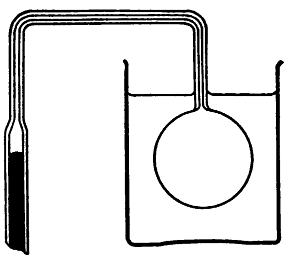

FIG. 222.

(1) With 50 cu. cm. of water in a thin beaker-glass mix about 10 cu. cm. of alcohol, taking both at the temperature of the room, and note the resulting change of temperature.

(2) Pour into a mass of snow or finely broken ice enough alcohol to moisten it, and note the resulting change of temperature.

The number of units of heat required to melt the unit mass of a given substance is called the *latent heat of fusion* of that substance. This quantity can most readily be determined by an application of the method of mixtures. It is illustrated in the case of ice in the following Exercise.

EXERCISE 44.

LATENT HEAT OF MELTING ICE.

Apparatus: Nos. 71, 80 (without the top), 82, 83, 89. About 150 gm. of ice in large clear lumps. (It is important to have the ice as dry as may be, wh n it is put into the water. If it is pounded fine long before being used, the melting, which is going on continually at the surface of each small lump in a room at the ordinary temperature, fills the spaces between with water, and serious error may result.) Placed in a canvas bag (No. 91) and struck smartly against any firm resisting surface, the large lumps may be quickly powdered

when the ice is needed. They should be kept in some convenient vessel—a saucer, for instance—until this time comes.

Half-fill the boiler with water, and place the flame beneath. Weigh the calorimeter. When the water in the boiler is near 50° C., pour about 200 gm. of it into the calorimeter, and weigh it with an accuracy of 0.5 gm.

Immediately transfer the lumps of ice to the bag, taking care not to *pour* them in lest water should go with them, and pound them fine. Stir the water in the calorimeter thoroughly, take its temperature, and then put into it about two thirds of the ice, avoiding the wetter portions. The weight of ice added need not be determined with accuracy at first, as it can be found by carefully weighing the vessel and its contents after the hurry is over.

Stir the water thoroughly, though not violently, with the thermometer, and record the temperature, as soon as all the ice is melted. If so much ice has been put in as to cool the water below 5° C., it is well to dip out the ice remaining unmelted at that temperature, taking as little water with it as possible.

Weigh the calorimeter and its contents, in order to find more exactly the weight of ice added.

Calculate the *latent heat of fusion* of ice, that is, the number of units of heat required to change 1 gm. of ice, taken at 0° C., into water at the same temperature. The thermal capacity of the calorimeter is to be taken into account in this and the following Exercise, as in the one preceding.

321. Record and Calculation of Results.—It must be noticed that the ice is first melted and then the water which results from the melting is raised to the final temperature. The heat gained in these two operations must equal that lost by the hot water and the calorimeter which contains it.

Let m_c = mass of the calorimeter;
m_w = " " water;
m_i = " " ice;
t_w = temperature of hot water;
t_m = " " mixture;
x = latent heat of ice.

Call the specific heat of the brass calorimeter 0.1.

With these directions the student should be able to form the necessary equation corresponding to, though not exactly like, that of § 312, and to find from it the value of x, the quantity which is to be determined.

PROBLEMS.

(1) If the melting-point of lead is 330° C., its specific heat .031, and its latent heat of melting 5.6, how many units of heat are required to raise 500 gm. of lead from 300° C. to the melting-point, and then melt it?

(2) If the latent heat of melting in the case of ice is 80, what temperature will result from melting 1 lb. of ice in 9 lbs. of water, the water being originally at 30° C.?

(3) If the latent heat of melting of ice is 80, when the Centigrade scale is used, what is it when the Fahrenheit scale is used? (Be sure of the definition of *latent heat of melting* before answering.)

Vaporization and Condensation.

322. Vaporization; Ordinary Evaporation.—Some solids, such as sugar and glue, and some liquids, such as olive-oil, cannot be vaporized, that is, converted into vapor, without suffering chemical change, by which they are split up into new substances which cannot be reunited by direct means. But a great many substances may, like water, exist in all three of the physical states—the solid, the liquid, and the gaseous.

Vaporization, or evaporation, is not for any particular substance confined to a particular temperature. Water vaporizes at all ordinary temperatures, even below its freezing-point. It is a fact well known to housewives that wet clothes hung out in very cold weather will "freeze dry," that is, dry without thawing; and it may be observed that icicles and patches of snow and ice waste away even in severe cold weather, when no thawing can occur.

Ordinary quiet evaporation occurs only at the free surface

of liquids or solids, and it may be greatly hastened by increasing this surface. In a process of salt-making formerly a good deal used the brine was concentrated by allowing it to trickle over piles of brushwood, on the surface of which it spread out and rapidly dried away. Increase of temperature, renewing the air in contact with the evaporating liquid, and rarefying the air, all aid evaporation.

323. Boiling.—The student is probably well aware that a liquid disappears more rapidly when boiling than when not boiling. He has seen, too, in some of the preceding Exercises that boiling, in the case of water at least, takes place at a definite temperature, which is, however, somewhat affected by the pressure to which the liquid is subjected. The following experiment is intended to explain the observed facts concerning boiling by revealing the nature of the process. It shows, too, some of the things that occur in the liquid while it is being heated, before it begins to boil.

EXPERIMENT.

Half-fill a flask (No. LXXI) with water, as in Fig. 223, and let the glass tube, about 10 cm. long, connected with the branch at the neck, dip one or two centimeters beneath the surface of cold water in a tumbler. The stopper at the top of the flask should fit tight. The tube leading through this stopper should dip two or three centimeters beneath the surface of the water and should be open at the top.

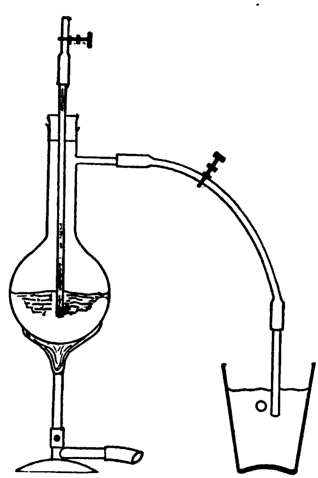

FIG. 223.

Apply a flame to the bottom of the flask, taking care not to let it rise higher than the water, and watch what occurs in the flask and in the tumbler as the temperature of the water rises.

Two kinds of bubbles may be looked for in the vessels: bubbles of air which will not disappear till they reach the surface of the water and burst; bubbles of water-vapor, which may rise to the surface if the water is hot, but will burst and disappear with considerable noise when they come into contact with colder water. The upper layers of water in the flask will be colder than the lower layers until the whole becomes violently stirred up by actual boiling.

Do any bubbles rise at first in the flask? If so, do they appear to be air, or to be vapor?

Do any bubbles flow over into the tumbler at first? If so, are they air, or vapor?

As the process continues, do the appearance and behavior of the bubbles in either vessel change? If so, what is the nature of the change?

What is the action in each vessel after boiling begins?

After boiling has continued for a minute or two, remove the flame and watch what happens, particularly in the tubes of the apparatus.

Does the water from the tumbler rise in the side-tube? If so, does it flow over into the flask? Can you explain the observed actions?

Before these actions cease, place one finger for a moment on the mouth of the tube at the top of the flask, watching the water in the side-tube at the same time. Does anything happen in the side-tube? * If so, can you explain it?

Again boil the water in the flask, close the rubber tube at the top by means of the pinch-cock, remove the tumbler, take away the flame, and immediately close the side-tube † by means of its pinch-cock.

What is now in the flask above the water—air or vapor?

Cool the upper part of the flask by cautious use of a wet cloth or sponge. Observe and explain the effect produced in the water by this cooling.

(If it is considered desirable that each student should perform this Experiment, apparatus like that of Fig. 224 can be used.)

* If the cold water is allowed to flow over into the flask, the latter may be broken.

† If this tube is closed before the flame is removed an explosion will probably occur.

After these experiments the student should have a clear notion of the phenomenon of *boiling.* It is *the formation of bubbles of vapor of the boiling substance* (*not of air, as many think*) *within the body of the liquid;* and *it cannot occur unless the expansive pressure of the vapor is great*

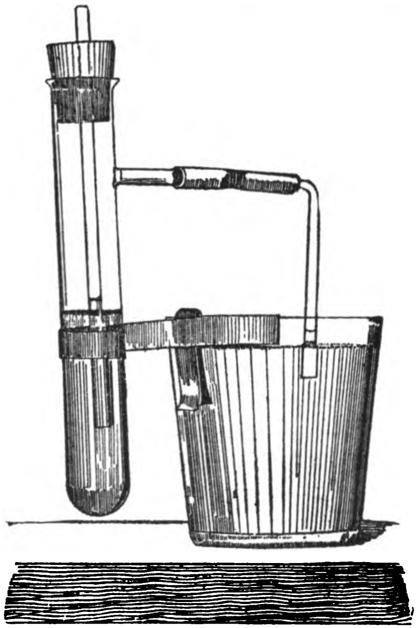

FIG. 224.

enough to withstand the pressure tending to collapse these bubbles.

324. Relation between Vapor-pressure and Temperature. —The pressure which the vapor of any given liquid can exert increases with rise of temperature, and so, with a given external pressure, there is for each liquid a particular temperature, beneath which this liquid cannot boil. The reason why the liquid cannot rise above this temperature while boiling, at the given pressure, is that the rapid forma-

tion of vapor absorbs, renders *latent*, the heat that is supplied by the flame. See Exercise 46.

The table in Appendix VII. shows that the maximum vapor-pressure of water is equal at 100° C. to the pressure of 760 mm. of mercury, the standard barometer pressure. Different substances vary greatly in their boiling-points, perhaps no two substances found in nature boiling at the same temperature. The extreme range between the highest and the lowest observable boiling-points has probably not yet been learned; two very wide apart are that of carbonic acid,—78° C., and that of zinc, about 1000° C.

325. "Non-saturated" and "Saturated" Vapors.—A century or two ago evaporation was explained as the absorption of liquid particles by the air, and the presence of air was supposed necessary for the process.* After a time it was shown that evaporation takes place with especial rapidity in a vacuum, as a later experiment will show, but the terms "saturated" and "non-saturated," as applied to vapors, continue to be useful.

The present theory of saturated and non-saturated vapors may be illustrated as follows: Let *A*, in Fig. 225, represent

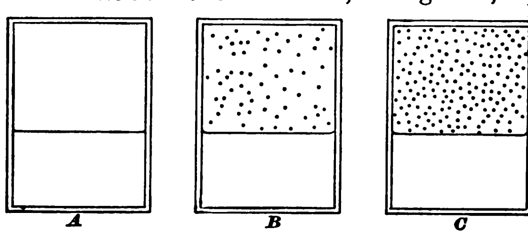

FIG. 225.

a closed vessel containing a liquid with a vacuum above it. The particles of the liquid are in a state of invisible motion; some of them are naturally more agitated than others. The more lively particles at the surface break loose and fly off into the vacuum above, which therefore ceases to be a

* See Whewell's *History of the Inductives Science*, vol. II.

vacuum (see *B*). These particles are now vapor. They fly about, bumping against each other and against the wall of the vessel, exerting pressure.

Some of the flying particles plunge back into the liquid, but at first others, more numerous, escape at the same time from the liquid. So the pressure in the space above the liquid increases. We have still a *non-saturated* vapor there.

After a time, if the temperature remains unchanged, the swarm of particles above the liquid becomes so dense (see *C*), that just as many go back into the liquid per second as escape from it. Now the vapor no longer increases in density and pressure. It is a *saturated* vapor.

EXPERIMENT I.

Arrange apparatus according to the indications of Fig. 226, where *F* is a small funnel containing common ether, and *C* is a pinch-cock closing air-tight the short rubber tube on which it presses.

Work the air-pump until the pressure-gauge indicates the best attainable vacuum in the flask, and then cut off connection with the pump by means of another pinch-cock, *C′*.

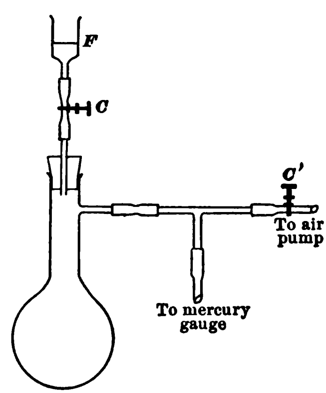

FIG. 226.

After making sure by watching the gauge that there is no leakage of air into the flask, open the pinch-cock *C* cautiously till a few drops of the ether descend into the flask, then close it again, and look to see whether the mercury level in the gauge has changed in such a way as to indicate any increase of pressure.

Let in more ether, a few drops at a time, watching the gauge meanwhile, till half a teaspoonful of liquid ether appears in the flask.

Then leave the apparatus to itself for an hour or two, all joints tight, and finally observe how much the pressure originally left in the flask by the air-pump has been increased by the evaporating

ether. This total increase should represent roughly the pressure of saturated ether vapor at the final temperature of the room.

The following experiment will illustrate still further the behavior of a saturated vapor:

EXPERIMENT 2.

Take a glass tube (No. LXXIII) about 80 cm. long and 0.8 cm. inside, sealed at one end, and fill it with mercury in the mercury-well (No. LXXIV), expelling the air-bubbles from this tube as fully as you can.

Take out the mercury for a distance of 1 cm. at the open end of the tube and fill this space completely with ether. Close this end with one finger, invert the tube, and plunge the end, still closed by the finger, beneath the surface of the mercury in the well.

After the ether has risen above the mercury, see whether there is a bubble at the very top. If so, it is air; but if the bubble is small, it will affect the experiment but little.

FIG. 227.

Observe that the ether being at the top of a mercury-column longer than that of the barometer, will be under very little pressure indeed as soon as the finger is removed from the unsealed end of the tube. Remove the finger and note what happens at the top of the tube.

Fix the tube in position, and then (see Fig. 227) measure the height of the mercury-column within it, above the general level. The difference between this and the length of the barometer-column will show roughly the pressure of saturated ether-vapor at the temperature * of the room.

To see how a saturated vapor acts when one tries to put greater pressure upon it, push the open end of the tube slowly down into the mercury and note the behavior of the mercury-column and the ether.

Does the mercury-column within the tube act, during this lowering of the tube, as it would if the vapor above it were replaced by air?

What becomes of the vapor as the space occupied by it is gradually lessened? Is it *compressed*, as air would be?

Try the effect of warming the ether, *e*, and the upper

* As the first violent evaporation cools the ether and the tube considerably, it is well to allow some minutes to elapse before measuring the height of the mercury-column.

part of the tube by means of a cloth wet with warm water. Try also the effect of ice-water upon it.

326. "Superheating" due to Compression.—Condensation, the reverse of evaporation, occurs whenever an attempt is made to increase the pressure upon a saturated vapor without rise of temperature, or to keep the pressure of such a vapor constant during a fall of temperature, as in the Experiment just preceding.

Evidently, an increase of temperature accompanying an increase of pressure might prevent condensation. Now compression heats most bodies, and it is a curious fact that when we try to condense certain saturated vapors by means of pressure, without giving them opportunity to discharge heat to other bodies, they become so much heated that the attempt to condense them fails, and they, on the contrary, become non-saturated, or *superheated*, vapors. Water-vapor is one that acts in this way.

327. Increase of Volume during Evaporation.—One cu. cm. of water at 100° would make about 1700 cu. cm. of steam, at the standard barometric pressure, at a temperature of 100°. When water is boiled at a temperature higher than 100° C. the saturated steam generated from it has a greater density, as well as a greater pressure, than saturated steam at 100° C.

Most liquids increase in volume less than water upon evaporating, so that their vapors are heavier than that of water.

328. Mixtures of Gases and Vapors.—When vapors and gases which have no especial attraction for each other are mixed, the pressure of the mixture is the sum of the pressures of its components. For example, if a cubic foot of oxygen at a certain pressure, a cubic foot of nitrogen at a certain pressure, and a cubic foot of aqueous vapor at a cer-

tain pressure, the temperature being the same for all, are crowded together into one cubic foot without change of temperature, the resulting pressure will be equal to the original pressure of the oxygen *plus* that of the nitrogen *plus* that of the vapor. Each of the three constituents is to be regarded as exerting still the same pressure that it exerted before the mixing. This is called *Dalton's law*, from its discoverer.

329. Atmospheric Vapor: Dew-point.—Our atmosphere is a mixture of gases and vapors, each bearing its part of the total pressure. When the air contains aqueous vapor in a nearly saturated condition it is commonly called moist air, although, strictly, aqueous *vapor* is not called *moisture.* The proportion of such vapor in the atmosphere near us may be ascertained by exposing a measured portion of air to the action of a weighed amount of some substance, calcium chloride, for example, which will absorb the water-vapor from the air, and then calculating the amount of this vapor from the increase in weight of the absorbing substance. A more rapid process consists in the use of some sort of *hygrometer*,* one of the commonest of which is the *dew-point* hygrometer (see Exercise 45). *Dew-point* is the name given to the temperature at which the water-vapor in the air, when cooled, begins to turn into liquid.

This temperature is higher the greater the amount of vapor in the air, and it differs greatly from time to time. If the room in which the following Exercise is performed is not very large, it will be interesting to compare the dew-point found after several boilers have been in operation for a considerable time with that found before boiling begins.

Observations of the dew-point are continually made by those whose business it is to foretell the weather.

* Greek ὑγρός, moist.

EXERCISE 45.

DETERMINATION OF THE DEW-POINT.

Apparatus: Nos. 82 and 89. Ice, or snow, and salt.

Pour a little water at the ordinary temperature into the cup, and into this put the bulb of the thermometer. Gradually cool this water by small additions of ice-water or ice, and finally of salt if necessary, watching the outside of the lower part of the cup after each addition, and continually stirring its contents. As soon as a mist begins to form on the bright surface near the bottom of the vessel, note the temperature, and cease adding ice.

It is likely that the temperature thus observed will be a little lower than necessary, a little below the true dew-point. Therefore reverse the process, making small additions of water, taken at the room temperature, until the mist begins to disappear. Then take the temperature again. This will probably be a little higher than the dew-point.

It is well to repeat the experiment. After one or two trials, the temperature found going down and that found coming up the scale should be pretty close together—not more than 1 or 2 degrees apart. The mean of the two may then be taken as the dew-poinit.

The experimenter must avoid breathing upon the outside of the vessel or holding any damp object near it, while looking for the mist to form.

If a thick layer of moisture forms upon the vessel before the reverse process is begun, it should be wiped off. A thin layer can be put on, when it is desired, by a mere breath.

The state of the weather and the out-of-door temperature should be noted in connection with this Exercise.

330. Condensation of the so-called Permanent Gases.— In the preceding Exercise all the gases near the metal cup are cooled. The oxygen, the nitrogen, and the carbon dioxide as well as the water-vapor, in every part of the neighboring air, shrink in cooling under the steady pressure of the atmosphere. The water-vapor alone succumbs to these depressing influences and turns into liquid, but we

know that still further applications of cold and pressure would at last condense the most elastic and perfect gases.

The English chemist and physicist Faraday was perhaps the first experimenter who succeeded in liquefying any of the gases which are permanent at ordinary temperatures. He was able by the joint use of cold and pressure to liquefy chlorine, and he afterwards managed by these means to reduce many gases to the liquid state. His method is indicated in Fig. 228. A strong closed glass tube contains in one end a solid or liquid which gives off at a high pressure, when heated, the gas that is to be condensed. The other end of the tube is surrounded by a freezing mixture, and in this end condensation occurs.

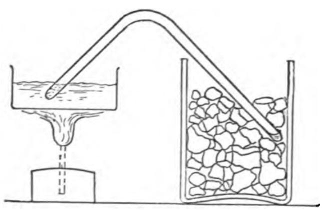

FIG. 228.

Six gases, however,—oxygen, nitrogen, hydrogen, marsh-gas, nitrogen dioxide, and carbon monoxide,—had not been liquefied up to the year 1877. In that year a Genevese physicist, M. Pictet, and a French one, M. Cailletet, working independently and by somewhat different methods, succeeded in liquefying all six of these permanent gases. Cailletet's method consisted in confining the gas experimented upon in a very strong glass tube closed at one end, which was kept very cold, then forcing mercury into the open end by means of a hydraulic press, and finally cooling the compressed gas to the point of liquefaction by a sudden withdrawal of pressure, thus allowing the gas to expand very rapidly (see § 336). The pressure during the experiment rose as high as 300 atmospheres, and the temperature at the moment of expansion fell to a point estimated to be about − 220° C.*

* Exact determinations of very low temperatu es are difficult to make.

Since air is mostly a mixture of oxygen and nitrogen, it is evident that air is now to be ranked among the condensable gases.

331. Distillation.—A most important practical application of the diversity of substances in regard to their boiling-points is found in the operation of distilling. In many cases a volatile liquid may be separated from less volatile solids or liquids with which it is mixed or combined by heating the mixture to boiling and cooling the escaping vapor until it condenses.

EXPERIMENT.

Color some hot water decidedly blue by stirring into it some powdered sulphate of copper. Half-fill a glass flask with this solution, close the top, as in Fig. 229, and connect with the side-tube in the neck a small glass tube about 80 cm. long. Carry this glass tube

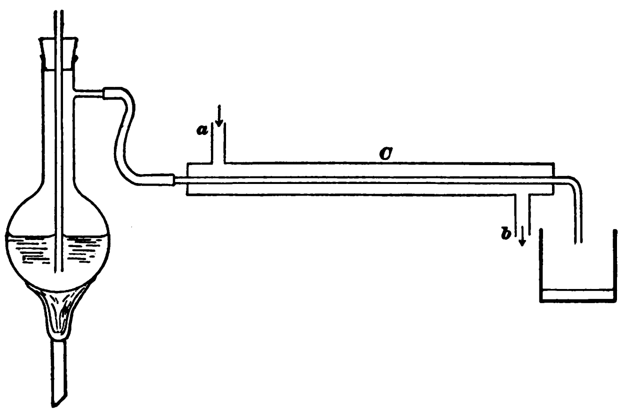

Fig. 229.

through the stoppered sheet-iron tube (No. 85) and pass a current of cold water through the latter.

Now boil the solution in the flask violently and catch in a beaker the water that condenses in the glass tube. Does the color of this water show that any of the sulphate of copper has passed over with the steam?

If a liquid, such as crude petroleum, is found to change its boiling-point upon long heating, it is highly probable that it consists of a mixture of substances of various boiling-points. These substances may be separated and collected by slowly distilling the mixture, with a thermometer in the path of the escaping vapor, and changing the receiving-vessel every time a rise in the thermometer is noted.

Sometimes two or more vapors distill over together, but in proportions different from those in the original mixture, and many successive distillations may be needed to make the separation complete.

332. Cooling by Evaporation: Latent Heat of Vaporization.—The absorption of heat by a boiling liquid has already been alluded to in explanation of the fact that liquids cease to rise in temperature when they begin to boil. A like absorption of heat occurs in the process of quiet surface evaporation, and if heat is not continually supplied in sufficient quantity to an evaporating liquid, the liquid itself and any object in contact with it is cooled.

The fact probably is that the more active particles break loose first from the liquid, leaving behind them a lower average of activity, a lower temperature.

Many important and curious effects are produced by this method of cooling.

EXPERIMENT I.

Place a watch-glass or (better) a thin, small, shallow, metallic vessel, on a drop of water on a large cork, as in Fig. 230. Fill the vessel a little more than half full of carbon disulphide, which is an extremely volatile liquid; blow on the surface of the latter with common hand-bellows or with the breath, and see if the drop of water beneath the watch-glass can be made to freeze by the rapid evaporation of the liquid.*

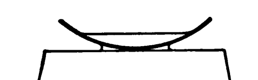

FIG. 230.

* This experiment should be performed where there is a good

Every one has noticed how much more the hands feel the out-of-door cold, especially on a windy day, when they are wet.* The sensation of cold is still stronger when they are wet with alcohol, which evaporates more readily than water. In the heat of summer the evaporation of perspiration from the skin tends to keep the body cool. When the air contains a great deal of water-vapor, when it is, in common speech, moist, this evaporation from the skin takes place but slowly, and we miss the cooling effect. Weather which is both hot and moist is therefore peculiarly oppressive.

It has already been stated that evaporation takes place with especial rapidity in a vacuum. Advantage is taken of this fact in the following experiment. In the glass bulbs and the connecting tube (see Fig. 231) there is no air, but water or water-vapor. When one of the bulbs is cooled by the freezing-mixture, much of the vapor within it is condensed, and this gives room for more, which is rapidly formed at the surface of the water in the other bulb. The apparatus (No. LXXVI) is called a *cryophorus*.†

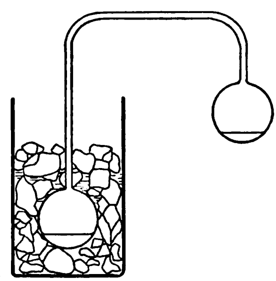

FIG. 231.

EXPERIMENT 2.

Get all of the water into the upper bulb of the cryophorus, wrap this bulb with cotton wool, place the other bulb in a good freezing-mixture, then leave the whole apparatus to itself for several minutes. When the covering is removed from the upper bulb the water within it will probably be found frozen at the surface.

draught of air and away from light or flame of any kind. The odor of carbon disulphide is peculiarly disagreeable; its vapor is highly inflammable, and it is poisonous to breathe.

* Sailors find the direction of a light wind by holding up a wet finger and noting upon which side the cool sensation is strongest.

† Greek: carrier of cold.

The absorption of heat by evaporation of water at the earth's surface or in clouds, and the giving up of heat in the reverse process of gathering dew or forming mists, cannot but have important atmospheric effects, which it is the duty of science to trace out and make useful in weather predictions and possibly in other ways.

In numerical terms, the *latent heat of vaporization* of a liquid is the number of heat-units (§ 309) required to evaporate unit-mass of the liquid without change of temperature. This quantity is different at different temperatures.

In the following Exercise we shall undertake to find the amount of heat required to evaporate one gram of water at 100° C., or as near that temperature as the atmospheric pressure prevailing at the time may permit the boiling-point to be. The actual process employed, however, in this Exercise is to *condense* the vapor at this temperature and find the amount of heat *given out* in this operation, which is the reverse of evaporation by boiling.

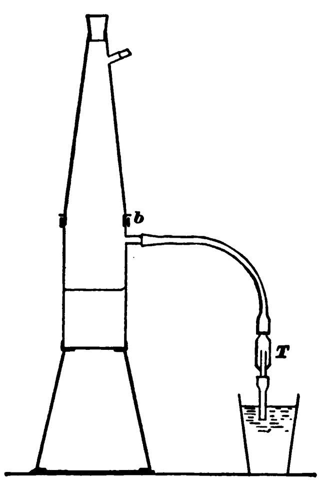

FIG. 232.

EXERCISE 46.

LATENT HEAT OF VAPORIZATION.

Apparatus: Nos. 71, 80, 82, 83, 89, and 92. Snow or ice. A board or sheet of pasteboard to serve as a shield between the lamp and the calorimeter; not shown in Fig. 232. A rubber band, *b*, to prevent leakage of boiler.

Fill the boiler to a depth of about 5 cm. with water, press down the cover with especial care, adjust the band *b*, close the two apertures at the top, and attach the glass "trap," *T*, to the side-tube, as in Fig. 232. Place a Bunsen flame beneath the boiler.

Weigh the calorimeter, and then weigh in it with much care

about 300 gm. of water, some fifteen degrees below the temperature of the room. See that the outside of the calorimeter is kept dry.

As soon as there is a strong flow of steam through the trap, put the shield in place, stir the cold water thoroughly, take its temperature, then dip the escape-tube of the trap about 2 cm. beneath its surface. If the boiler is reasonably tight and the flame beneath it is generous, the vigorous flow of steam through the trap should continue, and be shown by a noisy and incessant collapse of bubbles in contact with the cold water.

Stir the water with the thermometer as the temperature rises, and when it is about 30 degrees warmer than it was at the start, pull the mouth of the trap quickly away from the calorimeter, stir the water quickly and *thoroughly*, take its temperature, and then weigh it carefully without loss of time.

The increase of weight since the previous weighing will, if all has gone well, show the weight of steam condensed.

From the data thus obtained the *latent heat of vaporization*, which is equal to the amount of heat given out in the *condensation* (not including the subsequent cooling) of 1 gm. of steam, is to be calculated.

NOTES.

The great difficulty of this Exercise—and it is a serious difficulty—is to find accurately the weight of steam condensed. This weight is not large, and an error of 1 gm. affects the result greatly.

If the trap is not used a considerable amount of *water* trickles into the calorimeter, condensation having occurred in the tube. When the trap is used a much more serious difficulty *may* be caused by an occasional sudden collapse of the steam in the trap, making a momentary partial vacuum there, into which water rushes from the calorimeter. This accident is likely to occur if the mouth of the trap is plunged too far beneath the surface of the water, and it is probably due to the cooling of the trap itself. It may be prevented by the use of a short rubber tube added to the exit-tube of the trap.

Evaporation from the calorimeter may cause perceptible error if the condensation and subsequent weighing take a long time.

The thermometer, if removed several times from the calorimeter, will have carried off enough water to affect the result.

Water may spatter from the calorimeter during the condensation if the vessel is too full or the tube dips too short a distance beneath the surface.

333. Record and Calculation of Results.—

Let m_c = mass of calorimeter;
m_w = " " cold water;
m_s = " " steam;
t_w = temperature of cold water;
t_m = " " mixture;
x = latent heat of steam.

Call t_s, the temperature of the steam, 100° C.
Call the specific heat of the calorimeter 0.1.

Notice that there are two portions of heat yielded to the cold water; namely, that portion which proceeds from the condensation of the steam, and that which is derived from the resulting water, at 100°, cooling to the final temperature.

With these directions the student should form an equation between the amount of heat given out on the one hand and that taken in on the other, and from this equation find the value of x, the quantity to be determined. See §§ 312 and 321.

334. Steam-heating of Buildings.—The great latent heat of steam, shown in the preceding Exercise, makes this fluid an effective agent for the heating of buildings. A comparatively small flow through the radiating pipes is sufficient; for every pound of steam that condenses in these pipes yields a very large amount of heat. The main features of a simple steam-heating system are shown in Figs. 233 and 234.

Coal is burned in the fire-box G (see Fig. 233). The heated air and the hot gases given out by the combustion pass from front to back under the body of the boiler and from back to front through tubes, between and above which lies the water, the upper surface of which is at W. D is a damper to control the draft.

The steam, passing out through the main pipe, or *main*,

heated 10 Centigrade degrees by the condensation of 1 kgm. of water vapor at that temperature?

(9) If the latent heat of evaporation of water, boiling under atmospheric pressure, is 536.6 when the Centigrade scale is used, what would it be if the Fahrenheit scale were used?

CHAPTER XXV.

TRANSFORMATION OF HEAT.

335. Mechanical Equivalent of Heat.—The first precise measurements of the amount of work necessary to produce a given amount of heat were made by Joule, an English physicist, about 1845. His experiments were varied in several ways, but one method which he used was to cause a known weight *W* (Fig. 235), falling through a known dis-

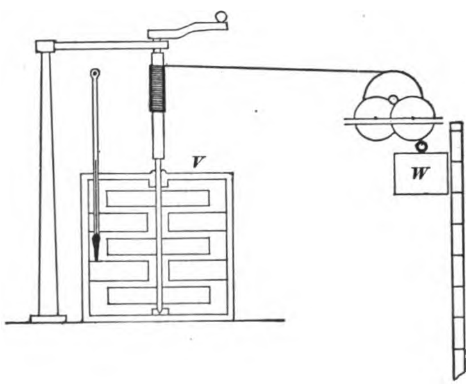

FIG. 235.

tance, to turn paddles inside a vessel, *V*, containing a weighed portion of water at a known temperature. The friction of the paddles raised the temperature of the water, and from the data obtained in this experiment the value of a thermal unit in foot-pounds could be calculated. According to Joule's observations, 772 ft.-lbs. would raise a pound

of water 1° Fahr.; that is to say, a pound of water falling 772 ft. would be raised 1° Fahr., if all the energy of its fall could be turned into heat and all the heat could be kept in the water. To raise a pound of water 1° C., about 1390 ft.-lbs., according to Joule's experiments, were required.

Later experiments by Professor Rowland of Baltimore have shown that the quantities 778 and 1400 are more nearly correct than 772 and 1390. Using the C.G.S. system (§ 247), we find, according to Rowland's experiments, that the mechanical equivalent of 1 gm.-deg. (Centigrade) is about 42,690 gm.-cm.

A striking example of the transformation of mechanical energy into heat is found in the following experiment, which requires a good piece of apparatus, good tinder, and considerable physical vigor.

EXPERIMENT 1.

Ignite tinder by heat generated by sudden compression of air in a "fire-syringe" (No. LXXVII).

As mechanical work can produce heat, so heat can, on the other hand, be used up in doing mechanical work. For instance, an expanding gas pushing something forward, and so doing work as it expands, becomes cooled by this act.

EXPERIMENT 2.

Wrap one bulb of the "differential thermometer" (No. LXXVIII, Fig. 236) with cotton wool, and then place the whole instrument under the bell-jar of a large air-pump.

Exhaust the air rapidly, and watch the thermometer for evidence that the unprotected bulb is cooled. (The air remaining in the bell-jar does *work* in expelling that which goes out.)

After a time let the air re-enter the bell-jar rapidly, and see whether the unprotected bulb gives evidence of heating.

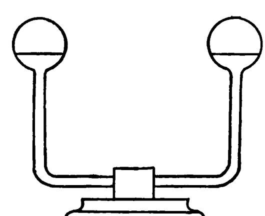

FIG. 236.

It is easier to turn other forms of energy into heat than to make the opposite transformation (see § 339), but experiment shows that one thermal unit wholly turned into work will yield just so many foot-pounds as in the reverse process would give one unit of heat. This is an illustration of the conservation of energy (§ 271).

336. How a Gas is Cooled in Expansion and is Heated in Compression.—The behavior of gases is best explained (§ 280) by supposing them to consist of very small elastic particles of matter which are flying about in very rapid motion in all directions, frequently bumping against each other and against the walls of the vessel that contains them, but rebounding in a very active manner. The tendency of a gas to expand, shown in the pressure which it exerts upon the walls of the containing vessel, is due to this activity of its particles. The energy of these motions is the heat-energy of the gas. To increase the velocity of the particles as a whole is to raise the temperature of the gas, and *vice versa.*

When the gas is expanding some part of the containing wall must be moving outward while the air-particles strike it. When it is being compressed, some part of the wall must be moving inward while particles strike it. Why does the gas become cooled in one case and heated in the other case? The following experiment may throw some light on the matter:

EXPERIMENT.

Suspend an ivory ball by means of a string several feet long. Hold in the hands a flat weight of two or three pounds. Give the ball a slight push, and on its return through the lowest point of its path let it strike the weight held at rest. Swing the ball again, and on its return let it at the same place strike the *receding* weight. Repeat, letting the ball strike the *approaching* weight. In each case note the effect of the collision upon the velocity of the ball, as indicated by the distance it swings after the collision.

When, *during the collision*, the weight recedes before the ball, the latter does *work* upon it and loses energy to it. When, during the collision, the weight keeps moving forward, pushing the ball back, the weight does work upon the ball and the latter gains energy.

337. Heat-engines.—In all heat-engines, whether they are ordinary steam-engines, hot-air engines, gas-engines, or other forms, we have mechanical work done at the expense of heat, the working substance, whatever it may be, using up heat as it expands, because it is doing work. The steam-engine is discussed at some length in Chapter XXVI.

338. The Sun our Main Source of Energy.—Nearly all our available sources of energy on the earth's surface are due to the energy of the sun. Wood is produced by the growth of trees, which growth is maintained by solar light and heat. Coal and petroleum are stored-up results of the solar activity of past geological ages. All animals depend for their food either directly or indirectly upon vegetable substances, and therefore upon solar energy. Finally, since running streams depend upon the evaporation of water and its subsequent fall in the shape of rain or snow, and since winds are due mainly to the unequal heating of different portions of the atmosphere, water-power and the motive power of windmills must be referred to the sun as their source.

The amount of heat which the earth receives from the sun in a day is enormous, and when we reflect that the earth, viewed from the sun, would look no larger than the planets appear to us, it is plain that we receive only a very minute part of the sun's radiation. How can the sun continue to give out this great flow of energy year after year and century after century without cooling?

Much ingenuity has been spent in attempting to answer

this question, and the following theories, with perhaps many others, have been proposed:

The Combustion Theory.—This assumes that the surface of the sun is a great furnace in which coal, hydrogen, etc., are burning, as these substances burn upon the earth,* it being known (see § 353) that the materials of the sun's surface are the same as those found on the earth. This theory must be rejected because, if it were true, the sun's diameter would be diminishirg rapidly, and no diminution is observed.

The Meteoric Theory.—In the space between the planets there is considerable solid material in pieces too small to be seen at any great distance. Such pieces, called meteorites, frequently plunge into our atmosphere and, becoming heated by friction with the air, glow and burn as the familiar objects which we call "shooting stars."

Even if there were no friction these bodies, falling from enormous heights under the influence of gravity, and striking the earth's surface with very great velocity, might be made red-hot by the shock. The sun's mass being vastly greater than that of the earth, meteorites approach its surface under a much greater attraction, and strike it with a correspondingly great generation of heat. In this way some have imagined the heat of the sun to be maintained.

It is now seen that a shower of meteorites sufficiently dense to produce this effect would, if not counteracted by some other agency, increase the diameter of the sun at a rate so rapid that the change would soon be visible. Therefore a meteoric bombardment, although it no doubt is maintained and may be of considerable importance, is not the only and is probably not the main source of the sun's heat.

* One gram of pure carbon burning with oxygen yields about 8000 thermal units; and one gram of hydrogen, burning in the same way, about 34,500 thermal units.

CHAPTER XXVII.

RADIANT ENERGY.

345. The Luminiferous Ether.—That medium (see §§ 90 and 287) by which light is transmitted through what we commonly call a vacuum is named by physicists the *luminiferous* (*light-bearing*) *ether*. This medium is a good deal of a mystery. We have overwhelming proof that it transmits energy by means of a wave-motion. It must, therefore, have inertia, like ordinary matter, but it is not, so far as we know, subject to gravitation like ordinary matter. Its wave-motion is much like that of an elastic *solid*, yet it does not, so far as we have discovered, impede the heavenly bodies in their motion through it. These various properties, positive and negative, do not harmonize very readily, and we have still much to learn about the ether.

All ether-waves are called *radiant waves*, or waves of radiant energy.

346. Nature and Length of Radiant Waves.—It is now well known that radiant waves are electrical and magnetic oscillations in the ether. We do not believe that the sun sends *electricity* to us, but we do believe that the heat-vibrations of the sun's particles disturb the electricity of the ether, and that this disturbance is transmitted to us, somewhat as a shock at one point of the earth sends a quiver to another point.

The length of radiant waves, the distance from the centre of one wave to the centre of the next (C to C_1 in Fig. 246)

is known to range from a few ten-thousandths of a millimeter to several meters. The shortest waves known to us are revealed by their photographic or fluorescent effects; those a trifle longer give us directly the sensation of light,

FIG. 246.

enabling us to see things; the longest waves are at present best shown by their electrical or magnetic effects.

All of these waves produce heat when they fall upon bodies incapable of reflecting or transmitting them as *waves*.

One surface may give off many different kinds (lengths) of vibrations at the same time. It may be said, in general, that the proportion of short waves given off increases as the temperature of the radiating surface rises.

347. Radiating Power.—Different surfaces at the same temperature do not radiate equally well. The best radiating surface known, for all temperatures, is the lustreless black of soot, called lampblack; though some other dull surfaces, that of white lead, for instance, may radiate equally well at low temperatures, such as that of boiling water. Polished metal surfaces are bad radiators.

EXPERIMENT.

Fill the cubical box (No. LXXIX) with hot water and place it some centimeters distant from one face of the thermopile (No. LXXX), the wires from which are connected with the reflecting galvanometer (No. LXXXI) in a somewhat darkened room. See Fig. 247.

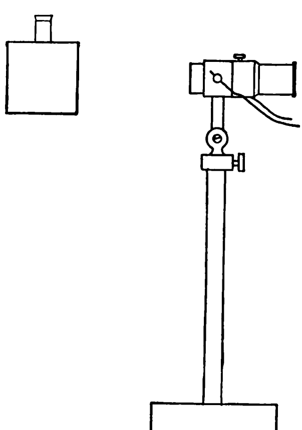
FIG. 247.

Present, at the same distance, the four vertical sides of the box in turn to the thermopile, and thus compare roughly the radiating power of their surfaces.

348. Absorbing Power.—The *absorbing power* of a substance is its power to take in radiant energy which falls upon its surface and turn it into heat. Absorbing power and radiating power go together; that is, any surface at a given temperature absorbs particularly well the kind (length) of vibrations which are most freely given off from it at the same temperature.

We have seen in the experiment of § 347 that a dull white surface may radiate nearly as well as a lampblack surface when both are at the same low temperature. The white surface would not radiate as well as the black at high temperatures, and the following experiment shows that white absorbs less readily than black the radiations from a very hot body, the sun.

EXPERIMENT.

Take a lens 4 or 5 cm. in diameter, and let the bright sunlight pass through it and fall upon a piece of white paper. Find by experiment at what distance from the lens the paper must be held in order that it may become charred very slightly without being consumed. Then blacken a spot upon the paper by rubbing it with a lead-pencil, and again present it to the lens, letting the light fall upon the blackened spot, the distance being the same as before. Is the paper now consumed?

349. Reflection of Radiant Energy.—The student is already familiar with the reflection of light, that is, of radiations giving the sensation of light. The following experiment shows that other radiations, the so-called *obscure* radiations coming from a body that is not hot enough to be luminous, are reflected in the same way:

EXPERIMENT.

Place a candle (see Fig. 248) in front of a spherical metal mirror (No. LXXXIV) in such a position that the image of the candle will fall

upon one face of a thermopile connected with a reflecting galvanometer. When the apparatus is in place, blow out the candle-flame, allow the effect produced upon the galvanometer by the candle to dis-

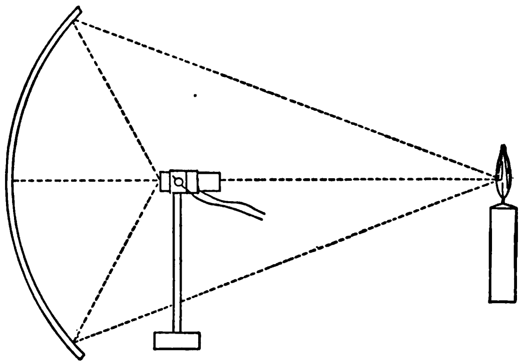

FIG. 248.

appear, and then put exactly in the place of the flame a metal ball (No. XLIII) somewhat hotter than boiling water. Note the effect upon the galvonometer. Move the ball a little to one side of its present position and again note the effect.

350. Exchange of Radiations.—The rate at which one body loses heat to another by radiation depends upon the difference in temperature of the two bodies. If the two are at the same temperature, there is an interchange of radiation between the two; but it is an equal interchange, neither, on the whole, gaining at the expense of the other. If the two are at different temperatures, the hotter loses to the cooler, and loses more and more rapidly if the difference in temperature is increased. When the difference in temperature is not large, the rate of loss from the warmer body to the other is nearly proportional to this difference; but when the difference of temperature rises to hundreds or thousands of degrees, the rate of loss by radiation increases much more rapidly than the difference of temperature.

Attempts have been made to estimate the temperature of

the sun by finding the rate at which we receive radiant energy from it, and calculating, by means of the law of radiation, how hot the sun should be to radiate to us as it does. But the law of radiation and the nature of the sun's surface are so uncertain that estimates so made have varied enormously. It now seems likely that if the sun's surface were of lampblack, or something radiating equally well, its temperature would have to be in the neighborhood of 10,000° C. to enable it to radiate as freely as it now does. The highest temperature obtained in furnaces operated by men is probably between 2000° and 3000° C.

351. Characteristic Radiations: "Spectra" of Different Substances.—When a substance is in the gaseous condition, so that its particles are free to vibrate in their own peculiar way, it gives off radiations or a combination of radiations different from those of other substances.

When, therefore, the light from any self-luminous, or *incandescent*, gas is allowed to pass through a prism (§ 133), under proper conditions, it forms beyond the prism a spectrum (§ 134), which is different from the spectrum of the sun.

The spectrum of any gas appears upon a screen (see § 162) as one or more bright lines parallel to the edges of the prism. In some gas-spectra there are scores or hundreds

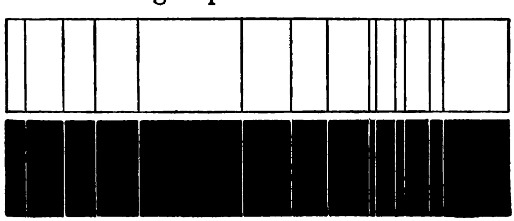

Fig. 249.

of such lines. The upper part of Fig. 249 shows a few of the lines in the spectrum of iron-vapor.

When a gas is very much compressed, its spectrum shows more lines. The explanation probably is that the more frequent collisions which its particles now suffer keep these particles vibrating in certain ways which would not be maintained if the particles were less disturbed.

EXPERIMENT.

Suspend a stick by a long string tied to one end. Strike the stick smartly at the lower end. Observe that the *quiver* of the stick dies out long before the *pendulum* motion, both of which are set up by the blow.

When compression goes so far that condensation occurs, giving either a liquid or a solid, the bright lines of the spectrum become so numerous that they meet each other, making a continuous band of light from one end of the spectrum to the other. All incandescent liquids or solids give such *continuous* spectra.

352. Absorption-spectra.—Every substance absorbs especially well the same kinds (wave-lengths) of radiation, as those given out especially well by itself.* Accordingly, if the light from an incandescent solid or liquid shines through a vapor which is not incandescent, the spectrum of the light that has passed through the vapor is not perfectly continuous, but shows certain comparatively *dark* lines. These lines correspond in position in the spectrum to the *bright* lines which the vapor in question would give if it were incandescent. A spectrum showing such dark lines is called an *absorption*-spectrum. It is just as good a witness to the presence of the vapor as the bright-line spectrum of that vapor would be.

Absorption-spectra may be obtained with incandescent gases, if the light shining through them from behind is

*** A similar fact is observed and discussed in the case of sound-waves. See Exp. 2, § 365.**

powerful enough so that the vapors get more than they give of their peculiar radiation.

353. Spectrum Analysis.—The spectrum, bright-line or absorption, of the vapor of a substance is often made use of to detect the presence of this substance in cases where chemical tests or other physical tests would be powerless. Certain of the chemical elements have actually been made known to man for the first time by means of their spectra, appearing under such conditions that substances already known could not account for them. The examination of a substance by means of the spectrum of its vapor, so as to find out what it is made of, is called *spectrum analysis*.

Spectrum analysis finds magnificent application in the study of the sun and the stars. The spectrum of the sun, examined under proper conditions, is found to be crossed by a vast number of dark lines. These lines, which are called Fraunhofer's lines, because Fraunhofer studied and described many of them very carefully, are now known to be absorption-spectra. They indicate the presence of certain vapors in the atmosphere of the sun, which is cooler than the inner part that radiates through it. Among these absorption-spectra many are recognized as corresponding to the bright-line spectra of substances on the earth. Fig. 249 is a case in point.

On such evidence as this we are persuaded that the sun and the stars are made up of just such elements as those which make our earth.

QUESTIONS AND PROBLEMS.

(1) Make a list of all the common substances that you know may readily be made to appear under all three states, the solid, the liquid, and the gaseous.

(2) What conclusion may be drawn from the fact that the inside of the upper part of a bottle partly filled with solid gum-camphor soon

becomes covered with a crystalline deposit? (Solid iodine acts in the same way.)

(3) Explain the difference between thermal capacity and specific heat.

(4) The climate of islands is more equable than that of the interiors of continents. Explain how this naturally follows from the physical properties of water.

(5) What is the error of the boiling-point in a thermometer that in freely escaping steam, with the barometer at 77.8 cm., reads 100°.1?

(6) Rewrite the formula for calculating specific heat of lead, on the supposition that the shot was cooled to 0° and put into water at 25°.

(7) Suppose that you had performed Exercise 43 exactly as you did, save for the substitution of oil of turpentine, sp. ht. 0.46, for the water actually used. Find the final temperature.

(8) From an inspection of the table of physical properties in Appendix VI what suggestion do you get in regard to the relation between specific gravity and specific heat?

(9) If n gm. of water taken at 0° C. and m gm. of kerosene taken at 20° C. attain when mixed a final temperature of 12° C., find the sp. ht. of kerosene.

(10) Tubs or barrels of water in a cellar in winter are said to "keep the frost away" from fruit and vegetables (whose freezing-point is a little below 0° C.). Explain how this can be true even if the water used is at 0° C.

(11) Taking as the value of the latent heat of fusion of ice 80 thermal units and that of the vaporization of water at 100° as 537 thermal units, what will be the number of units necessary to change a kilogram of ice taken at 0° into steam at 100°?

(12) How many inches of rain at 10° would be needed to melt a layer of ice 1 inch thick, the specific gravity of ice being 0.917?

(13) From the following data find the latent heat of melting for beeswax:

Weight of water	300 grams
" " wax	100 "
" " calorimeter	140 "
Sp. ht. of calorimeter material	0.1
Temperature of water just before wax enters	93° C.
" " wax just before it enters water	62° C. (its melting-point).
" " the whole after wax melts	62° C.

(Assume that no heat escapes from the vessel.)

(14) Let 100 gm. of ice at 0° C. be put into 300 gm. of water at 50° C., contained in a brass vessel weighing 80 gm. Calling the latent heat of water 80 and the specific heat of brass 0.1, calculate the temperature of the mixture.

(15) A kilogram of iron at 100° is immersed in a kilogram of ice and water at 0°. The final temperature is 3°. How much ice was present?

(16) What is meant by the phrase "the dew-point"? Describe carefully the method by which you determined the dew-point.

(17) Describe carefully the nature of boiling, showing how it differs from ordinary evaporation.)

(18) Describe as fully as you can the transferences of heat that occur in the process of heating a room by means of steam-pipes, beginning with the fire beneath the steam-boiler. Show why the steam in the pipes is more efficient in heating than an equal weight of air or water at the same temperature as the steam.

(19) Name all the familiar instances that occur to you of the conversion of mechanical energy into heat, omitting examples given in the preceding chapters.

(20) Assuming the average height of Niagara Falls to be 50 m., how much warmer should we find the water at the foot of the falls than at the top? (Assume that all the heat generated by the collision of the falling water with other water and with the river-bed below is imparted to the falling water.)

(21) How high could a 10 kgm. weight have been lifted by the heat given off during the condensation of the steam in Exercise 46?

(22) Explain as fully as you can the transformations of energy that take place when steam is generated in a locomotive engine, the engine employed to draw a train at a high rate of speed, and the motion finally checked by applying brakes to the train.

(23) A piece of iron weighing 15 kgm. is cut in two by a saw. The force required to push the saw through the iron is 8 kgm., the length of the stroke is 12 cm., and the number of cuts required to finish the work is 50. Supposing all the heat generated to remain in the iron, what will be its temperature at the close of the operation, if the temperature at first was 10° C.? *Ans.* 10°.006 + C.

(24) A loaded sled on ice at 0° C. is drawn by a pull of 25 lb. How much ice would be melted by the friction in drawing the sled 1 mile?

(25) An iron cannon-ball is shot against an iron target with an amount of energy sufficient to have carried it vertically upward ¾ mile.

If the cannon-ball weighs 50 lb. and the target 5 tons, how much rise of temperature would be noted in the target it it received all the heat produced by the blow ?

(26) Suppose a block of ice to fall from a height and strike the earth, retaining in itself all the heat produced by the blow. Neglecting the resistance of the atmosphere, what height would give just sufficient kinetic energy to melt the ice?

CHAPTER XXVIII.

SOUND.

354. Definition.—The word *sound* refers sometimes to the sensations we receive through the auditory nerves, sometimes to the external cause of these sensations. We shall use it mainly in the latter sense.

355. Transmission of Sound.—Light comes to us perfectly well through a vacuum. Can sound do the same?

EXPERIMENT 1.

Place a small alarm-bell, in operation, on a wad of cotton beneath the bell-jar of an air-pump, and exhaust the air as rapidly and as perfectly as you can.

Does the rarefaction of the air have any effect upon the loudness of the sound as perceived by the ear?

Let the air re-enter rapidly and note the effect.

If the alarm-bell used in the preceding experiment had rested directly upon the firm pump-plate, the sound would have made its way to the outer air by way of this solid medium. The following experiment illustrates still further the capacity of solids for carrying sound.

EXPERIMENT 2.

Hold one end of a slender rod of wood 3 m. or more in length firmly between the teeth while some one sounds a tuning-fork and then holds the stem against the free end of the rod.

It is a common and interesting experiment to hold the head under water while a comrade at a distance raps two stones together under water. The loudness of the sound

is painful. It may even be sufficient to injure the hearing at a distance at which the sound produced by rapping the same stones together in air would be only faintly heard.

EXERCISE 47.

VELOCITY OF SOUND IN OPEN AIR.

Apparatus: A pendulum beating seconds. A small spy-glass. A board. A hammer or stone.

One experimenter strikes the board sharply just when the pendulum-bob passes the middle point of its arc. Another places himself at the start about 900 ft. distant from the first, in a line at right angles with the plane in which the pendulum swings, and then, looking through his glass, seeks to place himself at such a distance that the stroke made when the pendulum-bob passes through the middle point of its arc in one direction reaches his ear just when the bob passes through the middle point of its arc in the other direction. The distance which the sound travels in one second in one direction is thus roughly determined. As the wind may either help or hinder this movement, the conditions should afterward be reversed by having the experimenter with the glass strike the board, while the other stays at the pendulum and listens for the stroke, signalling to the striker to come nearer or go farther away until coincidence occurs as before. The mean of the two distances now found should be taken as the distance which sound would travel in one second in still air.

This method is not likely to give very accurate results. Its merit lies in its directness and simplicity. A still more direct and simple method is to place two persons a long distance apart, but in sight of each other, and have each in turn discharge a pistol or gun while the other notes, as well as he can, the number of seconds between the sight and the sound of the explosion.

In case this Exercise is impracticable, as it would be in most city schools, the velocity of sound in air in a tube may be calculated from the results of Exercises 48 and 49.

Changes of temperature greatly affect the velocity of transmission in air and other gases, the speed in air increasing about 60 cm. per second for each centigrade degree of increase in temperature. Change of pressure without

change of temperature does not affect the velocity. In gases of different density, at the same temperature and pressure, the velocity of sound is found to be inversely proportional to the square root of the density. Experiment shows that the velocity of transmission of sound is greater in solids and in liquids than in gases.

356. Sources of Sound.—If we trace back to its source a sound that comes to our ears, we can usually find it in the movements, often very minute, of some body which is said to *give out* the sound. Often this motion is apparent to the eye, as in the case of vibrations of sounding strings or long tuning-forks. Frequently the sense of touch will show this state of motion in a sounding body, a short tuning-fork for instance, when sight does not. Sometimes, as in the case of faint sounds of high pitch coming from short thick metal bodies, we do not perceive the motion of the body directly in any way, and only by what we know of other sounding bodies convince ourselves that such motion exists.

357. Sound-waves.—But how is it that the air or any other medium brings the sound from its source to our ears? Do particles of the medium propelled from the sounding body move on until they reach our ears, as a baseball moves from the swinging bat to the hands of the player who catches it? Evidently this cannot be the case when the medium which transmits the sound is a solid, nor is it the case when the medium is a liquid or a gas. Particles of liquid or gas, if sent forth with the velocity of sound, about 340 m. per second, would soon be arrested by the resistance of their fellows, unless the latter were moving with similar velocity and direction. It is a *concussion*, or *jar*, sent through the air, that breaks glass miles away from an exploding powder-mill or dynamite manufactory.

Now a disturbance which travels through any medium without permanently displacing the parts of that medium,

which is, in fact, merely passed on from one set of particles to the next, is called a *wave*. There are various kinds of waves. With waves at the surface of water all are familiar. The waves which bring us energy from the sun have been alluded to in the preceding pages (§ 346). In these two kinds of waves there is a motion of the particles of the medium to and fro *across* the direction in which the waves move forward. A sound-wave consists merely of a compression traveling forward, followed by a rarefaction. Every particle of the medium affected by the wave is in turn crowded close to its neighbors and then withdrawn from them, making, on the whole, a slight excursion forward and backward, *parallel to* the direction in which the sound is traveling.

This theory of the nature of a sound-wave is sustained by our knowledge of the properties of bodies and by experimental evidence. We shall not enter upon the argument here, but the student will by himself probably find it easy to see, in a general way, how a wave of compression may be sent through a medium such as air. If we—as Professor Tyndall did in his lectures on Sound—explode a small balloon, at the moment of explosion the layer of air immediately about the balloon is violently compressed. By virtue of its elasticity this compressed air will almost instantly transmit the compression to another layer immediately outside of itself, and will at the same time itself become more rarefied than before the explosion. In this way a condensation and a rarefaction—in fact, a series of them—move outward in all directions from the exploded balloon. A single condensation and its accompanying rarefaction together constitute a sound-wave.

The sound-waves produced by a body in mid-air giving off sonorous vibrations, as in the case of a bell rung in a lofty steeple, have, at a distance from the source, nearly

the form of hollow spheres having the source as their common centre (see Fig. 250).

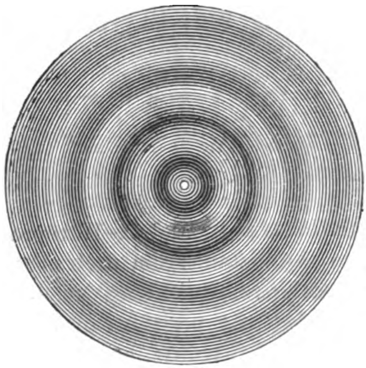

FIG. 250.

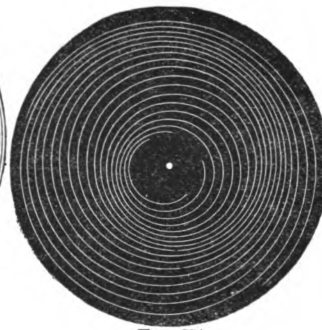

FIG. 251.

358. Analysis of Sound-waves.—In seeking to form a distinct idea of the motions of particles through which a sound-wave is moving the student will be greatly assisted by the use of *Crova's disk* (No. LXXXVII). This consists (Fig. 251) of a number of circles drawn so as to be nearly but not quite concentric.

EXPERIMENT.

The disk is attached to a rotating apparatus, and whirled; while across, in front of either half of it, is held a piece of cardboard, in which is a long slit 2 or 3 mm. wide parallel to any radius of the circles.

The short arcs of the circles seen through the slit may be taken to represent particles of air, and the way in which they crowd together and separate, as the disk rotates, gives a very vivid idea of the way in which the particles of air actually move when set in motion by a sound-wave. The places where many lines are crowded together represent the condensed portion, and those where the lines are widely separated, the rarefied portion, of a sound-wave. Observe how the condensations and rarefactions travel along the slit.

It is rather troublesome at first to see why a condensation or rarefaction in air should travel in one direction rather than another, why it should not go backward as well as forward. The following diagram (Fig. 252) may make the

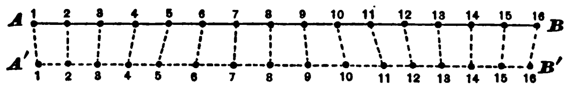

FIG. 252.

matter a little plainer: In the line AB the dots 1–16 represent particles of air which are equidistant from each other when undisturbed. A sound-wave coming along the line AB in either direction will change the positions of these particles without moving them out of that line, but to avoid confusion the particles in their disturbed condition are here represented on another line, $A'B'$.

On this new line particle 1 is displaced a little to the right from its natural position, 2 is not displaced, 3 is displaced a little to the left, 4 a little more to the left, 5 still more and farthest of all, 6 being displaced no more than 4, 7 no more than 3, and 8, like 1, not at all. No. 9 is displaced to the right, 10 still more, etc. Looking along the whole line we see a condensation with centre at 2, a rarefaction with centre at 8, another condensation with centre at 14. At the centre of each condensation or rarefaction is a particle in its natural position.

Is it possible to tell from this diagram whether the wave is moving from A toward B or in the opposite direction? No. That depends upon the present *motions* of the particles, concerning which nothing has been said.

Consider the particles 2 and 3. If they are both moving toward the right, 2 will in a moment be displaced just as 1 now is, and 3 will be in its natural position, as 2 now is; in other words, the condensation now central at 2 will have

moved to the right. The *wave* will be moving toward the right.

If we regard the other particles, we shall find everything consistent with this direction of the wave, although we shall not find all the *particles* moving toward the right. No. 4 is moving to the right, 5 is at its turning-point, 6 is moving to the left, and also 7, 8, 9, and 10. No. 11 is at its turning point, 12 is moving toward the right, and so on to 14, 15, and 16, which are merely repetitions of 2, 3, and 4.

We can, therefore, say, *When a sound-wave is moving toward the right the particles at the centre of condensation are moving toward the right and those at the centre of rarefaction are moving toward the left.*

359. Graphical Representation of a Sound-wave. — A diagram like the following (Fig. 253) is commonly used to *represent* a sound-wave.

The curve *ACD* is taken to represent the condensed part,

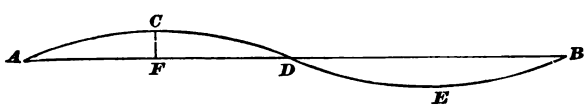

FIG. 253.

and the curve *DEB* the rarefied part, of the wave. Perpendiculars, such as *CF*, let fall from the curve upon the line *AB*, show the relative amount of condensation or rarefaction at any desired point *F* on the line *AB*.

This figure is not a *picture* of a sound-wave, but only a *symbol* for one. The wave is supposed to be traveling from *A* to *B* or from *B* to *A*, and the particles, originally lying on the line *AB*, still lie on that line while the wave-motion is going on.

360. Inverse Square.—Since sound-waves travel outward from their centres with uniform velocity in ever-enlarging hollow spheres, and since the surfaces of spheres are pro-

portional to the squares of their radii, it is plain that a given amount of energy in the shape of a sound-wave must, as it recedes from its source, occupy—if none of it is lost on the way—successive portions of space, which increase with the square of the distance from the source. Fig. 254 represents two of the positions occupied by a part of a wave-front included between certain radii.

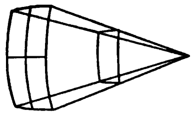

FIG. 254.

Accordingly, the intensity of the wave at any given point must lessen as the point is taken farther and farther away from the source of the wave. More exactly, *the intensity of a sound-wave (and its loudness to the ear) varies inversely as the square of its distance from the vibrating body which produced it.**

361. Reflection of Sound: Echoes.—Sound-waves may rebound or be reflected from surfaces against which they strike. They obey the same law of reflection as light-waves, that is, the angle of reflection is equal to the angle of incidence.

EXPERIMENT.

Place two large spherical mirrors (No. LXXXIV) facing each other at opposite sides of the room. Place a watch at the focus *A* of one mirror and listen for the tick with an ear-trumpet (No.

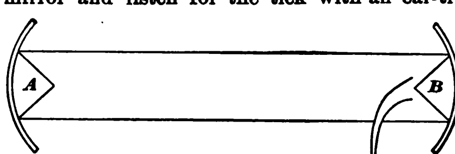

FIG. 255.

* This is true only when the distance is so large in comparison with the dimensions of the sounding body that the latter may be considered a *centre* from which the sound-waves proceed along radii. A person ten feet distant from the nearest point of a moving railroad train does not hear four times as loud a sound as one twenty feet distant from the train.

LXXXVIII) the mouth of which is held at *B*, the focus of the other mirror. See Fig. 255.

The usefulness of the ear-trumpet and of the speaking-trumpet is due to reflection of sound, one concentrating the sound as it approaches the ear, the other preventing it from spreading unnecessarily as it leaves the mouth.

A reflected sound is frequently called an *echo*. An echo occurs where a sound traveling through one medium comes abruptly against another medium of different density. Echoes have much to do with the ease or difficulty which a speaker finds in making himself heard in a large hall. If the echo is nearly simultaneous with the direct sound, it reinforces the latter and may be a great benefit. If it is too much prolonged, it causes much trouble. There is less echo from soft fabrics, like cushions or clothing, than from bare walls or benches. Hence, in a large hall, it is usually much easier to make one's self heard by a large audience than by a small and scattered one. The introduction of a gallery into an audience-room sometimes reduces the echo very much.

The long jarring rumble of a thunder-peal is maintained by echoes from strata of air, from clouds, or from the earth.

362. Refraction of Sound.—Sounds which *enter* one medium from another, or go from one place to another of different density in the same medium, suffer *refraction* somewhat as light-waves do. This sometimes occurs on a large scale in the open atmosphere, producing curious effects which are more or less troublesome in fog-signals.

363. Musical and Unmusical Sounds.—All observe that some sounds have a musical quality, while others are merely noise.

EXPERIMENT I.

Upon the whirling apparatus fasten a disk (No. LXXXIX) of metal or wood about 15 cm. in diameter carrying 2 circular rows of pegs,

each row being concentric with the disk. In one row the pegs are placed at equal intervals of about 1 cm. In the other row there is an equal number of pegs separated by unequal intervals.

Revolve the disk at a uniform rate several times a second, and hold the edge of a card lightly against the irregular row of pegs (see Fig. 256). Then hold the card against the other row of pegs. Is there any marked difference in the quality of the sound produced in these two cases?

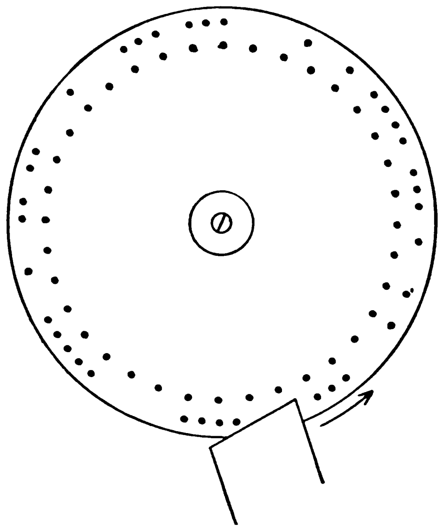

FIG. 256.

Although none of the effects obtained in this experiment are above criticism from a musical standpoint, they illustrate roughly the fact that air-pulses striking the ear at regular intervals, and with a frequency greater than a certain limit, produce the sensation of continuous musical sound. All musical instruments are capable of giving out sound-waves of regular intervals. The vibrating spring in the following experiment illustrates the action of a tuning-fork.

EXPERIMENT 2.

Take a long straight piece of clock-spring and fasten it in a vise or clamp, leaving about 40 or 50 cm. projecting horizontally. Set this part vibrating and note the regularity of its swings, observing that, like the beats of a pendulum, they take about the same length of time, whatever the length, or width, of the swing.

Shorten the vibrating part and observe the effect upon the quickness of the swing. Shorten it to 2 or 3 cm., and observe that now it gives out a good musical note.

A long piece of rubber tubing, fastened at the ends and stretched, illustrating the action of the strings in a piano or violin, would be found to vibrate regularly with a quickness depending upon the degree of tension.

The vibrations of musical strings or wires will be considered at greater length farther on.

364. Determination of the Length of a Sound-wave.—Since, therefore, a musical sound consists of a series of waves following each other at regular intervals of time, we may call the distance between similar portions of two consecutive waves the *wave-length* of that sound. We may take the distance from the point of greatest condensation in one wave to that of the point of greatest condensation in the next wave, or we may measure from rarefaction to rarefaction. The distance will be the same in both cases.

Suppose any kind of tuning-fork, for instance a common "A" fork of 440 complete* vibrations per second, to be set vibrating, and to remain vibrating for one second. In that time we may suppose that, with the ordinary temperature, the sound will have traveled about 340 m. Then a sphere of 340 m. radius will have been filled with alternate shells, or layers, of condensed and rarefied air. Each com-

* A complete, or double, vibration is a movement in which the vibrating body goes over its whole path twice, once from left to right and once from right to left, that is, performs a *round trip*. The word *vibration* used alone will, in this book, mean such a complete vibration.

plete vibration of the fork produced two layers, or one complete sound-wave. So the length of the sound-wave in this case is about $\frac{34,000}{440}$, or 77.3, cm.

Exercise 48 shows a simple and interesting method of measuring a wave-length, or known fraction of a wave-length, of sound in a tube. The principle of the Exercise may be discussed with the aid of Fig. 257, which represents a glass jar containing a column of air of such length as to reinforce the sound of the tuning-fork A.

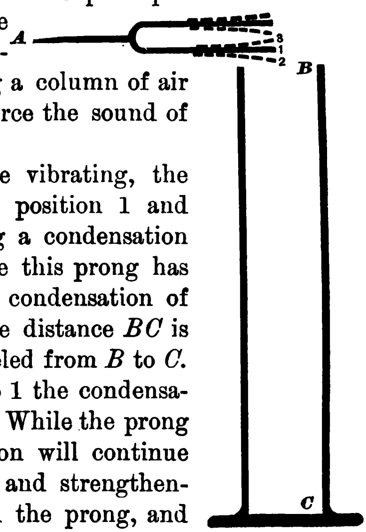

FIG. 257.

Suppose the fork to be vibrating, the lower prong being at the position 1 and moving downward, sending a condensation into the jar. By the time this prong has reached the position 2 the condensation of the sound-wave will, if the distance BC is properly chosen, have traveled from B to C. While the prong returns to 1 the condensation will travel back to 1. While the prong moves to 3 the condensation will continue to ascend, combining with and strengthening that sent directly from the prong, and so on.

The fact that BC is traversed by the wave while the fork is making one quarter of a vibration, that is, swinging from 1 to 2, shows that the distance BC is one quarter of a wave-length. It is found by experiment that the width of the tube makes some difference in the effective length of the air-column, so that in calculation one quarter of the diameter should be added to the length of the cylinder.

If the jar were deep enough, the condensation might travel to the bottom and back just in time to coincide with

the impulse sent out by the *second* up-stroke of the prong. The depth of the jar in this case would be about three quarters of a wave-length. The same correction for width of jar would have to be made in this case as in the preceding one; but if both depths were determined by experiment, one being one quarter of a wave-length minus a certain quantity, and the other three quarters of a wave-length minus the same quantity, the difference between the two depths would be just one half a wave-length. This method is followed in Exercise 48.

EXERCISE 48.

WAVE-LENGTH OF SOUND.

Apparatus: Articles 94 and 95.

Push the piston-head along the tube until it is not more than 20 cm. from one end. Strike one prong of the tuning-fork upon a piece of soft wood and then hold it at this end of the tube. While the fork is sounding draw the piston-head gradually away from the fork, listening for the rise and subsequent fall of the sound, which occurs when the air-column attains and then exceeds the proper length. Find in this way, and mark, one effective position of the piston-face.

Find in like manner a second effective position of the piston-face, which will give a mucn longer air-column than before.

The distance between the two positions of the piston will be equal to one half of a wave-length.

Varieties of Musical Sounds.

Musical sounds differ among themselves in *loudness*, *pitch*, and *quality*.

365. Loudness: Resonance.—At the start the loudness of a musical sound depends upon the width, or *amplitude*, of the vibrations of the body giving out the sound-waves, but the original loudness may be greatly reinforced by the vibrations of some other body, set into action by the sound

itself. Increase of loudness produced in this way is called *resonance.*

In order that resonance may occur it is necessary that the natural time of vibration of the assisting body be the same, or nearly the same, as that of the original sounding body. The second body is gradually set into action by successive impulses from the first, each impulse coming just at the right time to add to the previous effect. If the two bodies did not naturally vibrate in unison, one impulse might destroy the effect of the previous one, so that the second body would be affected but little. Vibrations of the second body produced in this way are called *sympathetic* vibrations (see § 369).

In Exercise 48 the reinforcing body was a column of air; but frequently solid objects serve a like purpose, as the following experiment shows.

EXPERIMENT I.

Take several unmounted tuning-forks of apparent pitch. Put each in turn into vibration and then press the end of the handle against the table-top, a door, a wooden box, etc., listening for resonant effects.

The fact that the same solid body may respond to several different notes is explained later, in § 372.

In the case of a tuning-fork mounted upon a sounding-box, as in Fig. 258, the vibration is communicated through the wood to the air-column within.

Musical instruments making use of vibrating strings or wires, like the piano and violin, are peculiarly dependent upon the resonance of the supporting wood.

The transference of a given note from one sounding body through the air to another body of the same rate of vibration is strikingly shown by the following experiment.

EXPERIMENT 2.

Place two tuning-forks of the same pitch, each mounted upon a resonance-box, some feet apart on a table, the mouths of the boxes being turned toward each other. Set one fork into vigorous vibration by means of a bow, and, after some seconds, stop this fork and notice whether the sound is now given out by the other one.

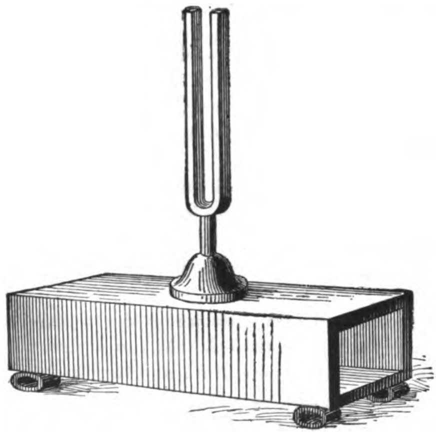

FIG. 258.

To make this experiment a complete success it may be necessary to attune the forks. This can be done, if their difference is not great, by loading equally with wax both prongs of the higher fork.

366. Pitch.—The next characteristic of musical sounds to be considered is *pitch.*

It is easy to show, by turning with varying speed the rotating disk used in § 363, that the degree of *sharpness* changes as the vibrations producing the sound become more or less frequent. A like fact was noticed in the experiment with the vibrating clock-spring. An increase of sharpness in a musical sound is called a rise of *pitch.*

It is evident that an increase of frequency in the vibrations raises the pitch of a sound. We may naturally inquire how many vibrations per second are needed to yield a tone

of given pitch, for instance some familiar note of a musical instrument.* This question is taken up in the next Exercise.

EXERCISE 49.

NUMBER OF VIBRATIONS PER SECOND OF A TUNING-FORK.

Apparatus: Nos. 95 and 96.

The method of this Exercise is to draw the smoked glass along in a straight line beneath the style of the vibrating tuning-fork, thus tracing on the glass a sinuous curve, and simultaneously to cut this curve crosswise by means of traces made by the style of the pendulum.

Arrange the pendulum and fork so that the horizontal part of the pendulum motion shall be parallel to the motion of the fork-prongs, and so that the two styles may, when at rest, be in the same vertical plane, with their points as near each other as practicable (see Fig. 259).

Place the piece of smoked glass under the tuning-fork and pendulum, and make such adjustments that the styles will bear lightly upon the glass. Set both pendulum and tuning-fork into vibration, and then draw the glass quickly along beneath them in a direction at right angles to the vibrations.

Count the number of vibrations of the pendulum per minute, and find from the smoked plate the number of vibrations of the tuning-fork corresponding to one vibration of the pendulum. Then calcu-

* The pitch of musical instruments has changed very greatly in Europe from time to time and from place to place. A certain note, "A," has ranged all the way from 375 vibrations per second to 567. In Germany during the seventeenth and eighteenth centuries the pitch of this particular note was about 425, but it has now risen considerably. The following quotations are from the History of Musical Pitch given by Ellis in the appendix to his Translation of Helmholtz's *Sensations of Tone:* "As this [preceding this century] was the period of the great musical masters, and as their music is still sung, and sung frequently, it is a great pity that the pitch should have been raised, and that Handel, Haydn, Mozart, Beethoven, and Weber, for example, should be sung at a pitch more than a semitone higher than they intended. The high pitch strains the voices," etc. . . . "The rise of pitch began at the great Congress of Vienna, 1814, when the Emperor of Russia presented new and sharper wind instruments to an Austrian regiment of which he was colonel. The band of this regiment became noted for the brilliancy of its tones."

late the number of complete double vibrations of the tuning-fork per second.

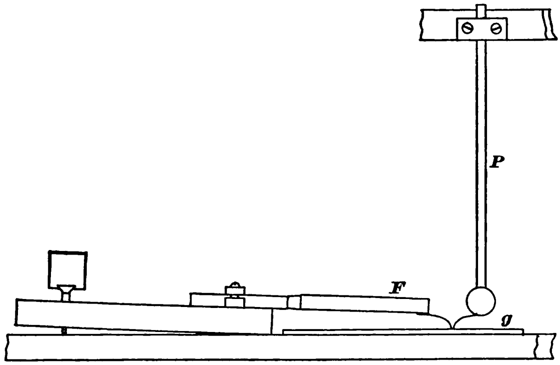

FIG. 259.

367. Combinations of Sounds Differing Slightly in Pitch: Beats.—When one is within hearing of two sounding bodies differing a little in pitch, it will happen at certain instants that the condensations of the two sets of sound-waves, and also the rarefactions, coincide at the ear, giving an effect which is the sum of the two effects produced by the bodies acting singly. At other instants the condensations of one set of waves will coincide with the rarefactions of the other set, giving an effect which is the difference of the two single effects. The result is a succession of changes in the combined sound, from strong to weak and back again. These fluctuations are called *beats.*

EXPERIMENT.

Take two tuning-forks of the same pitch, mounted on sounding-boxes, and set both into vibration by means of a bass-viol bow drawn across the ends of the prongs. If the two forks are really of the same pitch, the two sounds will appear to be one, which will gradually die away.

Load both prongs of one fork, near the end, with a small quantity

of wax, thus diminishing its rapidity of vibration, and then sound the two together, listening for the beats.

Nearly every musical instrument gives out sounds of different pitch at the same time, and these different sounds are sometimes related to each other in such a way as to cause beats. Gongs and bells nearly always give perceptible beats when sounding undisturbed.

If sound-waves are represented, as they may be for convenience, by a diagram resembling water-waves, the nature of beats is well expressed by Fig. 260, where the regions of

FIG. 260.

high crests and deep troughs stand for phases when the two waves reinforce one another, and the more level intermediate portions for interference-phases.

368. Harmony and Discord.—When two or more musical sounds are simultaneously produced, their effect upon the ear may or may not be agreeable. It is certain to be agreeable if there are only two tones, whose vibration-numbers have a very simple ratio. For instance, if the two tones are those produced by a "violin-A" fork, of 440 vibrations per second, and another fork of 220 vibrations per second, so that the ratio is 2 : 1, the interval between the two tones is what is called an *octave* and the effect is pleasing. Such a pleasing effect is called *harmony*.

But if the fork of 440 vibrations were sounded at the same time with another of 420 or 460 vibrations, for example, beats would be produced, and though these beats would be too rapid to be counted, or even recognized as such, by the ear, the result would be unpleasant. It would be called *discord*.

Tones an octave apart are produced by any two sonorous bodies in vibration when their vibration-numbers have to

each other the ratio 2 : 1. An octave is the simplest possible musical *interval*, and this is subdivided, in what is called the *natural scale*, or *gamut*, into seven smaller intervals. The vibration-numbers of the whole series of tones, comprising a given tone, its octave above, and the intermediate tones of the natural scale, bear to each other relations indicated by the following representative numbers *:

do		re		mi		fa		sol		la		si		do
C	:	D	:	E	:	F	:	G	:	A	:	B	:	C
24	:	27	:	30	:	32	:	36	:	40	:	45	:	48
1	:	$\frac{9}{8}$	:	$\frac{5}{4}$	:	$\frac{4}{3}$	:	$\frac{3}{2}$	:	$\frac{5}{3}$	:	$\frac{15}{8}$	:	2

The lower line of numbers shows what the vibration-frequencies of the various notes would be if the vibration-frequency of the lowest note were called 1. Thus, the number of vibrations per second in *sol* is $\frac{3}{2}$ times as great as that in *do*.

Any person trained in music will see that the notes which go best with *do*, giving the best harmony, are those for which the vibration-ratio, with reference to *do*, can be expressed by the use of small whole numbers. The simplest ratio, from this point of view, is that of the lower and the higher *do*, which go particularly well together. The next simplest ratio is $\frac{3}{2}$, that of *do* and *sol*, which notes also go very well together. Especially bad ratios are the $\frac{9}{8}$ and $\frac{15}{8}$.

The connection between simple ratios and harmony has been known for a long time. People used to try to explain it by saying that the human mind or soul delighted in simple ratios as such, but Helmholtz pointed out that many persons have excellent perception of musical harmony who never even heard of vibration-numbers. He explained *discord* by means of *beats*.

* The note *re* is called the *second* of the *do* below it; *mi* is called the *third* of the same *do*, *fa* the *fourth*, etc.

He found that beats are especially unpleasant when they occur about thirty times a second.

Keeping the above-given numerical relations in mind and knowing the vibration-number of any particular note of the scale, we can calculate the number of vibrations that will be required to produce any given tone. Suppose we are asked to find the number of vibrations per second that will produce the tone G next preceding that of the "violin-A" fork of 440 vibrations. Then we have the proportion $36 : 40 :: x : 440$, from which $x = 396$ vibrations.

369. Quality.—Aside from differences in the loudness of sounds and in their pitch, there is a well-known difference in *quality*. It is this, in part, which enables one to distinguish the voices of acquaintances in the dark, or to pick out a familiar voice singing among many others. Still more marked are the differences between the sounds of certain musical instruments, the violin and the flute for example, even when both are producing sounds of the same pitch. The fact is that a musical sound is usually a combination of several notes differing in pitch, and what is called the pitch of the sound is merely the pitch of its loudest component.

Helmholtz invented a system of *resonators*, consisting usually of hollow brass bulbs or cylinders with an opening at one or at each end, each responding, by means of the air-column within it, to some particular note when any sound containing that note is produced near by. By the help of such instruments he analyzed the sounds of the human voice and explained, for example, the difference between the different vowel-sounds when sung by the same voice at the same pitch. His explanation can be understood better after certain experiments on vibrating strings, etc. (§§ 370 and 373), have been given, but the student in advance of all further discussion will do well to try the

following experiment, which is described in the great work of Helmholtz on *Sensations of Tone.**

EXPERIMENT.

Lift the top of a piano so as to expose the strings to view, push down the loud pedal so as to leave the strings free to vibrate and give out their full volume of sound, then sing *into* the piano a prolonged *ä*, giving the sound as in *ah*. Keep the pedal down after the voice ceases, and listen to the response given by the strings. Then try the sound of *ō*, as in *note*, in the same way.

The piano returns the sound in each case with almost startling distinctness. With most of the other vowel sounds the effect is much less striking.

370. Fundamental Tone, Overtones, and Harmonies.—It has already been observed (§ 365) that a single body may give out several different notes. It is now time to study the behavior of vibrating bodies more closely. We may well begin with the transverse, sidewise, vibrations of strings, or wires.

EXPERIMENT I.

Stretch one of the sonometer-wires (No. XCIV) pretty tightly, until it gives a clear musical sound on being stroked at the middle with a bass-viol bow. While it is still sounding, drop a little ʌ-shaped rider, made of stiff paper, on the wire near the middle. Note the behavior of the rider.

Sound the wire again, and look at it in a strong light, while it is sounding. Repeat the experiment with the rider, dropping it upon various portions of the sounding-wire, until the trials have been carried quite to one end of the wire.

Then, holding the finger-tip lightly against the exact middle of the wire, stroke the latter midway between the point where the finger is applied and either end. Then quickly remove the finger and put the rider in its place; then put it on some other portion of the wire.

Once more place the finger-tip on the wire, twice as far from one end as from the other, sound it by stroking it half-way between the finger and the nearer end, remove the finger and put the rider in its place; then put the rider on another portion of the wire.

* Longmans & Co.

Measure off the wire accurately into four equal parts, as in Fig. 261. Place a colored paper rider at point *2*, and another at point *3*. Place a white paper rider at each of the points *a*, *b*, and *c*. Hold the

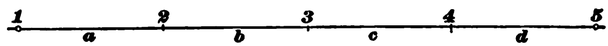

Fig. 261.

wire lightly at *4*, and stroke it at *d*. Observe which riders are thrown off and which are not.

These experiments will doubtless show that the vibrating wire has regions of greatest vibration and points of little or no vibration. The regions of greatest vibration are called

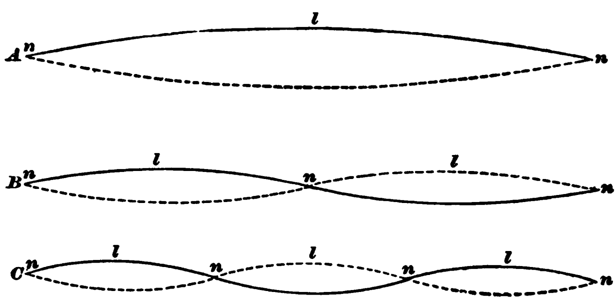

Fig. 262.

loops; points of no vibration, *nodes.* In Fig. 262 the loops and nodes of some vibrating wires are shown. The centre of each loop is marked *l*, and each node is marked *n*.

EXPERIMENT 2.

Pluck one of the sonometer-wires strongly at the centre, then at various intervals toward either end, until finally the end is almost reached. Note the clanging tones of high pitch that are produced as the end of the wire is approached, and observe how these quickly die away, leaving the lowest tone of the wire still sounding.

This lowest note is called the *fundamental note*, or simply the *fundamental*, of the wire. It is given by the form of vibration shown in Fig. 262, *A*. The higher tones produced by plucking the wire toward the end, in the way just described, are called *overtones*.

There are with some instruments two kinds of overtones: those which unite agreeably to the ear with the fundamental, and those which do not. The former are called *harmonic overtones*, or *harmonics*. The overtones of freely vibrating wires are harmonics.

371. Laws of Vibrating Strings.—We can hardly undertake in this course of study to find by experiment the *laws of vibration* of strings or wires. They may be stated as follows: The number of vibrations per second is—

(1) *inversely proportional to the length of the string;*

(2) *inversely proportional to the square root of weight of the string per unit length;*

(3) *directly proportional to the square root of the stretching force.*

The application of the first of these laws to cases like those exhibited in Fig. 262 shows that the fundamental and the overtones of a string have vibration-frequencies bearing to each other the ratios of the numbers 1, 2, 3, 4, etc.

The fact has already been stated that the vibration-frequencies of any two notes that unite pleasantly to the ear bear to each other some ratio that can be expressed by small whole numbers.

372. Chladni's Figures.—Strings are by no means the only objects that can vibrate in parts. Thin plates of metal or wood may give out a great variety of sounds, corresponding to different modes of vibration. If this were not so, such instruments as the violin and piano would be practically useless.

Plates of glass or of metal may be fastened at the middle by a clamp, as in Fig. 263, or in any other convenient way, and set vibrating by a violin-bow. If sand is sprinkled on the surfaces of such plates while they are vibrating, it will arrange itself in lines which mark the position of the nodes. The figures formed by the sand are known as *Chladni's figures*, in honor of the investigator who first studied and described them.

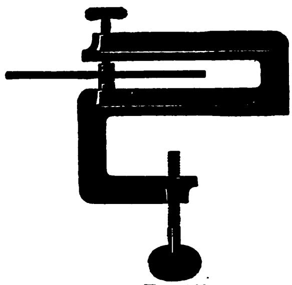

FIG. 263.

EXPERIMENT.

Sprinkle fine sand and lycopodium powder over a square Chladni plate, and then, using a violin or bass-viol bow, produce as many as practicable of the sand-figures shown in Fig. 264. In each case the

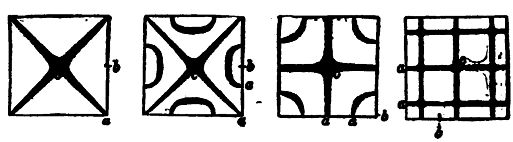

FIG. 264.

letter *b* shows where the bow is applied to the edge of the plate; *a* indicates the position of the finger held in contact with the plate, to produce nodes; and *c* is the position of the clamp.

Observe the behavior of the lycopodium while the plate is vibrating, and the positions where it finally settles.

The *nodal lines*, marked by the sand-figures, are those parts of the plate which have little or no up-and-down motion while the plate is vibrating. The sand-particles are apparently tossed toward these lines by the blows they receive from the quivering parts of the plate.

The very light lycopodium is, however, not easily tossed through the air. It is controlled largely by the movements

of the air above the plate, and these movements are such as to keep the powder at the points of greatest agitation of the surface. In a vacuum the lycopodium acts like the sand.

373. Vibrations in Organ-pipes.—A column of air in a pipe does not necessarily vibrate as a whole. This fact accounts for the variety of pitch obtainable from a fife, flute, or other similar wind-instrument.

In every pipe giving out a musical sound there is at least one cross-section where the air is alternately compressed and rarefied without movement *through* the section. This is called a *node*.

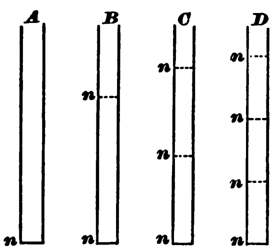

FIG. 265.

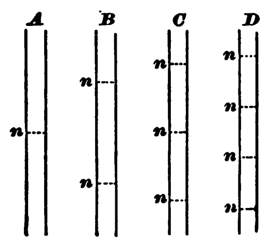

FIG. 266.

A In Fig. 265 indicates the condition of a pipe, closed at one end, giving out its lowest note. There is only one node, and this is at the closed end. The wave-length (Exercise 48) of the sound is four times the length of the pipe.

B of the same figure shows the nodes in the same pipe giving its next higher note. The wave-length of this note is four-thirds times the length of the pipe. In *C*, the next case, the wave-length is four-fifths of the length of the pipe. In *D*, the next case, it is four-sevenths, etc.

All this shows that in a closed pipe the vibration-numbers of the fundamental tone and the overtones (§ 370) are to each other as the numbers 1, 3, 5, 7, etc.

Fig. 266 shows the nodes in an open pipe giving out (*A*) its fundamental tone, (*B*) its first overtone, (*C*) its second overtone, etc.

The wave-length of the fundamental tone is equal to twice the length of the pipe, that of the first overtone is the length of the pipe, that of the second overtone is two-thirds the pipe-length, etc. The corresponding vibration-numbers are to each other as the numbers 1, 2, 3, 4, etc.

If there are two, and only two, nodes in a pipe, there is condensation at one when there is rarefaction at the other. If there are three, and only three, there is condensation at the middle node when there is rarefaction at the other two, etc.

Midway between the nodes the air moves alternately up and down a very slight distance with very little, if any, change of density.

If a hole in the side of a vibrating pipe is opened half-way between two nodes, the sound is not changed. If a hole is opened at one of the nodes, condensation and rarefaction can no longer occur there; the place ceases to be a node, and the note given by the pipe suddenly changes pitch.

Many wind-instruments have side openings, which are closed at will by the player. Another device for changing the pitch of the note from such instruments is to blow with greater or less force at the mouthpiece.

These facts may be illustrated by use of an organ-pipe with properly placed openings and stops (No. XCVI).

374. Quality of Sound due to Overtones.—The quality of any musical sound is due to the number, kind, and relative strength of the overtones which, together with the fundamental, constitute the sound. This has been proved both by analyzing sounds such as those of the human voice, of organ-pipes, etc., into their component tones, and by

uniting many separate simple sounds to form a compound sound. Any sound which is caused almost entirely by vibrations of a single rate, that is, sound which is nearly destitute of harmonics, is comparatively thin and lacking in mellowness and richness.

EXPERIMENT.

Sound a tuning-fork, for example the "middle-C," or the "violin-C," fork, alternately with the sonometer-wire tuned to the same pitch. The wire should be plucked about one-eighth or one-ninth of its total length from one end, in order to give a full series of harmonics. Compare the rich sound of the wire with the thin sound of the fork.

QUESTIONS AND PROBLEMS.

(Assume a temperature of 0° C. in the following problems if nothing is said to the contrary.)

(1) How long after the flash of a gun is seen would its report be heard if the temperature were 0° and the distance of the observer from the gun were 1.5 kilom.? Light will traverse this distance almost instantaneously.

(2) Find the temperature at which sound will travel in air 3400 meters in 10 seconds, the velocity at 0° C. being 332 m. per second? See § 355.

(3) How much louder is a sound at a point 40 ft. from its origin than it is at a point 100 ft. distant?

(4) On a day when the temperature is 20° C. a man sets his watch by the striking of a clock 2000 meters away. Determine to the $\frac{1}{10}$ of a second the error due to distance.

(5) A cannon 2 miles distant from an observer is fired and he sees the flash. The wind is blowing 10 miles an hour from the man to the cannon. How many seconds after the flash does the sound of the report reach him, the temperature of the air being 30° C.?

(6) In order to ascertain the distance of a cliff a gun was fired and the time taken until the echo was heard. The time was found to be 50 seconds. The thermometer stood at 10° C. Determine the distance of the cliff.

(7) Describe carefully any method which you have used for measuring the velocity of sound.

(8) Do sounds of high or low pitch travel faster? How does the sound of distant music illustrate your answer?

(9) What will be the length of the resonant air-column that will respond most loudly to a fork of 220 vibrations per second? Neglect the influence of the diameter of the tube.

(10) If the velocity of sound in hydrogen is 4 times what it is in air, what must be the length of a tube filled with hydrogen which will respond to a fork making 480 vibrations per second?

(11) An open pipe l ft. long is found to respond to the note C. How much must be cut off to cause it to respond to the note G?

(12) If the pipe in the previous example had been closed, how much must have been cut off?

(13) A person pumping water from a well sometimes detects among the various sounds that attend the operation one that changes in pitch as the water rises in the pipe. How can this be accounted for?

(14) If a tube 4 cm. in diameter and 0.5 m. long responds most loudly to a certain fork, what is the wave-length for the tone of that fork?

(15) Why does the sound of a circular saw, cutting through a board, grow lower in pitch as the saw enters the board?

(16) Just how could the sonometer-wire be set vibrating in four loops? in five loops? How many nodes would there be in each case?

(17) In a sonometer-wire 1 m. long, find the lengths of wire to give the first five harmonics of the fundamental tone of the whole wire.

(18) Why are violin-strings bowed, and piano-wires struck, near an end rather than at the middle?

(19) If a certain tone is sung loudly over the sounding-board of a piano, what wires will respond? If there is any difference in the loudness of the responses, what will be the order as regards loudness?

(20) A certain musical note is caused by 256 vibrations per second. How many vibrations will be necessary to produce its *fourth* (§ 368, foot-note)? its *fifth?* its *octave?*

CHAPTER XXIX.

MAGNETISM.

375. Magnets, Natural and Manufactured.—There is a certain iron ore of which lumps are occasionally found having the power of attracting particles of iron. Such lumps of ore are called *load*-stones, or *lode*-stones, a word equivalent to *leading*-stones. They are called also natural magnets.

A piece of iron acted upon by a loadstone becomes in turn a magnet, and will attract particles of iron as the loadstone does. Magnets can be made in other and better ways, and are familiar objects to most students. Cobalt and nickel have strong magnetic properties, but are inferior to iron in this respect, and magnets made of them are curiosities rather than articles for real use. Hard steel magnets retain their power better than those made of common iron, and therefore most magnets are made of steel *tempered* very hard.

376. Induced Magnetization; Temporary and Permanent Magnetization.

Magnetization produced in a piece of metal by the action of magnetic force is called *induced* magnetization. A part of such magnetization usually disappears when the producing force is withdrawn. This part is called *temporary* magnetization. The part that remains is called *permanent* magnetization.

EXPERIMENT.

Take several small pieces of very soft wrought iron, horseshoe-nails for instance, and test them among themselves to see whether they will exhibit any magnetic power. Then lift one of them by means of a strong bar-magnet and apply one end of a second nail to the lower end of the one so lifted. If the second nail remains suspended by magnetic action, attach a third to it, and so on until the chain so formed breaks. Finally, after removing all the nails from the neighborhood of the magnet, test them again among themselves for evidences of magnetization.

377. Magnetic Needle: Magnetic Compass.—A slender magnet suspended by a flexible fibre or balanced upon a sharp point, so as to be free to turn in a horizontal plane, is called a magnetic *needle*.

It has been known to Europeans for about seven centuries, and possibly to the Chinese for some thousands of years, that a magnetic needle in coming to rest after any disturbance always tends to a position in which its length will be in a general north and south direction. At some parts of the earth's surface the needle points somewhat to the west of north, in others somewhat to the east of north; and its exact direction of pointing at any one place on the earth's surface varies from century to century.

EXPERIMENT.

By means of an observation of the North Star, or any other convenient method, lay off upon a table in the laboratory a true north and south line.* Then note how many degrees to the west or east of this line the north end of the magnetic needle points.

An instrument in which a magnetic needle is placed within or over a graduated horizontal circle, so that the

* For methods of doing this see books on Surveying, e.g., Johnson's *Theory and Practice of Surveying*, John Wiley & Sons, New York.

turning of the needle from its normal position may be at once read off in convenient divisions of the circle, is called a *magnetic compass*. The *mariner's compass*, by which the sailor steers his course in the open ocean, is such an instrument, having usually several small magnets fastened to a cardboard support balanced upon a point.

378. Magnetic "Dip"; the Earth's Magnetic Poles.—If a symmetrical magnetic needle is suspended by its middle so as to be free to turn in a vertical plane as well as in a horizontal plane, one end of the needle will, at most parts of the earth's surface, hang lower than the other end. (See Fig. 267.) At most places in the Northern Hemisphere the north-seeking end of the needle will hang low, at most places in the Southern Hemisphere the south-seeking end. In either case the needle is said to *incline*, or *dip*.

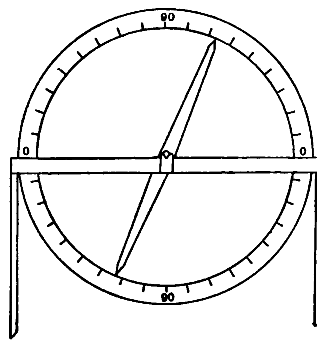

FIG. 267.

In general, as one goes north the north-seeking end of the needle will dip more and more, and at some place in the far north will point straight down. This place, which is called the north *magnetic pole* of the earth, is not at the geographic north pole. Somewhere in the far south is a corresponding south magnetic pole, where the south-seeking end of the magnet would point straight downward.

EXPERIMENT.

Hold a bar of very soft iron, *I* (Fig. 268), about 50 cm. long, in a vertical position, and move the upper end of it slowly toward the side of a delicately pivoted magnetic needle, *M*, noting the direction in which the needle is thereby deflected.

Then move the lower end of the bar toward the side of the needle, and note the direction of the effect.

Invert the bar, and repeat.

The process of magnetization can be helped on by tapping the bar smartly with a heavy stick while it is in the vertical position.

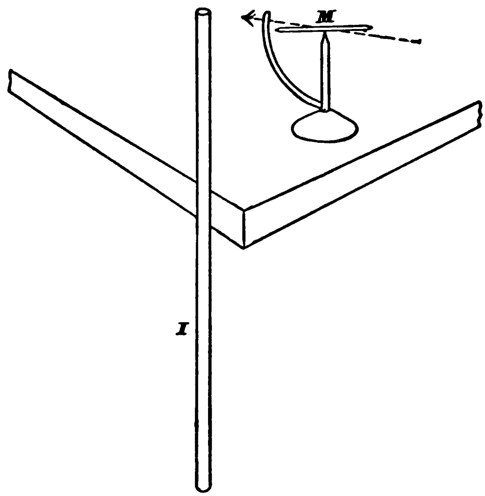

FIG. 268.

To show this, hold one end of the vertical bar at such a distance from the needle as to produce a rather small deflection before the bar is struck, and then note the increase of deflection produced by the striking.*

This experiment shows that the earth has the power of magnetizing iron. The earth is, in fact, a magnet, and the "natural" magnetic power with which the loadstone is found endowed has, no doubt, been imparted to it by the magnetic action of the earth as a whole.

What makes the earth magnetic is not certainly known,

* When a magnet is under the influence of forces tending to *demagnetize* it, jarring promotes *demagnetization*.

although various theories to account for its condition have been proposed.

379. Poles of Ordinary Magnets; Magnetic Repulsion.—If one takes an ordinary bar-magnet and presents different parts of it in turn to one end of a magnetic needle, it will be found that the ends have much more effect than the middle. As in the case of the earth we speak of two magnetic *poles* from which the magnetic influences appear to come, so in the case of the bar-magnet we call the two regions, usually near the ends, in which the peculiar power of the magnet seems to lie, the *poles* of the magnet.

If we present the two ends of a bar-magnet in turn to the magnetic needle, we shall find that one end attracts the north-seeking point of the needle and repels the south-seeking point, while the other end of the magnet repels the north-seeking point of the needle and attracts the south-seeking point.

If we now float the magnet on water, on a board just large enough to carry it safely, or suspend it properly, we shall find that the pole which repelled the north-seeking pole of the needle will itself point north. We conclude, then, that *poles which repel each other are alike*, and that *poles which attract each other are unlike.*

380. Law of Inverse Square.—By means of careful and delicate experiments upon the attractions and repulsions exerted by the poles of magnets, it has been found that these attractions and repulsions are, other things being equal, *inversely proportional to the square of the distance between the mutually acting parts.*

For example, if two poles one foot apart attract each other with a certain force F, the same poles when two feet apart will attract each other with a force $F \times \frac{1}{2 \times 2} = \frac{1}{4}F.$

Every magnetic pole that we have to do with is subject to an attraction exerted by one of the earth's poles and a repulsion exerted by the other. Both the attraction and the repulsion tend to carry a north-seeking pole north and a south-seeking pole south. Any movement toward the earth's north magnetic pole strengthens the effect of that pole, but weakens the effect of the south pole. If the movement is not very great these two opposite effects will nearly neutralize each other.

PROBLEM.

If two unlike but equally powerful magnetic poles are 200 ft. apart, and if each of the two exerts upon a third pole midway between them a force F, how much would the total force felt by this third pole be increased if it were moved 1 ft. toward the attracting pole?

Solution: **The present total force urging the middle pole toward the attracting pole and away from the repelling pole is $2F$. The attracting force, F_a', after movement of pole to the new position, can be found by means of the proportion**

$$F_a' : F :: \frac{1}{99^2} : \frac{1}{100^2}, \qquad \text{whence} \qquad F_a' = \frac{100^2}{99^2} F.$$

The repelling force after the movement is $F_r' = \dfrac{100^2}{101^2} F$.

The sum of F_a' and F_r' is about $2.0006F$, so that the increase is about $.0006F$.

It is evident that, if the attracting and repelling poles, instead of being 100 ft. distant from the third pole, were some thousands of miles from it, as in the case of the earth's poles, any small movement toward the north or south would make no appreciable difference in the total force exerted upon the third pole.

381. Opposite Kinds of Magnetism.—The property by virtue of which a magnet attracts or repels is called *magnetism.* To give it a name does not *explain* the property, but the name is a convenience.

As there are two kinds of magnetic poles, so we must

recognize two kinds of magnetism. The north-seeking pole of a magnet is that part in which the north-seeking magnetism, frequently called *north* magnetism, is more abundant than the south-seeking magnetism, and prevails over it. The south-seeking pole is the part in which the south-seeking magnetism, frequently called *south* magnetism, prevails over the north-seeking magnetism.

Does a magnet have equal amounts of the two kinds of magnetism? The following experiment will help to answer this question.

EXPERIMENT 1.

Returning to the floating magnet (§ 379), note whether it tends *as a whole* to *drift* either toward the north or toward the south.

Try this experiment with a variety of magnets, and note whether any of them move as a whole in either direction, taking care in all cases to have the magnet at rest when the experiment begins, and having the surface of water as large as practicable.

The float should be the lightest that will bear the magnet with security.

If, on experimenting carefully in this way, we find a magnet which always floats toward the north, we cannot attribute such behavior to the fact that one pole is nearer the north than the other, for we have seen that this would make no perceptible difference in the total horizontal force to which the pole is subjected. We shall have to explain it by the supposition that the particular magnet used has more north-seeking magnetism than south-seeking magnetism. But in fact no such magnets are known.

This is an interesting and rather surprising fact. One would naturally expect, after breaking at the middle a long magnet with a well-defined pole near each end, to find each half possessed of only one kind of magnetism and one pole, but this does not happen.

EXPERIMENT 2.

Take some long, thin piece of hard steel, an old metal saw-blade for instance, and magnetize it by means of the most powerful mag-

net at hand, stroking one end of the steel with one end of the magnet, and the other end with the other end of the magnet. It will now be found that one end of the bar will attract the north-seeking end of a magnetic needle, and the other end repel it.

Now break the bar in two at the middle. Test each half separately by means of the magnetic needle. Does each half have both north-seeking and south-seeking magnetism?

Break one of the halves in two, and repeat the test. Continue thus till the parts become too short to be readily broken.

All poles are not near the ends of magnets. A saw-blade that has been touched by one end of a strong magnet near the middle will have three poles, one at the middle and one at each end, the end poles being like each other and unlike the middle pole. The magnetism of the one middle pole will be equal to that of the two end poles.

382. Magnetic Field; Lines of Magnetic Force.—Any portion of space in which magnetic force is found is called a *magnetic field.*

If, starting at any point of a magnetic field, one notes the direction in which the north-seeking end of small magnetic needle points, and moves the needle bodily in this direction, and if one continues this process, changing the direction of motion, if need be, continually so as to make it at all times agree with the direction of pointing, the centre of the needle will trace out what is called a *line of magnetic force.*

EXERCISE 50.*

LINES OF FORCE NEAR A BAR MAGNET.

Apparatus: A bar magnet (No. 97). A small compass (No. 98). A sheet of paper about 50 cm. square.

Fasten the sheet of paper on a table, and lay the magnet on the middle of the sheet, the north-seeking pole, pointing north. Place

* Jarring magnets or touching like poles together may weaken the magnetism. When the magnets are not in use, it is well to keep them in a rack like that shown in Fig. 269, the north-seeking ends pointing *downward.*

the small compass at the extreme northeast corner of this magnet, and then move it away in the exact direction in which the compass-needle points. Continue this movement, changing direction in such a way as to follow continually the changing indication of the compass-needle, until the path reaches the edge of the paper, or returns to the magnet. Trace upon the paper the line thus followed by the middle of the compass, putting arrow-heads at several points to indi-

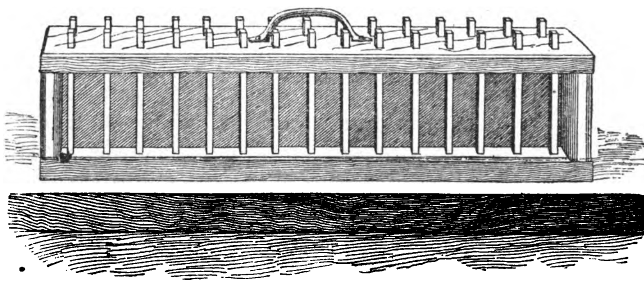

FIG. 269.

cate the direction in which the north-seeking end of the compass-needle points at these places.

Then place the compass a very little distance from the northeast corner of the large magnet toward the middle, and, starting anew, trace another line, and mark it as before.

Then beginning still farther toward the middle of the magnet, do as before.

Finally, start not more than 3 or 4 cm. from the middle of the magnet, and trace a line.

Trace an equal number of lines on the western side of the magnet.

Trace upon the paper the outline of the magnet as used, marking the position of its north-seeking end. Trace also an arrow showing the direction *north*.

The student should study the diagram obtained in the preceding Exercise, endeavoring to explain all its peculiar features in accordance with what he has learned about magnets and the earth's action upon them. He should note whether any one of the lines turns in such a way as to cross itself or lead back to its own starting-point.

Nails or screws in the table-top may affect the lines in places.

The following method shows in a striking way some of the general features of the lines of force around a magnet.

EXPERIMENT.

Place one of the bar magnets under a sheet of paper, and then slowly sprinkle iron-filings on the paper.

383. Theory of Magnetism.—Putting together what we have learned in the preceding sections, we can form some theory of the nature of magnetism. The fact that the lines of force of an ordinary bar magnet come from or return to its ends, the middle having little or no effect upon the needle, seems at first to show that the ends only are magnetized; but when we find that such a magnet if broken at the middle at once shows opposite poles at the break, and when we find that every magnet has equal quantities of the two kinds of magnetism, we are led to conclude that a magnet, instead of having all its magnetism confined to the poles, is magnetized with opposite poles in its smallest particles,—is made up, in fact, of particles, or molecules, each one of which is endowed with equal quantities of the two opposite kinds of magnetism.

If we conceive of a bar magnet as made up of such particles (see Fig. 270), with their *north* poles all pointing in the same general direction, we can see that in the middle of

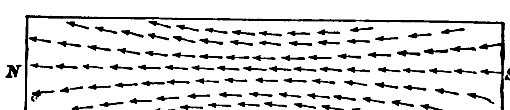

FIG. 270.

the bar the north pole of any particle is sure to be very near the south pole of some other, so that this part of the bar will have little or no action upon outside bodies. At

either end of the magnet, however, there is a surface made up of poles all of the same kind, and so from the ends comes the power which affects the needle.

Breaking the bar at the middle merely separates two surfaces, which, one being made up of north poles and the other of south poles, neutralize each other while in contact, but have each the power of attraction or repulsion when separated.

As to the magnetization of the molecules themselves, it seems likely that it is natural or permanent in them, so that the act of magnetizing a bar is merely a process of *arranging* the molecular magnets of which it is composed.

It is thought that this magnetism of the molecules may be due to electric currents *within* them, not passing from molecule to molecule. This theory cannot well be discussed until after the student has become somewhat familiar with the common properties of electric currents.

384. Strength of Magnetic Pole and of Magnetic Field. —The *unit magnetic pole* is defined as a pole that would act upon an equal magnetic pole at unit distance with unit force.

We will take the centimeter as the unit of distance and the *dyne* as the unit of force.

A magnetic field (§ 382) is said to be of *strength*, or *intensity*, h, if a unit magnetic pole placed in it requires a force of h units to prevent it from moving in the direction of the lines of force of the field.

PROBLEMS.

(1) How great is the repulsion of a unit pole upon a like unit pole at a distance of 5 cm.?

(2) How great is the attraction of a unit pole upon an unlike 5-unit pole at a distance of 10 cm.? How great is the attraction of the 5-unit pole upon the unit pole in this case?

(3) How great is the action of a 5-unit pole upon a 10-unit pole at a distance of 20 cm.?

(4) Four like 5-unit poles are placed at the corners of a square 10 cm. on a side, the diagonals of the square extending east and west and north and south. How great is the resultant repulsion of the east and west poles upon the north pole, and what is its direction? (Refer to the Parallelogram of Forces.)

(5) With the arrangement just described, how great is the resultant repulsion of the east, the west, and the south pole upon the north pole?

(6) If a magnet having two poles 20 cm. apart, each pole having a strength of 100 units, is suspended in a horizontal magnetic field of intensity 0.2, describe some combination of forces that will hold the magnet at right angles with the lines of the field.

(7) If the magnet in the preceding problem were allowed to point in its natural direction in the field, how great a force would be required to prevent it from moving, as a whole, in the direction of the lines of force of the field?

CHAPTER XXX.

ELECTRICITY.

385. Historical.—It was known to the ancient Greeks and Romans that certain substances acquire, when rubbed, the power of attracting light objects. Amber was one of these, and from the Greek name of this substance, ἤλεκτρον, the name *electricity* was formed by the English philosopher Gilbert, who was born in 1540 and died in 1603.

It appears that Gilbert was the first to make systematic and extensive observations of electrical phenomena. From his time the subject has grown in interest and importance through the invention of frictional electrical machines, of the Leyden jar, Franklin's discovery of the identity between lightning and electricity, the invention of the *galvanic*, or *voltaic*, *cell* (§ 402) about the beginning of this century, the work of Oersted, Ampère, Faraday, and a host of others more recent, until the name *electricity* is now the most popular and fascinating in the whole vocabulary of physical science.

386. Electrification by Friction. — Experiments with electricity produced by friction are very beautiful, and of great theoretical interest, but many of them are troublesome to perform, and their practical importance is comparatively small. Those which follow are selected mainly for the light which they throw on the phenomena of electric currents.

EXPERIMENTS.

(1) On a cold dry day rub a rod of gutta-percha or hard rubber with a catskin, and then present the rubbed part to small light pieces of paper or bits of thread lying on a table.

(2) Fasten two small pith-balls to the ends of a dry silk thread about 15 cm. long and suspend them by the middle of the thread from any convenient support.

Touch these balls with the freshly rubbed rod of gutta-percha. Note the behavior of the balls with respect to the rod just before they are touched and just after. Note also their behavior with respect to each other after they are touched by the rod.

If they act in an unusual manner, it is because they have become electrified, or "charged with electricity," by the rod.

(3) Rub a smooth glass rod vigorously with a piece of silk, and present the rubbed part to the suspended pith-balls still charged from the sealing-wax. Note their behavior before and after being touched by the glass.

What evidence do you find in these experiments that there are two kinds of electrification? Is there attraction or repulsion between bodies similarly electrified? between bodies oppositely electrified?

387. Nature and Kinds of Electricity.—There has been a difference of opinion as to whether electricity is or is not a *substance.* A century ago, when heat and light were believed to be weightless fluids, electricity was classed with them as a substance. Later, when it was shown that heat and light were not substances, but mere "modes of motion," in which the particles of matter are involved, the notion gained currency that electricity was a mode of motion, rather than a substance by itself. During recent years belief in the existence of electric substance, or substances, has been growing again.

It is shown by the experiments preceding that there are two kinds, or states, of *electrification*, and that bodies may be oppositely electrified, but this does not prove that there are two kinds of *electricity.*

It has been held by some physicists that there is only one kind of electricity. According to their theory all bodies in their normal, apparently unelectrified, state are endowed with a certain quantity of electricity, any addition to which produces one kind of electrification, and any subtraction from which produces what we call the opposite kind of electrification. This is the so-called *one-fluid theory.*

Other authorities have held that there are two kinds of electricity, and that in the normal state a body is endowed with equal quantities of the two kinds, which neutralize each other. According to this theory a state of electrification is produced by making either kind of electricity on the body exceed the other in amount, the nature of the electrification being this or that according as one kind or the other of the two electricities is in excess. This is called the *two-fluid theory.*

The nature of electricity, and the question whether there are two kinds, is still in debate, but in describing electrical phenomena there is great convenience in using the language of the two-fluid theory, and such language will be freely used in this book.

388. Positive, or Vitreous, and Negative, or Resinous. —We shall, following custom, speak of the state into which sealing-wax or any other resinous substance is brought by rubbing with catskin as being due to a charge of *resinous*, or *negative*, electricity, and the state of a glass rod rubbed with silk as due to a charge of *vitreous*, or *positive*, electricity.

It is customary to arrange a considerable number of articles in a list, and state that any one of them becomes electrified positively when rubbed with another farther down the list, and negatively when rubbed with one farther up the list.

If the lists given by different books are compared they

will probably be found not to agree. The fact is that the surface condition of a body of given material has much to do with its electrical behavior when rubbed. For example, as the following table* indicates, silk is negative with respect to smooth glass, but is sometimes positive with respect to roughened glass. The two sides of a glass plate, ground on one side and smooth on the other, may be used to show this fact.

1. Fur of cat.	4. Feathers.	7. Silk.
2. Polished glass.	5. Wood.	8. Shellac.
3. Woolen stuffs.	6. Paper.	9. Rough glass.

389. Conductors and Insulators.—If one attempts to repeat Experiments 2 and 3 of § 386, using a very thin wire or a cotton thread or a wet silk thread for suspending the pith-balls on a metal support, in metallic connection with the ground, it will be found that the balls do not retain their charge as they did before. The explanation is, that the charge escapes along the thread or wire to the support, and so is lost. *Materials which, without motion of their own, can serve as avenues of escape for an electric charge, are called* CONDUCTORS *of electricity. Materials which cannot serve this purpose are called* NON-CONDUCTORS, *or* INSULATORS.

Metals are the best conductors, and resinous and vitreous substances are among the best insulators. No perfect insulator is known, and, on the other hand, there is, apparently, no perfect conductor,—no conductor which is not somewhat heated by the passage of electricity through it, thus showing that it offers a certain amount of resistance† to the movement.

* From Deschanel's *Natural Philosophy*, Appleton & Co.

† It is thought, however, that molecular currents of electricity, not passing from one molecule to another, may exist without friction and so continue indefinitely after they are once set up.

390. The Gold-leaf Electroscope.—The pair of pith-balls suspended by a silk thread used in § 386 is sometimes called an *electroscope*, for the reason that it enables us to detect, or make evident, an electric charge; but for more delicate experiments a more sensitive instrument is needed. This is found in the *gold-leaf electroscope* shown in Fig. 271.

G may be an open-topped glass *receiver*, such as is frequently used with air-pumps, fitted at the top with a cork, *c*; *B* is a disk * of metal, usually brass, six or eight inches in diameter, from which a metal rod, *r*, reaches downward through the cork. The upper ends of the strips of thin gold-foil, *l l* (the thinnest used by dentists), are crowded into a narrow slot sawed in the lower end of *r*, so that there is good metallic contact from *B* to *l l*.

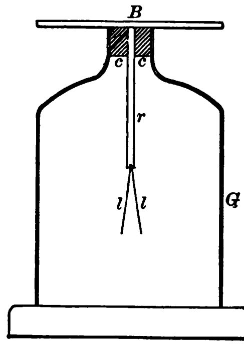

Fig. 271.

EXPERIMENT.

Hold a charged rod of sealing-wax or glass over the electroscope, and lower it gradually, stopping an inch or two above *B*. Then raise and lower the rod alternately several times. Are the leaves *l l* affected when the rod approaches the plate ? Do they continue to show any peculiar action after the rod is removed ?

(If the leaves become permanently charged, they may be restored to their normal condition by touching the edge of the plate *B* with the finger. One must take great care in dry cold weather not to rub the glass of the electroscope, even with the hand, lest it acquire a charge which will be difficult to remove, and which will greatly disturb the proper course of the experiments. Moreover, one must guard care-

* The instrument is more sensitive, and therefore better for certain experiments, if a *ball* about 3 cm. in diameter is used instead of the disk *B*. But the disk is *necessary* for certain experiments that come later. The instrument can easily be made with replaceable tops, one a disk, the other a ball.

fully against the action upon the electroscope of possible charges on the clothing. The experimenter should frequently test for a charge upon his sleeve by holding his arm, *without the charged rod*, over the plate *B*, the *electroscope being previously discharged.*)

391. Electric Induction. — In § 386 pith-balls were charged from an electrified rod by direct communication.

In § 390 the electroscope was charged, temporarily, without contact with the electrified rod; but the fact that its charge disappears when the rod is removed indicates that it really receives nothing from the latter save a certain *influence* which throws it, for the time being, into a peculiar state of electrification without really changing the amount of electricity upon it. This operation is called electrification by *induction*.

One can readily understand that if every body is, in its normal condition, endowed with a certain quantity of positive electricity and a certain quantity of negative electricity, the approach of a charged body may produce in the electroscope, previously uncharged, a redistribution of its electricities, one kind being drawn toward the approaching charged body, while the other is repelled into the leaves. If, on the other hand, one holds that there is only one kind of electricity, one may suppose that the approach of a body having more than its normal amount will repel the electricity of the electroscope into the leaves, and that the approach of a body having less than its normal amount will attract the electricity of the instrument into the plate *B*, either event putting the leaves into condition to repel each other. Whichever theory one adopts, experiment shows that a redistribution of electricity, and nothing more, does take place at the approach of the charged body.

But can we now find some means of charging a body permanently by the aid of such induction?

EXPERIMENT.

Put the plate of the electroscope, *B* (Fig. 271), into good electrical connection with any large conductor, the body of the experimenter, for example, or the earth itself by means of water-pipes or gas-pipes, and then bring the charged rod toward *B*. Break the connection at B while the charged rod is still held near. Do the leaves now show any evidence of a charge after the rod is removed?

If any charge remains, is it of the same kind as that on the charging-rod? To answer this question, bring the charged rod * again toward *B*, and notice whether the *first effect* of its approach is an increase or a decrease of the divergence of the gold-leaves. If it is an increase, the charge on the leaves is like that on the rod; if a decrease, the charge on the leaves is unlike that on the rod.

The explanation of the permanent charge obtained in this experiment is that the charge on the approaching rod drives electricity of its own kind from the metal of the electroscope to the larger conductor, and attracts electricity of the opposite kind from the larger conductor to the electroscope.

392. The Electrophorus.—This instrument, the action of which depends upon induction, is very convenient for supplying electricity with which to charge conductors of moderate capacity. It consists usually of a shallow metal pan (*P*, Fig. 272), which may be about 25 cm. in diameter, containing, *r*, a quantity of resin or other similar material (which has been poured in while hot and has cooled in place, forming a smooth hard surface), and a flat circular plate of metal, *C*, somewhat less in diameter than the pan, furnished with an *insulating* handle of glass or hard rubber.

* If there is a charge on the electroscope, the approach of an *unelectrified* conductor—the hand of the experimenter, for instance—toward the plate *B* may cause the leaves to approach each other, for the charge already on the electroscope induces a charge upon the approaching conductor, and is itself somewhat changed in consequence. To avoid error from this cause the charged rod should be a long one, so that the hand need not come near *B*.

To prepare this apparatus for use, the resinous surface is rubbed with a dry catskin, and thus acquires a charge of negative electricity. The metal disk is then placed upon this surface. If the two fitted each other perfectly the metal plate would become charged with the same kind of electricity as the surface upon which it rests; but the two surfaces really touch at certain points only, and so

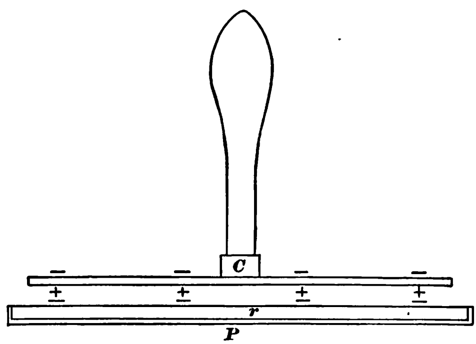

FIG. 272.

the negative charge upon the resinous plate *induces* a positive charge on the lower surface of the metal plate and repels some of the negative electricity to the upper surface.

If this upper surface is now touched for an instant by the hand of the experimenter, negative electricity escapes, and when the plate is lifted it carries a positive charge, which may be used to charge other conductors. The resinous plate meanwhile has suffered little loss, and the operation may be repeated many times without recharging it.

393. Self-repulsion of Electrical Charge: Discharging Action of Points.—The self-repulsion of an electrical charge spreads it over the *outer* surface of the conductor upon which it rests.

EXPERIMENT.

Charge an insulated hollow metal sphere (No. CX ; see Fig. 273) by means of the metal plate of the electrophorus, making several contacts if necessary.

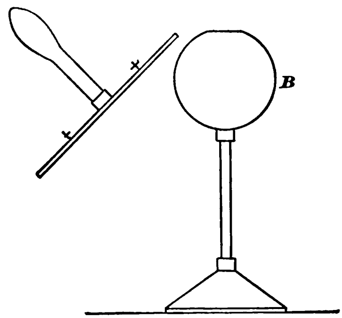

FIG. 273.

Take a metal ball, *b*, about 1.5 cm. in diameter, mounted on an insulating handle about 15 cm. long, touch it to the outer surface of the hollow charged sphere, and then to the plate of the gold-leaf electroscope, repeating the operation a number of times if the effect upon the electroscope is not at once evident.

Discharge the electroscope, by touching it with the hand, and then try to charge it by touching it with the little ball after this has been in contact with the *inner* surface of the meta' sphere, repeating the operation as many times as before.

Does the ball appear to get any electricity from the inner surface of the sphere ?

The self-repulsion of the charge tends to drive it off from the conductor, and great care must be taken to enable a conducting body to retain its charge for any great length of time. The escape takes place with peculiar readiness from sharp points, especially when they are directed toward other neighboring conductors.

On the other hand, a conductor provided with such points receives electricity from neighboring bodies more readily than one not so provided.

394. Electrical Potential; Electrical Capacity.—There is a limit to the electrical charge which a given electrical agent can produce upon a given body.

If the upper plate of an electrophorus, after being charged as in the preceding article, is brought into contact with a piece of metal *M* supported on a glass or other *insulating* stand, the insulated hollow sphere already used, for example, the self-repulsion of the electricity on the plate will cause some of it to flow over upon *M*; but presently the self-repulsion of the charge thus imparted to *M* prevents further flow. The plate and *M* are now at the same electrical *potential*, as it is called.

Definition.—*Two conductors are said to be at the same electrical* POTENTIAL *when the potential energy of a quantity of electricity on one is just as great as the potential energy of an equal quantity of electricity on the other, so that there is no flow of electricity from one to the other when they are connected by a conductor.*

If the plate is repeatedly charged and as often touched against *M*, the spark at contact of the two will become less and less noisy until it is almost or quite imperceptible. *M* is now charged about as highly as it is possible to charge it from the electrophorus in its present condition.

If a larger piece of metal than *M*, of the same shape, were used, more repetitions of the charging operation would be necessary before the spark became equally small. That is, the larger body would take more electricity than *M* in being charged to the same potential.

The larger body is said to have a greater *capacity* for electricity than the smaller.

Definition.—*The* ELECTRICAL CAPACITY *of a body is measured by the amount of electricity which, when given to the body, raises its potential a certain amount.* (But see § 395.)

Twice as much electricity would raise its potential twice as much, and so on.

395. Condensers: the Leyden Jar.—The electrical capacity of a conducting body does not depend upon its size and shape alone. It is much affected by the nearness or distance of other conductors, as the following experiment will show.

EXPERIMENT.

Take the inner coating, *I* (Fig. 274), from a dissecting Leyden jar and place this coating upon a glass support. Charge it by means of the electrophorus plate, counting the number of repetitions necessary to make the charge complete, so that the spark at contact shall be very slight.

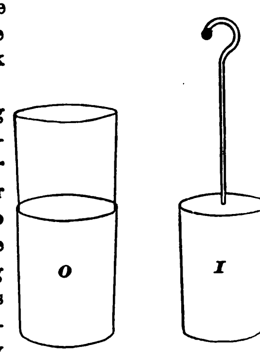

FIG. 274.

Discharge the coating, by touching it with the finger or otherwise, and replace it in the jar. Connect the *outer* coating of the jar with the gas-pipes or water-pipes, or have some person keep his hand against it, and repeat the operation of charging the inner coating from the electrophorus plate; count, as before, the number of contacts necessary to make the charge practically complete, if this can be done.

Compare the number of contacts in the two cases, and decide whether the capacity of the inner coating is greater or less when it is in the jar.

Discharge the jar by means of a discharging-rod (*D*, Fig. 278), touching first one knob to the outer coating, and then, without breaking this contact, bringing the other knob to touch the knob of the inner coating.

The Leyden jar is the most familiar form of electrical *condenser*. Its invention was suggested by an accident.

In 1746, Cuneus, a student of physics at Leyden, undertook to electrify some water contained in a glass vessel which he held in one hand. He used an iron wire to carry the

electricity to the water, and was quite unconscious that the charge communicated to the water was inducing upon his hand which held the vessel an equal charge of the opposite kind, and that the mutual attraction of these two charges was making the water take up a much greater quantity of electricity than it would have done by itself. When he considered the water sufficiently charged, still holding the vessel in one hand he attempted to remove the iron wire with the other hand. The positive and negative charges united through his arms and body, giving him a shock that frightened him greatly. He kept his wits, however, sufficiently to know how the accident had happened, and soon all the leading scientific men of Europe were repeating the experiment.

Before long coatings of tin were substituted for the hand and the water, and in this form the *Leyden jar* has been used ever since.

There are many other forms of electrical condensers, but they are all essentially alike: *two conductors separated by a non-conductor, the positive charge upon one conductor and the negative charge on the other mutually attracting each other, so that the capacity of each conductor is increased by the presence of the other.*

The definition of electrical capacity given in § 394 does not hold for the whole condenser, used as such. *The* CAPACITY OF A CONDENSER *is measured by the amount of electricity, positive or negative, which goes to either half of the condenser while the difference of potential between the halves is increased by a certain amount.*

396. Specific Inductive Capacity.—The capacity of a condenser depends upon the nature of the non-conductor, or insulator, as well as upon the dimensions and shape of its parts. Thus, two metal plates separated by paraffin

have a capacity about twice as great as that of two similar plates separated an equal distance by air.

The ratio of the capacity of a condenser using any given insulator to the capacity of a like condenser using air is called the *specific inductive capacity* of that insulator.

The specific inductive capacity of most, if not of all, solid or liquid insulators is greater than that of air.

397. Machines for Production of Electrical Charge.—We have seen that electrical charges can be produced by friction and by induction. The apparatus we have used thus far in either method is very simple, and very slow in operation if any large charges are to be given. But "machines" have been invented and are in common use which make much more convenient and effective use of both friction and induction.

398. Friction-machine.—This usually consists of a glass plate, mounted upon a horizontal axis and made to revolve between two soft pads which press it on both sides, covering only a small part of it at once. Electricity of one kind appears upon the pads and electricity of the other kind upon the glass, as a result of this rubbing.

The pads are covered with a conducting preparation by which their electricity is steadily carried away. The part of the glass which has just been rubbed by the pads is by the revolution brought near a number of sharp metal points which take off its electricity. Thus the action is continuous, and comparatively rapid.

399. The Induction-machine.—This machine appears in several forms, but each of them may be described as a continuously acting electrophorus, or a combination of such instruments.

Every such machine has one part or more than one, which corresponds to the resinous plate of the electroph-

orus, and is, like this plate, charged at the beginning of operations, by friction or otherwise. Such parts are called *armatures*.

Every such machine has something corresponding to the movable metal plate of the electrophorus. This something is usually a glass plate, sometimes provided with strips or disks of metal, mounted so as to revolve readily upon a horizontal axis. We shall call this the *carrier*.

The particular form of machine which will now be described is called sometimes the Voss, and sometimes the Toepler-Holtz machine. It is equivalent to two electrophoruses.

It has two armatures, AB and $A'B'$ (Fig. 275), each con-

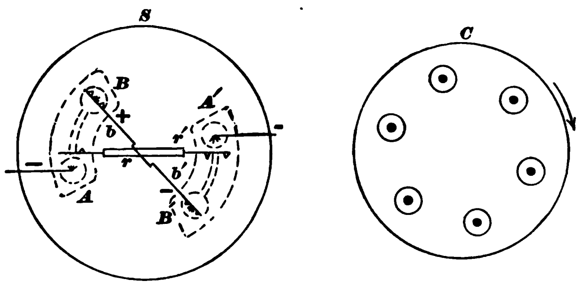

FIG. 275.

sisting of a strip of tin-foil protected by a covering of paper, fastened to the back of a stationary glass plate, S, fixed in a vertical position.

A small metallic brush in *front* of the plate is in metallic connection, around the edge of the plate, with A, and a similar brush is similarly connected with A'.

Just above the first brush there is a horizontal metal rod with projecting short wires, which is called a comb. Just below the other brush there is a similar comb. These two combs are both supported by a rod of hard rubber, rr', but are not metallically connected with each other.

The left-hand comb is metallically connected with the inner coating of a small Leyden jar, J (Fig. 277); the other comb is similarly connected with the inner coating of the jar J'.

The brass rod bb' has at each end both a brush and a comb. One end of this rod is placed in front of B, the other in front of B'.

The filaments of all the brushes and the teeth of all the combs are turned *toward* the fixed plate S, but are at such a distance from it that the revolving carrier glass plate C (Fig. 275) is placed between, the axle on which it revolves passing through a hole in the centre of the plate S.

On the front side of C are six disks of tin-foil, each having a small boss at the centre. Each of these bosses is touched by each of the four brushes at every revolution of the plate C.

The process of charging the machine for operation is, or may be, as follows: Separate the balls, b and b' (Fig. 277), two or three centimeters, and then rub the paper of the armature AB (Fig. 275) gently with a catskin, the plate C meanwhile remaining at rest.

If now, before C is started, different metal parts of the apparatus are tested by touching them with the small insulated "proof-ball," b (Fig. 273), and then bringing this ball near a charged gold-leaf electroscope, the sign of the charge given to the ball at any one of the several points touched will probably be that indicated by the sign $+$ or $-$ placed near that point in Fig. 275. At most of these points the charge is an induced charge, which can be traced back to the charge imparted to AB by the rubbing.

When C is now started each carrier-disk in turn touches the brush connected with A and thus acquires a $-$ charge. It then passes behind the comb just above A and gives off to this comb, as the cover on the electrophorus gives off to

the finger, its — charge, and takes on a + charge. At the comb opposite B the disk may have this + charge increased. Then, going on to the brush connected with A', the disk imparts a + charge to the left-hand armature.

After a few revolutions of C, if the machine is in good condition, the process of charging is complete, and frequent sparks pass between the balls b and b' (Fig. 277).

The state of charge can now be tested while the carrier C is in motion by means of the proof-ball provided for the occasion with a small metal brush. This state is probably

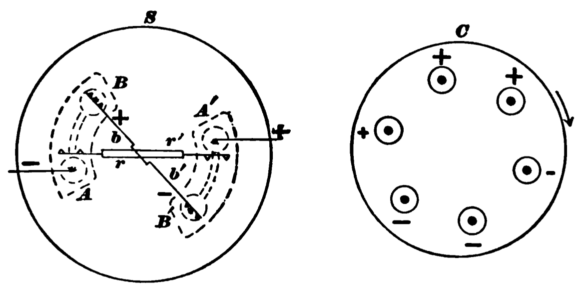

FIG. 276.

that indicated by the + and — signs in Fig. 276, although it is not very easy to determine by experiment whether the magnitudes of the + and — *charges* on the carrier correspond to the magnitudes of the + and — *signs* here shown.

Exploration with the proof-ball after the carrier is stopped is likely to give a false idea of the state of things which exists when the carrier is in motion.

It has been already stated that a pair of Leyden jars (see Fig. 277) make a part of this machine. The outer coatings of these jars are connected together by a wire W. The inner coatings are connected with the balls b and b', the distance between which can be varied.

When the charge upon the jars becomes great enough, a

spark leaps between the balls, uniting the opposite charges of the inner coatings. When this occurs the positive and negative charges on the outer coatings, which charges have been kept apart by the attractions of the inner charges, rush together along the wire *W*.

So great is the rush of this external discharge, that if a second wire, *w*, is made to touch one of the outer coatings and almost to touch the other outer coating, leaving a narrow gap, some of the current will pass by this wire at every discharge, and a small spark will occur at the gap. This fact

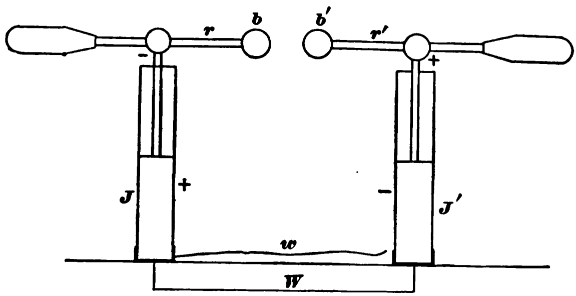

FIG. 277.

has attracted much attention, and will be referred to later in connection with the discussion of protection from lightning.

400. Charge on Glass of Leyden Jar.—The induction-machine makes it easy to charge Leyden jars heavily. A common method of doing this is to connect one coating of the jar with the rod *r* (Fig. 277), and the other coating with the rod *r*′, while the disk is kept in rotation.

The following experiment with a *dissecting* Leyden jar is interesting and instructive, as it shows that the coatings are merely a convenient means of *charging* and *discharging*, but are not essential to the action of the jar as a reservoir or *condenser* of electricity.

EXPERIMENT.

Charge the dissecting Leyden jar heavily and then lift out the inner coating by means of a glass rod. Place this coating on the table, then lift the jar by its top, and push off the outer coating.

Touch the two coatings together, noting whether any spark occurs. Then put the jar back into the outer coating, drop the inner coating into place, *taking care now not to touch both coatings with the hands at the same time,* then connect the two coatings in the usual way by means of the discharging-rod, to see whether any charge remains.

Repeat the experiment, rubbing the glass inside and out with the hand after the coatings have been removed.

401. Protection from Lightning.—Since the time of Franklin it has been customary to use certain arrangements of metal rods for the purpose of protecting buildings from lightning-strokes. The most common device is an iron or copper rod having one end buried in the ground and the other attached to a chimney and extending some feet above the chimney-top.

A common theory of such rods is: *first*, that they tend to *prevent* lightning-strokes, by assisting the *quiet* passage of electricity between the earth and the clouds; *second*, that, if the stroke occurs, it will pass along the rod rather than through the building over which the top of the rod is raised. The first of these uses is probably very slight. The second may be very effective, *if the rod runs down several feet into water, or is attached to a large sheet of metal buried in damp earth.* Otherwise the rod may be useless, or worse than useless.

Under some conditions the discharge current is not wholly confined to the rod, a part of it passing in a spark between the rod and some neighboring piece of metal—the gas-pipes or water-pipes of the building, for example. (See the discussion of discharge of outer coatings of the Leyden jars on p. 497.)

Sometimes the lightning pays little attention to the rod, but strikes some other elevated object connected with the building.

In short, one or two rods on a large building do not afford sure protection from lightning. The only certain means of protection is to inclose the building in a network of rods or heavy wires, some vertical and some horizontal, like the bars of a cage. The following experiment will illustrate the efficiency of such an arrangement:

EXPERIMENT.

Place the gold-leaf electroscope on a sheet of metal near the induction-machine. Cover the electroscope with a wire cage, the bottom of which shall rest upon a metal sheet (see Fig. 278). Connect the

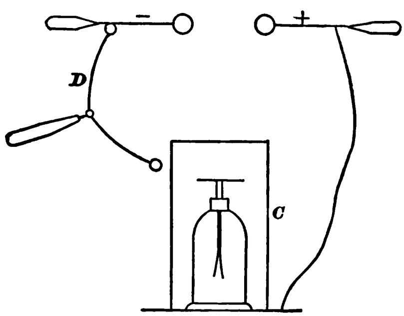

Fig. 278.

metal sheet with one of the rods of the machine. Lead a metal rod from the other rod of the machine almost to the top of the wire cage, leaving a gap of 1 cm. or more.

Work the machine so as to allow sparks to pass to the cage, and watch the behavior of the gold leaves.

Take away the cage and the metal sheet, and note the behavior of the leaves when a rod connected with one of the poles of the machine is brought near (not too near) the top of the electroscope.

The interior of an iron ship, having the iron of its masts and rigging connected with the outer shell, is a peculiarly safe place, so far as danger from lightning goes.

It is hardly practicable to cover a building with a close network of wires for protection, and such a network is hardly necessary. The meshes of the net may be many feet square, and yet give effectual protection.

The electric currents of lightning-strokes last for an exceedingly short time—a millionth of a second, let us say. It is now known that such transient currents flow along the outer layer of a conductor rather than along the interior, and it is considered well to use flattened strips of copper, rather than round rods, for lightning-conductors. Convenience, however, is in favor of the round rod, and this is more often used.

It is a false proverb which declares that lightning never strikes twice in the same place. Some localities are peculiarly liable to lightning-strokes.

A forest probably affords very good protection from lightning. The shelter of a single tree standing alone in a field is comparatively dangerous.

A person compelled to stay in a large open field during a violent thunder-storm had better lie flat on the ground, since, other things being equal, the lower a body is the less likely it is to be struck by lightning.

Indoors, one would do well to avoid the immediate neighborhood of water-pipes and gas-pipes. But really, very few people indoors are killed by lightning. Aside from the danger of being frightened to death, a thunderstorm is about the safest excitement a community can have.

CHAPTER XXXI.

THE GALVANIC CELL AND ELECTRIC CIRCUIT.

402. Electricity developed by Chemical Action: the Vol taic, or Galvanic, Battery.—"In 1780, Galvani, professor of anatomy at Bologna, studying the influence of electricity upon nerves, observed by chance, in the skinned legs of a frog recently killed, convulsions which occurred at the moment when an electric machine was discharged near by. . . . He set himself at once to study the circumstances of the phenomenon with various animals, warm-blooded and cold-blooded, with the desire of proving the identity of the nervous fluid with electricity. He devoted six years to this work.

"In the course of these researches, in 1786, wishing to see what effects the discharge of thunder-clouds would produce, he suspended on a balcony of the terrace of the Zambeccari palace the hind legs of a frog, by means of a copper hook, which passed through the spinal cord, and saw, with keen surprise, these members convulsively agitated, though no thunder-cloud was near. He observed soon that this took place at the moment when the limbs accidentally touched the iron of the balcony. He could thenceforth repeat the experiment as often as he wished, and he found himself in possession of a new and unexpected fact, which has become the point of departure of a long series of brilliant discoveries, and the origin of one of the most extended and most important parts of physics." *

* Daguin's *Traité Elémentaire de Physique*, tome troisième.

In the year 1800, Volta, another Italian, following the hint given by Galvani's discovery, showed how to maintain a continuous supply of electricity by the aid of chemical action.

The most famous arrangement which he employed for this purpose consisted of a column made up of disks of copper, zinc, and cloth moistened with acidulated or salt water, the order of arrangement being copper, zinc, cloth, copper, zinc, cloth, etc., the column beginning with one metal and ending with the other. This was called *Volta's pile*. It has the advantage of great simplicity of construction and portability, but loses its power as the liquid evaporates from the cloth disks, and is not so easily cared for as another arrangement (see Fig. 279), also devised by Volta, in which the liquid is contained in vessels, each vessel con-

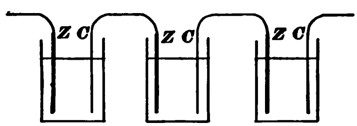

FIG. 279.

taining also a copper and a zinc strip. The copper of one vessel is joined to the zinc of the next, and so on through the whole series of *cells*, which was for a long time called, like its predecessor, a *pile*.

By means of such a pile, or *battery*, as it is now generally called, it is easy to produce effects resembling in kind those shown in the preceding experiments upon electricity, thus showing that the electricity obtained from chemical action is just like that obtained by rubbing bodies together. The intensity of charge obtained from a battery of any moderate number of cells is, however, much less than that obtained by friction, and some pains must be taken to make the electrification produced by the battery evident.

EXPERIMENTS.

(1) Take a battery of fifty small cells, each containing water, a strip of zinc, and a strip of copper, connected according to the description given above, the one free zinc at one end of the series and the one free copper at the other end of the series being provided with a copper wire about 50 cm. long.

Now take the electroscope and cover the plate with a sheet of thin paper, *S* (Fig. 280), which has been soaked in melted paraffin. Place on this paper the metal cover of the electrophorus, taking care that the paper shall prevent all contact between the two plates of metal. Then touch the lower plate with one wire leading from the battery and at the same time touch the upper plate with the other wire. Remove the wires, and at once lift the upper plate and the paper. Do the leaves show any evidence of an electric charge?

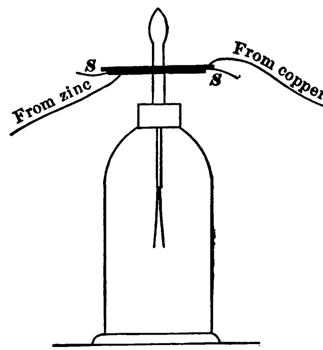

FIG. 280.

(The two metal plates with the paper between them serve as a *condenser* (§ 395), thus enabling the electroscope to take a larger amount of electricity from the battery than it otherwise could. When the plates are separated the *capacity* of the electroscope is diminished and the same quantity of electricity produces a higher state of *charge*.)

(2) Charge the electroscope again, touching the lower plate with the wire leading from the copper end of the battery and the upper plate with that leading from the zinc, and then, after removing the upper plate and the paper, test the charge upon the electroscope, in the way already described (§ 391), to find whether it has received positive or negative electricity from the battery.

Then repeat, touching the lower plate with the wire leading from the zinc end of the battery and the upper plate with that leading from the copper.

Which end of the cell furnishes positive electricity?

The following Exercise will show some of the important features of a very simple cell:

EXERCISE 51.

SINGLE-FLUID GALVANIC CELL.

Apparatus : Materials for a small copper-zinc cell (without porous cup), using clean dilute sulphuric acid, about 20 parts in volume of water to 1 part in volume of concentrated acid.* A galvanoscope (Fig. 281). The zinc is at first unamalgamated.

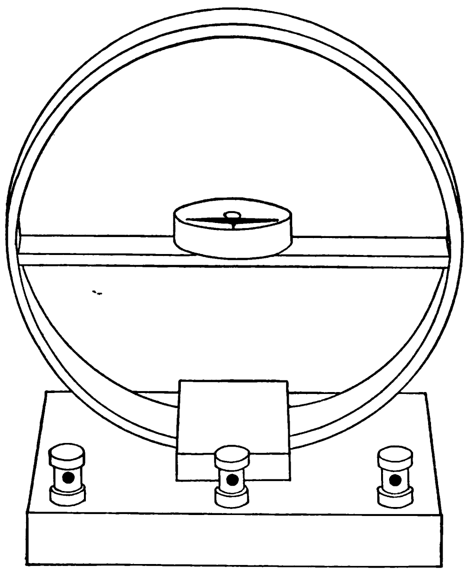

FIG. 281.

(In this and every other Exercise in which galvanic cells are used, whenever wires have to be connected, by binding-screws, clamps, or

* In mixing *pour the acid slowly into the water*, stirring the mixture. Pouring water upon the acid is dangerous. A small quantity of acid sinking at once in a large mass of water is speedily cooled in spite of the strong chemical action. A small quantity of water floating on the top of a large mass of acid is heated to boiling, and then spatters the acid.

mere twisting together, take care to make the connection firm, with good metallic contact.)

Place the galvanoscope so that each circle of wire upon it shall be in a vertical north and south plane.

Fill the glass with the dilute acid to a level about 1.5 cm. below the top.

Scour the strip of copper with emery-paper till so much of it as will be immersed in the acid is clean and bright.

Adjust the zinc strip and the copper strip in the clamps attached to the block at the top of the glass jar, so that each strip shall reach straight downward almost to the bottom of the jar.

ZINC UNAMALGAMATED; *Circuit Open*.—Observe for a short time and record what happens at the surface of each strip, all metallic connection between them being avoided.

Circuit Closed.—Then put the two strips into metallic connection with each other through the 15 turns of the galvanoscope, and again observe and record what happens at the surface of each strip. Note, too, the behavior of the galvanoscope-needle, and record the position in which it comes to rest, tapping the instrument lightly to prevent the needle from being detained by friction in a wrong position.

ZINC AMALGAMATED; *Circuit Open*.—Remove the zinc strip from the cell and *amalgamate* its surface by dipping it for an instant in mercury. Shake off or wipe off the superfluous mercury that clings to the strip and then replace the zinc in the cell. Take care that no mercury shall touch the copper.

Again observe and record the action at the surface of the zinc, the wires remaining unconnected.

Circuit Closed.—Again connect the strips through the 15 turns of the galvanoscope, read and record the position of rest of the needle as soon as convenient, and read and record every minute thereafter for five minutes.

Observe what occurs at the metal surfaces.

If no decided change in the size of the deflection is noticed in the course of five minutes, take out the copper strip, which, *if the acid solution is in the right condition*, will still look bright, rinse it, and again scour it with emery-paper. Then replace it in the cell,*

* It is better to use a fresh solution for this part of the Exercise; otherwise confusing results may follow.

take a reading as soon as convenient, and continue reading for several minutes.

The Electric Circuit with a Galvanic Cell.

403. Open Circuit and Closed Circuit; Current.—A galvanic cell and the external wires, or other conductors, connecting its plates, or strips, of metal, is one form of *electric circuit*. When the plates of the cell are not connected by an unbroken line of conducting material, the circuit is said to be *open*, or the cell is said to be *in open circuit*.

If the conducting line is unbroken, as when the wires from the cell are connected with the galvanometer in the preceding Exercise, the circuit is said to be *closed*.

It is consistent with the " two-fluid theory " of electricity (see § 387) to suppose that what we call an *electric current* is made up of two streams of equal strength, one of positive and the other of negative electricity, moving simultaneously in opposite directions in every part of the circuit.

The possibility of this double current is often recognized in phrases relating to electrical circuits, but *commonly the current is spoken of as single; and, when it is so spoken of, it is regarded as flowing in the direction of the positive current of the two-fluid theory.* It is frequently called expressly the *positive* current.

It is the common practice to speak of the electricity as coming out from the cell by one wire, passing through the external part of the circuit to the other wire, entering the cell by this wire, passing through the liquid, and emerging again by the first wire; or, in other words, to say that an electric current is a continuous stream going around and around the circuit—as water circulates from a heater, up some of the pipes of a building and down others to the heater once more, and so on over and over again, none escaping.

This theory of the galvanic circuit is no doubt somewhat crude, especially in regard to the cell itself; but it is incomplete rather than incorrect, and we need not quarrel with it or discard it in an elementary course.

404. Terms Relating to the Cell.—Michael Faraday (1791–1867), who was one of the greatest investigators in electricity that the world has known, invented, with the help of a friend learned in the ancient tongues, several terms from the Greek, which he applied to various parts of the galvanic cell. These terms are now in very common use.

Thus, the conductor, usually a solid, by which the current flows out from the cell is called the *cathode* (*c*, Fig. 282); that is, the *way down* out of the cell.

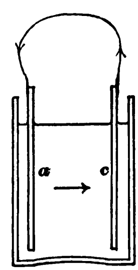

FIG. 282.

The conductor, usually a solid, by which the current flows back into the cell is called the *anode* (*a*, Fig. 282); that is, the *way up* into the cell.

These terms are apt to be confusing, because it may seem that the "*way down*" should lead *into* the cell, but Faraday fixed the use once for all.

The cathode is frequently called the positive pole, or plate, of the cell, and the anode, the negative pole, or plate.

In a storage-cell (§ 406) the set of plates by which the current enters during the "charging" process is called the positive pole, because during the real use of the cell the current leaves the cell by this pole.

The anode and the cathode together are called *electrodes.*

The passage of electricity through a cell is accompanied, as we shall see in the next article, by decomposition of some chemical compound in the cell. This kind of decomposi-

tion is called *electrolysis*, and the compound so broken up is called an *electrolyte*.

The parts into which the compound is broken up are called *ions*. Some of these make their appearance at the anode, and are called *anions*. Others appear at the cathode, and are called *cathions*, or *cations*.

405. Chemical Action of Single-fluid Cell.—A *molecule* * of sulphuric acid, we are taught by chemists, consists of two *atoms* of hydrogen (H_2), an atom of sulphur (S), and four atoms of oxygen (O_4). The symbol showing the composition of the molecule is H_2SO_4.

When sulphuric acid comes in contact, under proper conditions, with zinc, chemical action takes place, and the two atoms of hydrogen are replaced by one atom of zinc (Zn). The hydrogen is thus set free as a gas, while the

* **Atoms and Molecules.**—*Chemistry and Physics.*—Of all the substances known to us, about seventy are called *elementary substances*, or *elements*. An elementary substance is one which, so far as science, has yet ascertained, is not composed of other substances.

The smallest particles of an elementary substance that science has any knowledge of are called *atoms*, the name, which means *undivided*, or *indivisible*, implying something that science does not know how to divide. The atoms of any one substance are generally supposed to be exactly alike.

There is reason for believing that in most elementary substances each atom is, in its ordinary condition, joined in some definite manner to one or more other atoms. These definite combinations of atoms are called *molecules*, and in any particular substance all molecules are supposed to be exactly alike.

The atoms of one element may unite in definite combinations with those of one or more other elements. A group of atoms so formed is also called a molecule, but it is a molecule of some new substance, different from either of the elements that go to make it up. Such a substance is called a *compound substance*. It has molecules, but does not have atoms of its own.

The science of *chemistry* deals with atoms and the ways in which they combine to form molecules. *Physics* takes molecules ready-made, and studies their behavior and the phenomena which bodies can show without suffering a break-up of their molecules. Even so great a change as that from ice to water or from water to steam leaves the molecule unchanged in composition, the substance used remaining the same throughout.

compound that remains is $ZnSO_4$, called zinc sulphate, or sulphate of zinc. This change, or *reaction*, as it is called, may be represented briefly by the following equation:

$$Zn + H_2SO_4 = H_2 + ZnSO_4,$$

in which the sign = is equivalent to the word *produces.*

If this reaction takes place while a battery is not in use, it is evidently wasteful of zinc, which is a rather expensive metal. Chemically pure zinc, not in contact with other metals, is but little affected by sulphuric acid, but its high cost forbids its use in batteries. Impure zinc when amalgamated acts much like chemically pure zinc. (See Exercise 51.)

Copper is, under ordinary circumstances, but little affected by sulphuric acid. Any action seen at the surface of the copper strip in Exercise 51, even when the circuit is closed, is mainly due to hydrogen bubbles, freed by chemical action which leaves the copper unaffected.

In the cell of Exercise 51, SO_4, called *sulphion*, is the anion and H is the cation.

406. Polarization in a Galvanic Cell; Storage-cell.— The gradual weakening of the current probably observed in Exercise 51, when the circuit is closed for a considerable time, and its recovery when the copper strip is thoroughly rubbed, are phenomena well worthy of attention. The weakening is not due to chemical exhaustion of the solution, but rather to the condition which the action of the cell produces at the surface of the copper strip. A similar but more striking effect is shown in the following experiment:

EXPERIMENT.

Nearly fill a battery jar with such liquid as is used in Exercise 51, and place in it, near to but not touching each other, two pieces of sheet lead, as large as can be conveniently used, each having soldered to it a copper wire 50 cm. or more long.

Connect these two wires for a moment with the binding-posts of the lecture-table galvanometer (see Fig. 283) to show that the cell in its present condition gives no perceptible current.

Leave one of the wires connected to the galvanometer, and attach the other to one pole of a battery of four or five Daniell cells (see Exercise 52) arranged as in Fig. 283. Connect the other pole of the Daniell battery with the other binding-post of the galvanometer.

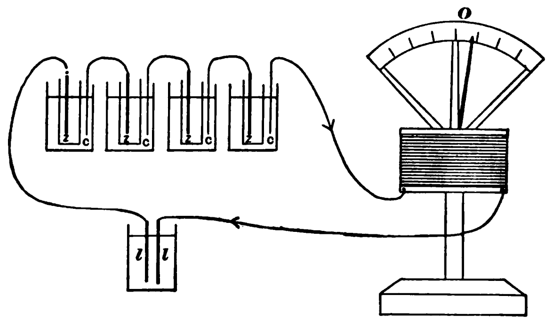

FIG. 283.

Observe the direction of the deflection produced, and note whether this deflection increases or diminishes during the next two minutes.

Remove the Daniell battery from the circuit and test the lead-cell as at the beginning to see whether it can now produce a perceptible deflection of the index. Note the direction of this deflection. Is the current passing through the lead-cell as now used in the same direction as that which was sent through it by the battery? Is the current constant, or does it lose strength?

Any current which the lead-cell, used alone, may yield is due to the peculiar state into which its plates are thrown by the action of the current from the other cells. This state is called battery *polarization*. It does not merely offer resistance to the electric current by which it is produced, for it tends to send a current backward through the cell, and will do so, as the experiments just described make evident, when opportunity is given. It was such polarization that gradually reduced the strength of the current in Exercise 51.

Polarization is a serious disadvantage in many galvanic cells, but it is not without its uses; for upon it depends the action of all the so-called *storage-batteries*, which since the year 1891 have come to fill an important place in the manifold applications of electricity. It is to be noted that the thing stored in such cells is not, according to the usual terms of science, electricity, but chemical energy in a form peculiarly available for the production of an electric current.

EXERCISE 52.

STUDY OF A TWO-FLUID GALVANIC CELL.

Apparatus and Materials: The solid parts of a small Daniell cell, the zinc well amalgamated. Saturated solution of sulphate of copper, and a sulphuric acid solution of the same strength as that used in Exercise 51. A galvanoscope. The platform balance (No. 71).

Place the zinc in the porous cup and place this cup and the copper in the glass jar. Fill the jar with sulphate of copper solution to a level about 2 cm. from the top and fill the porous cup with the dilute acid to the same level, or a little higher. (See Fig. 284.)

Lift the copper plate from the solution, let it drip 15 seconds, then weigh it carefully with all the liquid that goes with it. Then replace it in the cell. Wipe the liquid from the pan of the balance.

Lift the zinc from the acid, let it drip 15 seconds, then weigh it carefully with all the liquid that goes with it. Replace the zinc in the cell and wipe the balance dry.

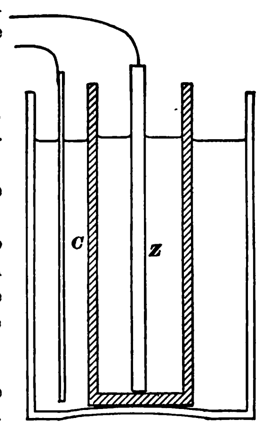

FIG. 284.

Connect the poles of the cell with the terminals of the 5-turn section of the galvanoscope, and note the time at which this is done. Read and record the position of the needle as soon as it comes to rest, tapping the galvanoscope lightly as in Exercise 51.

Read and record the position of the needle every five minutes thereafter for half an hour, if the length of the Exercise permits.

Break the circuit, noting the time; then weigh each of the metal plates just as before, after allowing each to drip 15 seconds.

Find how much each plate has lost or gained in weight per *minute* of the time during which the circuit was closed.

NOTE.—Each plate carries about the same amount of liquid to the balance both times, so that the difference of its two weighings should tell, pretty nearly, its own change of weight. Wiping the plates might rub off some loose metal, and it is therefore not advisable. The zinc should not be freshly amalgamated at the beginning of this Exercise; for freshly amalgamated zinc carries superfluous mercury, which would be likely to drop off during the Exercise, making the weighings of the zinc useless.

If the current increases in strength for several minutes after the circuit is closed, this is probably due to the gradual driving out of obstructing air from the pores of the clay cup. If the cell is to be put to any use requiring a very constant current, the porous cup, kept outside the jar, should be filled some little time before the work begins. When a film of moisture appears on the outside of the cup it may be placed in the jar.

407. Discussion of Exercise 52.—Any galvanic cell in which the active parts are copper, sulphate of copper, zinc, and sulphuric acid or sulphate of zinc, is called a Daniell cell.

The chemical action in the Daniell cell is somewhat more complicated than that in the single-fluid cell of Exercise 51, but if we consider only the *results* of the reactions they are not difficult to understand.

Within the porous cup we have, as in Exercise 51, zinc replacing the hydrogen of sulphuric acid. The hydrogen thus freed does not appear in the form of bubbles, but in the outer part of the cell we find that hydrogen is replacing the copper of the sulphate of copper, and the copper thus freed is deposited upon the copper plate. It is not to be supposed that the identical atoms of hydrogen replaced by the zinc necessarily find their way to the outer cell, but the

outcome of the reactions is represented by the following equation:

$$Zn + H_2SO_4 + CuSO_4 = ZnSO_4 + H_2SO_4 + Cu.$$

It will be seen from this account that the polarization which occurs in Exercise 51, cannot occur with the two-fluid cell of Exercise 52, the plates of which are not necessarily changed in chemical condition by the operations that increase or diminish their weight.

Various injurious actions are likely to take place in a Daniell cell when it is left for any long time, over-night, for instance, in open circuit. Even when the plates are taken out, the porous cup may be spoiled by the deposition of copper upon it, owing to the presence of metallic particles in its wall. The cell should therefore be taken apart when not in use.

408. Other Galvanic Cells.—There are many other cells in common use. In nearly all of them, except the storage-cells, zinc is employed as one of the electrodes (§ 404), and whenever used it is the anode. The zinc is usually, but not always, placed in sulphuric acid, and from this frees hydrogen. We have, in § 407, seen one chemical device for preventing this hydrogen from accumulating upon the other electrode. There are various other devices, the general plan of which is to render the hydrogen harmless by bringing it into combination with oxygen.

One of the most effective oxidizing agents is nitric acid, and one of the most powerful galvanic cells, called *Bunsen's* cell, is that which is formed by replacing the sulphate of copper in the Daniell cell with nitric acid, and the copper plate with a plate of carbon. This carbon is not affected by the chemical action of the cell. The choking and corrosive fumes that come from the nitric acid make the Bunsen cell objectionable, and it is now used but little.

Another cell is that which may be formed from the Bunsen by replacing the nitric acid with a mixture of sulphuric acid and bichromate of potash. This is known as the *Poggendorff* cell, and by various other names.

The so-called *volta-pavia* cell is like the Poggendorff cell with this exception, that it employs bichromate of soda instead of bichromate of potash. It is more enduring in its action than the Poggendorff cell.

All the cells just described are, when in good condition, more powerful than the Daniell cell, but none of them equal it in constancy of behavior.

The *Leclanché* cell uses zinc in a solution of sal-ammoniac for the anode, and for the cathode a bar of carbon packed in crushed carbon mixed with peroxide of manganese. This cell polarizes rapidly when the circuit is closed, but is very useful in furnishing occasional currents of short duration, such as are needed for ringing door-bells and sounding alarms.

Storage-cells usually have for the positive electrode, plates, or "grids," of lead loaded with oxides of lead, and for the negative electrode similar plates with less oxide of lead. These plates are placed in sulphuric acid of a certain strength, contained usually in a glass or hard rubber jar.

In the "charging ' of such a cell a current is sent in from some dynamo (see Chap. XXXIV) by way of the positive electrode and out by the negative electrode. Oxygen is thus sent to the positive plates, adding to the oxides which are already there, and meanwhile hydrogen, going to the negative plates, unites with and takes away some or all of the oxygen there. The original difference between the positive and the negative plates is thus increased. When the charging is nearly complete, bubbles of hydrogen, no longer absorbed by the oxygen at the negative plates, rise

in large quantities from those plates to the surface of the liquid.

The method of arrangement of the plates in a storage-cell makes them equivalent to two very large plates, one positive and one negative, very close together (see Fig. 285, which shows the top view of a cell). This makes the so-called *resistance* (see Chap. XXXIII.) of the cell very small, and it is for this reason, mainly, that a storage-cell, newly charged, can give a much more powerful current than a Poggendorff cell, for example.

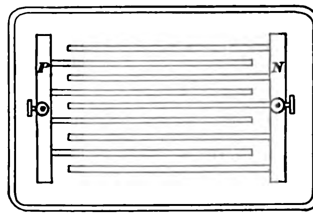

FIG. 285.

After a time of use, the storage-cell becomes reduced in "charge," and the current from it speedily grows less. The cell is said to be "run down," and must be charged again.

The positive plates of a storage-cell can usually be distinguished from the negative by their brownish color, due to the heavy oxides with which they are more or less loaded, even when the cell is run down. Another means of distinguishing between the positive and negative plates is the fact that, usually, the positive group contains one less plate than the negative group, as in Fig. 285.

Storage-cells are called also *secondary* cells, because they have to be charged from other cells or from a dynamo before use. Cells like those described in the first part of this article, which from the start yield a current from their own action, are called in distinction *primary* cells.

CHAPTER XXXII.

CURRENT-STRENGTH; ELECTROMOTIVE FORCE; ELECTRICAL WORK.

409. Current Strength.—*The ordinary unit of current-strength is called the* AMPERE, from Ampère (1775–1836), the great French investigator of electrodynamic laws.

We have already seen that an electric current is accompanied by chemical change in passing through a liquid containing a chemical compound. We have seen, also, that such a current affects a magnetic needle. Each of these properties may be made, and is made, the means of measuring what is called the *strength* of the current.

410. Chemical Measurement of Current; Voltameter.—The ampere is defined * as a current of such strength as, when passed through a solution of nitrate of silver in water, in accordance with certain specifications, deposits silver on tho cathode at the rate of 0.001118 gramme per second.

The same current, passing through a solution of sulphate of copper in water, would deposit copper on the cathode at the rate of 0.0003277 gramme per second.

A current depositing $n \times 0.001118$ grams of silver per second from a nitrate of silver solution, or $n \times 0.0003277$ grams of copper per second from a sulphate of copper solution, would be called a current of n amperes. In short, a current is regarded as proportional to the amount of chemical change it causes per second in a given cell.

* *Proceedings of the International Electrical Congress*, Chicago, 1893.

Many other substances beside those just mentioned might be used in defining the ampere.

A cell used for the purpose of measuring an electric current by the amount of chemical action is called a *voltameter*. It may or it may not furnish the motive power of the current which it measures.

If a dozen voltameters were introduced as links of the same electric circuit, with conductors of any sort between them, they would all indicate the same strength of current. It is upon such facts as this that the description of an electric current as a continuous stream, having everywhere the same strength, is based (see § 403).

A common form of voltameter consists of two plates of copper in a solution of sulphate of copper. Such a cell does not *maintain* the current. It merely transmits the current which is driven by some other means.

The anode of this voltameter loses weight and the cathode gains weight, and one might expect the loss of one to balance the gain of the other; but the chemical reaction at the anode is not a mere dissolving of copper. Other changes take place there and so complicate matters that the loss of weight is not a reliable measure of the current. The gain of weight of the cathode is the quantity depended upon.

Another well-known voltameter consists of two strips of platinum immersed in water containing a very little sulphuric acid. The acid in some way assists the passage of the current, but the visible chemical change is the decomposition of the water, H_2O, which is resolved into hydrogen, collected in a test-tube over the cathode, and oxygen collected in a similar tube over the anode (see Fig. 286).

EXPERIMENT.

Decompose acidulated water by means of the current from a battery of three or more Daniell cells arranged as in Fig. 286, collecting the gases in tubes above the electrodes.

Observe the relative bulk of the two gases, and show the combustible character of the hydrogen by igniting it, holding the test-tube, *filled* with the gas, mouth downward. Before the flame is applied it is well to wrap the tube in a cloth to gaurd against danger from a possible explosion.

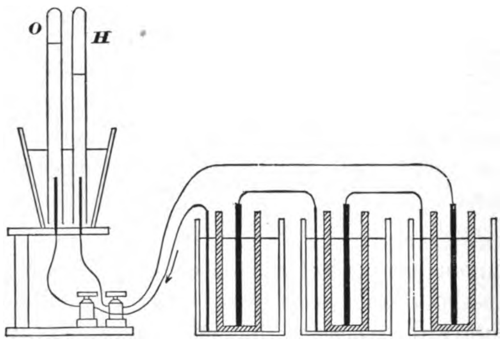

FIG. 286.

The length of time required for the use of a voltameter makes any such instrument unsuitable for determining the strength of a variable current at any particular instant; but in many cases, in the commercial use of electricity, the quantity to be arrived at is not the strength of the current at any one moment, but the total amount of electricity that passes in a given time—a day or a week, for example. For this purpose voltameters are particularly well suited and have been much used.

A voltameter, properly used, is a very sensitive and satisfactory means of measuring a current of unchanging strength lasting for a considerable time, and voltameters are very often used in "standardizing" other instruments, called galvanometers, which, after being standardized, are more convenient for general purposes of current measurement.

The proper use, however, of a voltameter is a somewhat delicate art, and one should consult some book on electrical

measurements before attempting accurate work with this instrument.

PROBLEMS.

(1) How much copper can be deposited from sulphate of copper in 1 hour by a current of 5 amperes?

(2) How strong must a current be to deposit 10 gm. of copper from copper sulphate in 3 hours?

(3) Calculate from the data obtained in Exercise 52 the strength, in amperes, of the current there used.

(4) In electroplating with copper a deposit of excellent quality is obtained by using a current of 1 ampere for 50 sq. cm. of surface upon which the deposit occurs. If the density of copper is 8.9 gm. per cu. cm., how long must such a current run to make a deposit 0.01 cm. thick?

411. Magnetic Measurement of Current; Galvanometer. —It is customary in the study of pure physics to define the strength of an electric current by reference to the force which a certain length of the conductor transmitting it exerts upon a magnetic pole of a certain strength at a certain distance. On this basis currents are measured by means of the galvanometer, an instrument similar to the galvanoscope of Exercises 51 and 52, but more carefully made.

An extended discussion of the measurement of electric currents by their magnetic action is beyond the scope of this book, and it seems best for our purpose to define the unit strength of current by reference to chemical action, as we have already done in the preceding Article. We may therefore regard the voltameter as the fundamental instrument, by means of which a galvanometer can be tested and "standardized," that is, so studied that from its reading the strength of a current passing through it can be found.

By definition (p. 516) the strength of an electric current is proportional to the amount of chemical change it produces per second in a voltameter. *By experiment* it is found that *the magnetic force of a current is, other things*

being equal, proportional to the strength of the current as measured by the voltameter, but this does not imply that the deflection of the needle of a galvanometer is proportional to the strength of the current.

We have, in the data obtained from Exercise 52, the means of calculating roughly the strength of current which, in the magnetic field of the laboratory, will produce a particular deflection of the needle of our galvanoscope when the 5-turn section is in use. By means of similar data, obtained with currents of different strength, it would be possible to find the significance, in amperes, of any deflection within a considerable range, but this instrument is hardly worthy of so much study.

An accurate galvanometer must have a well-constructed scale of some sort, and its magnetic needle is usually suspended by a very slender fibre, subject to troublesome torsion and disastrous breaking, or poised upon a very delicate point, which is almost sure to be ruined in the hands of a novice.

The object of the following Exercise is to give the student some further experience with the magnetic action of electric currents and, in particular, to make him familiar with the general character and course of the lines of magnetic force due to a current running through a galvanometer-coil.

EXERCISE 53.

LINES OF MAGNETIC FORCE ABOUT THE GALVANOSCOPE.

Apparatus: The galvanoscope with detachable compass. A Daniell cell. A commutator (Fig. 287). Short pieces of wire for making connections.

The lines of force to be examined in this Exercise are not due entirely to the current in the galvanoscope. They represent the combined action of the earth's magnetism and the current, and the character of the lines must depend, to some extent, upon the position of the galvanoscope with respect to the earth's lines of magnetic force. Something might be said in favor of studying the lines when the windings are placed north and south, as in the ordinary use of the

instrument, but, on the whole, it seems better to place the windings east and west for this Exercise.

At some points the force due to the current will be in exactly the same direction as the horizontal force due to the earth. At such points the resultant force will be greater than that due to the earth alone, and the needle, if disturbed, will vibrate *more quickly* than it would if the current were not acting.

At other points the force due to the current is exactly opposite in direction to the horizontal magnetic force of the earth. At such points, if the force due to the current is greater than the other, the needle will be reversed and point south. If the earth's magnetic force prevails, the needle will still point north, but will vibrate *less quickly* than if the current were not acting.

In the diagram of the lines, points where the lines run north are to be marked "strong" or "weak," if the resultant force at these points is greater or less than the earth's horizontal magnetic force.

Put the Daniell cell in circuit, through the commutator, with 15 turns of the galvanoscope, and place the latter so that the plane of its circle shall be east and west.

Make in the note-book two lines, *E* and *W* (Fig. 288), to represent half-size a horizontal cut through the middle of the galvanoscope.

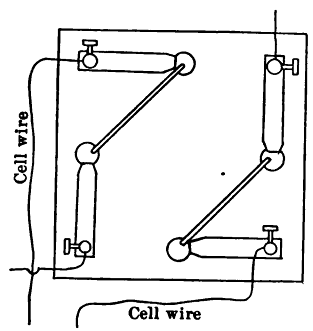

FIG. 287.

Adjust the commutator so that the current shall flow from east to west on the top of the galvanoscope circle.

FIG. 288.

Hold the compass about 15 cm. south of the centre of the circle on the same level, and record with an arrow-head on the diagram the direction of pointing of the needle. Move the compass north six stages of 5 cm. each, recording on the diagram the direction of the needle at each stopping-place.

Then place the compass inside the circle, near to the eastern side, and mark the direction of the needle. Follow the needle, as in Exercise 50, and mark its direction at three or four other points several centimeters apart, until the line followed appears to be completed.

Make similar observations and record with respect to the western side of the circle.

Find, and record verbally, the direction of pointing just over and just under the top of the circle.

Reverse the current through the galvanoscope, by operating the commutator, and then repeat all the observations just indicated.

412. Lines of Magnetic Force around a Straight Current. —*The lines of magnetic force due to a current in a long straight wire are circles, each in a plane at right angles with the wire, which passes through the centre of each circle.*

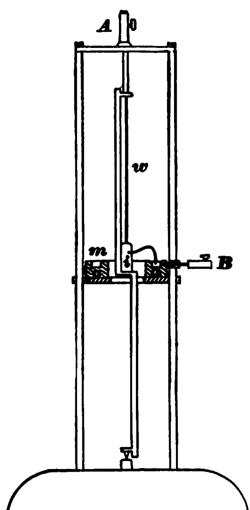

FIG. 289.

The direction of the magnetic force along each line depends upon the direction of the current in the wire, and is in all cases such that, *to a person looking along the wire in the direction of flow of the current, the magnetic needle would lead around each circle in a clockwise direction.*

The diagram of Exercise 53 should prove to be consistent with this statement, although the closed curves found in that Exercise are not circles.

The direction in which the north-seeking end of the needle *points* is that in which a detached north-seeking *pole* would *move*, if it were free. The following well-known experiment makes use of an ingenious device for causing one pole of a magnet to rotate around a current while the other pole does not.

EXPERIMENT.

Send the current first one way and then the other through the rotation apparatus shown in Fig. 289, and compare the direction of motion of the affected pole, in each case, with the rule given earlier in this Article.

413. The Earth's "Directive Force;" Astatic Galvanometer.—It is evident that in the ordinary use of a galvanometer, with its windings placed north and south, the current in the coil tends to make the magnet at its centre point east or west, while the horizontal force of the earth's magnetism, called the earth's "*directive force*," tends to make it point north. The position the needle takes is the result of a compromise between these two partly opposing influences. If the earth's directive force could be weakened, other things remaining unchanged, the deflection of the needle would be increased.

In fact, the earth's directive force, under which title is commonly included not merely the magnetic force of the earth proper, but also that of any magnetic bodies, such as iron gas-pipes or steam-pipes, that may be near the galvanometer may be very different at different parts of the same room. This fact must be taken into account in careful galvanometric work.

For many kinds of work (see Exercises 56 and 57) a far more sensitive galvanometer is needed than the instrument thus far employed in this course. So-called "*astatic*," that is, *unstable*, *sensitive*, galvanometers make use of a device which practically reduces the earth's directive force almost or quite to zero.

This device consists of two magnetic needles of nearly equal power, placed parallel to each other and so connected that they must turn as one, but pointing, with like poles, in opposite directions. These magnets are usually placed one above the other, one being inside the coil of the galvanometer and the other outside, as in Fig. 290.

Such a pair of magnets, if they are perfectly equivalent to each other, will have no more tendency, as a whole, to point north and south than to point east and west. It will point in any horizontal direction under the influence of a very slight force.

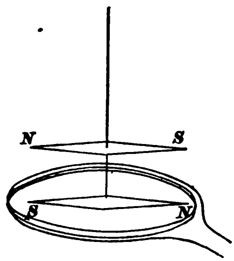

FIG. 290.

In fact, one of the magnets is usually a little stronger than the other, so that the pair points naturally in some particular direction, but much less decidedly than one needle alone would.

When a current goes through the galvanometer-coil this current has its full effect upon the magnet inclosed by it. The effect upon the outside magnet is less, but by recalling the observations of Exercise 53, and remembering that the two magnets point with like poles in opposite directions, one can see that the action upon the outside magnet works *with*, not in opposition to, the action upon the inside magnet.

The windings of the astatic galvanometer-coil are usually very many and very near the inner magnet, for the purpose of increasing the sensitiveness. If the poles of an ordinary galvanic cell were applied directly to the ends of such a galvanometer-coil, without any other resistance, the current through the coil would probably be powerful enough to change the magnetism of the inner needle, thus destroying the equilibrium of the two needles, and greatly reducing the sensitiveness of the instrument. Care must be taken to prevent accidents of this kind.

Electromotive Force.

414. Electromotive Force of a Battery.—The power which a galvanic cell has of charging one of its terminals positively and the other negatively (p. 503), so that a current

of electricity will flow from one to the other when they are connected by a wire, is called its *electromotive force*.

The electromotive force, which we shall now call the e. m. f., of a cell or battery of cells in open circuit can be measured by the difference of potential (§ 394) which it can produce between two metal plates connected with its positive and negative poles respectively. (See Fig. 291.)

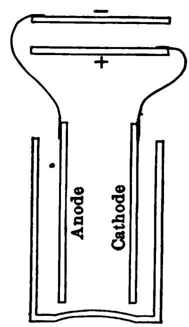

FIG. 291.

The e. m. f. of a cell or battery in closed circuit can be measured by the strength of the current which it can maintain in a circuit of given *resistance* (§ 420).

Owing to polarization (§ 406), and perhaps to some other causes, the e. m. f. of a battery in closed circuit is usually somewhat less than its e. m. f. in open circuit.

The e. m. f. of a cell depends upon its chemical composition and, to some extent, upon its physical condition. The first of the following experiments will tend to show whether it depends upon the size of the cell.

EXPERIMENTS.

(1) Take one very small Daniell cell and one very large one (see

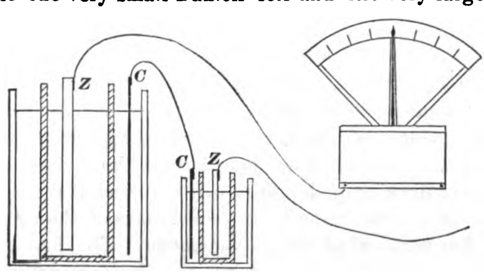

FIG. 292.

Fig. 292). Join together the copper terminals of the two cells. Then connect the zinc terminal of one cell with one terminal of the

lecture-table galvanometer, and the zinc terminal of the other cell with another terminal of the galvanometer, thus forming a circuit consisting of the two cells, *opposed to each other*, and the galvanometer-coil windings. Is there now evidence of any current through the galvanometer? What does the experiment indicate as to the comparative electromotive force of the two cells?

The following experiments will give some idea of the electromotive force of various combinations of galvanic cells:

(2) Take three similar Daniell cells. Connect the copper of one with the zinc of a second, and the copper of the second with the copper of a third, as in Fig. 293. Connect the zinc of the first cell

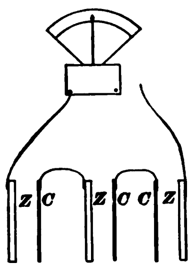

FIG. 293.

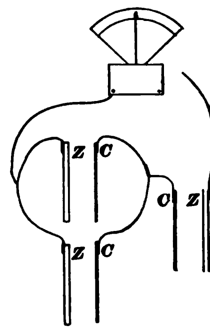

FIG. 294.

and the zinc of the third with the terminals of the galvanometer. With this arrangement the first and second cells, *connected in series*, as the phrase is, and so working in the same direction, are opposed by the third cell. Do they prevail over it?

(3) Join the zinc of one cell to the zinc of a second, as in Fig. 294, and the copper of the first cell to the copper of the second. Connect the two coppers thus joined with the copper of a third cell. Connect the zinc of this third cell with one of the terminals of the galvanometer, and connect the joined zincs of the first two cells with the other terminal of the galvanometer. The first and second cells are now working together, but side by side, rather than, as before, end to end. They are now said to be joined in *parallel*, or *in multiple arc*, or *abreast*. They are opposed by the third cell. Do they prevail over it?

Try four or five cells joined abreast against a single cell, and note the result.

The preceding experiments point to several conclusions which more careful experiments would justify:

1. *The e. m. f. of a cell does not depend upon its size.*

2. *The e. m. f. of n similar cells in series is n times as great as that of one cell alone.*

3. *The e. m. f. of n similar cells in multiple is equal to that of one of the cells alone.*

415. Unit of E. M. F.: the Volt.—*The ordinary unit of electromotive force is called the* VOLT, in honor of Volta (§ 402). The electromotive force of an ordinary Daniell cell is about 1.1 volts.

Instruments now very commonly used to measure e. m. f. are called *voltmeters.*

416. Other Sources of Electromotive Force.—The battery is not the only seat of e. m. f.

Thermo e. m. f. is due to differences of temperature in a circuit composed of more than one material. (See the experiment of § 347.) The e. m. f. of a dynamo is due to *induction*, so called. (See § 437.)

Work of the Electric Current.

417. Analogy between Water-power and Electric Power.—When two points some distance apart along a wire through which an electric current is driven by a battery are connected with a sufficiently sensitive electrometer, or voltmeter, it is found that they differ in potential.

A steady electric current flowing from the point of higher potential to the point of lower potential is in some respects like a stream of water flowing down-hill with unchanging velocity. In such a stream gravity does work upon the descending water, and this work is spent in overcoming friction, thereby producing heat, or, perhaps, in driving

machinery. So upon the electrical stream work is done between the two points, and this work may take the form of heat, in the wire, or mechanical energy in an electric motor (§ 432).

One very important practical difference between water-power and electric power must be borne in mind. The former power we find ready made by the natural processes of evaporation and rainfall. The latter power we do not find ready to hand in any usable form and quantity. We have to manufacture it at the expense of other power when we would use it. (See § 439.) A thermo-electric current uses up heat.

418. The Watt; Electrical Horse-power.—An electric current of one ampere, flowing between two points the difference of potential of which is one volt, absorbs electrical energy between those two points, and yields heat or some other form of energy, at a rate called one *watt* (from Watt, see § 344). *Seven hundred and forty-six watts are equivalent to one horse-power* (§ 266).

This method of reckoning electrical work is very common among those who have much to do with dynamos and motors.

PROBLEMS.

(1) A battery, the e. m. f. of which is 40 volts, sends a current of 100 amperes through a certain circuit. What is the number of watts? What is the rate of work in horse-powers?

(2) A battery working at the rate of 3 horse-power has an e. m. f. of 80 volts. What is the strength of the current?

(3) What must be the e. m. f. of a battery which maintains a current of 10 amperes, and works at the rate of 21 watts?

XXXII

CHAPTER XXXIII.

RESISTANCE.

419. Ohm's Law.—Ohm, a German (1787–1854), showed that, other things remaining equal, the current through a given conductor is proportional to the electromotive force which is applied to the conductor. In other words, if E stands for the electromotive force driving a current C through a given conductor, *the ratio* $E \div C$ *is a constant quantity for that conductor, so long as its physical condition remains unchanged.*

This ratio was taken by Ohm, and has been taken generally since his time, as the numerical measure of the conductor's *resistance.*

The numerical relations of the three quantities L, C, and R may be expressed by the equation

$$R = \frac{E}{C},$$

which, with its other forms,

$$C = \frac{E}{R} \quad \text{and} \quad E = R \times C,$$

is called *Ohm's Law.*

For example, if a battery having an electromotive force of 5 volts is placed in a circuit the total resistance of which is 100 ohms, the strength of current will be $5 \div 100 (= \frac{1}{20})$ ampere.

420. Unit of Resistance, the Ohm.—*The ordinary unit of resistance is called the* OHM, in honor of the investigator Ohm.

The so-called *legal ohm*, which probably differs a little from the correct unit, is a resistance equal to that of a column of pure mercury, at 0° C., 106 cm. long and 1 sq. mm. in area of cross-section.

421. Resistance of a Conductor; Specific Resistance.—The resistance of a conductor depends upon its material, its dimensions, and, in most cases, upon its temperature.

The *specific resistance* of a given material is the name given to the resistance of a centimeter cube of that substance at a particular temperature. A measurement of the specific resistance of copper will be attempted in Exercise 56.

A measurement of the rate of change of the resistance of copper wire with rise of temperature will be attempted in Exercise 57.

The resistance of a conductor is proportional to its length, other things being equal. In Exercise 54 the effect of length upon resistance will be observed in a certain fashion, German-silver wires of various lengths, but all of the same cross-section, being introduced into the circuit in turn.

In Exercise 55 a German-silver wire of greater diameter will be used, and by a comparison of Exercises 54 and 55 the effect of area of cross-section upon diameter should be indicated. Exercise 55 will consider also the resistance of two similar German-silver wires placed side by side and connected at their ends.

The method of comparison used in Exercises 54 and 55, where the various resistances are used in turn, other things remaining unchanged, is called the method of *substitution*, one resistance being substituted for another.

EXERCISE * 54.

RESISTANCE OF WIRES BY SUBSTITUTION: DIFFERENT LENGTHS.

Apparatus: A Daniell cell. A galvanoscope. A commutator. Five pieces of No. 30 German-silver wire, all of the same quality, one 200 cm. long, one 160 cm., one 120 cm., one 80 cm., one 40 cm., each suitably wound on a spool (see 103A). Two double binding-screws.

Put the zinc into the porous cup, and fill the latter with dilute acid to the usual level. *Let this cup stand outside the glass jar till a film of moisture appears upon its outside, showing that the liquid has filled its pores* (see *note* at the end of Exercise 52). Then finish putting the cell together in the usual way.

Place the galvanoscope, as usual, with the plane of its windings north and south, the needle pointing to the 0-pt. of the compass.

Connect the cell, the 15 turns of the galvanoscope, and the longest German-silver wire, in circuit, introducing the commutator in such a

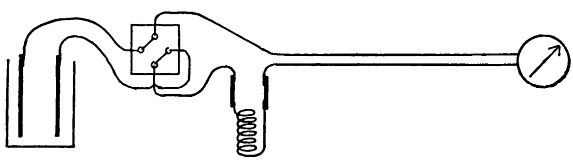

FIG. 295.

way that the current through the galvanoscope may be easily reversed (see Fig. 295).

Read very carefully and record the position of the needle, tapping the instrument lightly, as usual; then, by means of the commutator, reverse the current through the galvanoscope, and again read and record.

Make similar arrangements and readings with each of the other pieces of German-silver wire in turn, proceeding from the longest to the shortest.

If time permits, after making observations with the shortest wire, use each of the others again in turn, ending with the longest.

Take the mean of all the deflections obtained with each wire as the true deflection with that wire.

* The present form of this Exercise and the next is due mainly to Mr. C. C. Hyde, at one time my laboratory assistant.—E. H. H.

On a piece of coordinate paper, ruled in very small squares, measure off horizontal distances, as in Fig. 296, to represent the various lengths of wire, and above the points marked 40, 80, etc., place dots at vertical distances representing the deflections corresponding to the various wires. Then through these points draw a curve of such a character, *A* or *B*, as the facts require.

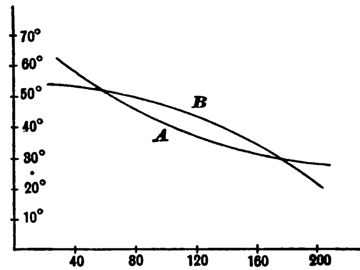

FIG: 296.

By measurement from any point on the base-line up to this curve one can now find what deflection would correspond to a wire of any given length between 40 cm. and 200 cm. This diagram will be of use in connection with the next Exercise.

NOTE.—Each porous cup used in this Exercise should be marked, so that the student who uses it may find and use the same cup in the next Exercise. The Daniell cells will probably differ mainly in the quality and thickness of their cups; and it is highly important to have the cell in the same condition, as nearly as may be, in Exercises 54 and 55.

EXERCISE 55.

RESISTANCE OF WIRES BY SUBSTITUTION: CROSS-SECTION AND MULTIPLE ARC.

Apparatus: The Daniell cell used in Exercise 54. A galvanoscope. A commutator. A spool with 200 cm of No. 28 German-silver wire. Two spools with 200 cm. each of No. 30 German-silver wire. Two triple binding-screws (though double ones will serve, if the holes are large enough to take in two of the spool terminals side by side). Screw calipers.

Set up the cell as in Exercise 54, and arrange the circuit as in Fig. 295, introducing the spool of No. 28 wire. Make careful observations, with the commutator in both positions, and take the mean of all the deflections.

Find by examination of the diagram of Exercise 54 what length of No. 30 German-silver wire would correspond to the same deflection,

that is, would have the same resistance as the 200 cm. of No. 28 German-silver wire.

Measure carefully by means of the screw calipers the diameter of the No. 30 and of the No. 28 wire, and calculate the ratio of their areas of cross-sections. Compare this ratio with the ratio of equivalent lengths of these two wires.

Remove the No. 28 wire from the circuit, and put in its place the two spools of No. 30 wire joined in *parallel*, or *multiple*, as in Fig. 297, so that the current, as indicated by the arrow-heads, will be divided equally between them. With this arrangement make observations as in the preceding case, taking finally the mean of the deflections recorded.

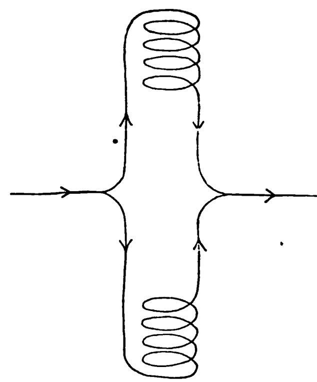

FIG. 297.

Find by examination of the diagram of Exercise 54 what length of No. 30 wire, single, corresponds in resistance with the two 200-cm. pieces of No. 30 wire, as now arranged.

422. Dimensions and Resistance.—The relations already stated or intimated between dimensions and resistance may be summarized as follows: *Other things being equal, the resistance of a conductor is proportional to its length and inversely proportional to its area of cross-section.*

PROBLEMS.

(1) If the resistance of 45 m. of copper wire 1 sq. mm. in cross-section is 1 ohm, what is the resistance of 90 m. of like wire 4 sq. mm. in cross-section.

(2) If the resistance of 2.5 m. of G. s. wire 3 sq. mm. in cross-section is 0.4 ohm, what is the resistance of 10 m of like wire 0.5 sq. mm. in cross-section ?

(3) If the resistance of 10 m. of wire 1 mm in diameter is 3 ohms, what is the resistance of 25 m. of like wire 2 mm. in diameter ?

(4) How long must a piece of wire of the same quality as that of

the preceding problem be, in order that its resistance may be 15 ohms, the diameter being 0.15 cm.?

423. Conductance: Resistance of Conductors in Parallel.—It may have been inferred from Exercises 54 and 55 that two equal wires joined "in parallel," or "in multiple," have a resistance one half as great as that of either wire alone.

More extensive and accurate experiments confirm this inference, and show that, *If n equal resistances are joined in parallel with each other, the resistance of the combination is equal to one of the single resistances divided by n.*

PROBLEMS.

(1) If the resistance of a certain wire is 12 ohms, what would be the resistance of three such wires joined in *series?* joined in *parallel?*

(2) If the resistance of an electric arc lamp is 5 ohms, what would be the resistance of six such lamps joined in *series?* joined in *multiple?*

We must now consider what the result is if the various resistances joined in parallel are not equal among themselves. We will attack the subject in the form of problems.

(3) If two wires, one having a resistance of 20 ohms, the other a resistance of 5 ohms, are joined in parallel, what is the resistance of the combination?

Solution: The 5-ohm wire is equivalent to four 20-ohm wires joined in parallel. The whole combination is therefore equivalent to five 20-ohm wires joined in parallel. Its resistance is, therefore, $20 \div (1 + 4) = 4$ ohms.

Observe, for use below, that this is the same as $1 \div (\frac{1}{20} + \frac{1}{5})$.

(4) What is the combined resistance of two wires, in parallel, of 17 ohms and 4 ohms respectively?

Solution: The 4-ohm wire is equivalent to four and one-quarter 17-ohm wires, joined in multiple, the *quarter* meaning a wire as long as the 17-ohm wire and one-fourth as large in cross-section. The whole combination is equivalent to five and a quarter of the 17-ohm wires joined in multiple. Its resistance is, therefore, $17 \div (1 + 4\frac{1}{4}) = 3.24 -$ ohms. Observe that this is the same as $1 \div (\frac{1}{17} + \frac{1}{4})$.

In both of these cases, as we see, the correct answer can be obtained *by dividing 1 by the sum of the reciprocals of the single resistances*, and if we were to consider more complicated cases, with any number of wires, we should find the same statement to hold true in all of them.

It is evident, then, that some shorter name for "the reciprocal of a resistance" is needed. The name used is the *conductivity*, or *conductance*. *The* CONDUCTIVITY, *or* CONDUCTANCE, *of a conductor or combination of conductors is the reciprocal of the* RESISTANCE.

The resistance of any number of conductors joined *in series* is merely the sum of their separate resistances. The resistance of any number of conductors joined *in parallel* is found by taking the sum of their separate conductances and dividing 1 by this sum.

(5) What is the resistance of two conductors in parallel, the separate resistances of which are 4 ohms and 12 ohms? 6 ohms and 11 ohms?

(6) What is the resistance in multiple of four wires, the separate resistances of which are 4, 6, 8, and 10 ohms? *Ans.* 1.56— ohms.

424. Resistance-coils. — It is frequently necessary in electrical work to introduce resistance into a circuit, and so-called *resistance-coils* are part of the ordinary equipment of a physical laboratory. They consist of spools upon which are wound pieces of silk-covered or cotton-covered wire, long or short, thick or thin, according to the particular service each coil has to perform. To avoid magnetic action from these coils the wire is wound upon them in such a way that half the turns carry a current in one direction and half in the other direction. This can be easily done if the wire is doubled before it is wound on the spool.

Much attention has been given to the choice or invention of materials for resistance-coils. The qualities most needed are constancy of resistance (through changes of temperature,

lapse of time, etc.), absence of troublesome thermoelectric * qualities, high specific resistance (§ 421), and convenience of working.

Certain alloys of two or more metals are found to have very great specific resistance and to change in resistance far less with change of temperature than most pure metals. Accordingly, alloys are generally used in the construction of resistance-coils. German-silver, the specific resistance of which is about thirteen times as great as that of copper and increases about 1 per cent, with a rise of 23° C., has been, and still is, very commonly employed. But there are other alloys, notably *manganin*, composed of copper, manganese, and nickel, that are now preferred to German-silver for the best work. The resistance of manganin changes far less than that of German-silver with change of temperature.

A set of resistance-coils is usually arranged in a wooden case with some convenient means for putting any or all of them into or out of the circuit. A set of resistances so encased is called a "resistance-box."

425. Theory of Wheatstone's Bridge.—The comparison of resistances by "substitution," as in Exercises 54 and 55, is simple in theory and is sufficient for some purposes; but far more accurate measurements of resistance can be made by means of a device called *Wheatstone's bridge.*

This latter method requires the use of an astatic galvanometer (§ 413), and of one or more conductors the resistance of which is already known. Such conductors are usually in the form of resistance-coils of any convenient number of ohms.

The following experiment with a simple form of Wheatstone's bridge is intended to develop the theory of its use.

* A beam of sunlight falling upon a junction of dissimilar metals may utterly derange delicate electrical experiments.

EXPERIMENT.

(Preliminary to Exercise 56.)

Stretch a piece of No. 30 uncovered copper wire from *a* to *b* of No. 106 (Fig. 299), fastening it carefully beneath the washers under the lower nuts.

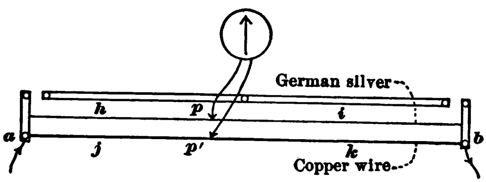

Fig. 298.

Connect with the astatic galvanometer (Fig. 299) two thin covered wires each about 1 m. long.

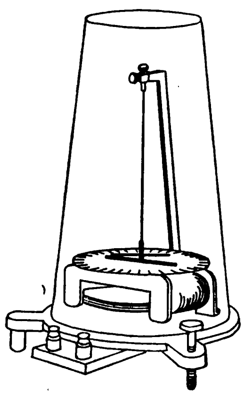

Fig. 299.

Send the current from a Daniell cell in at *a* and out at *b*. This current divides, a part going through the G. s. wire, but the larger part through the parallel copper wire.

Hold one of the galvanometer-wires against any point of the G. s. wire, at *p* for instance, and then touch the other galvanometer-wire to various points on the copper wire, until a point of contact, *p'*, is found that gives no current through the galvanometer, the contact at *p* being all the while maintained.

The two points *p* and *p'*, between which no current flows when they are connected, are called a pair of *equipotential points* (§ 394). Let the parts into which *p* divides the G. s. wire be called *h* and *i*, and the parts into which *p'* divides the copper wire be called *j* and *k*.

Experiment with various pairs of equipotential points (none of which should be very near the ends of the wires, because of certain imperfections of the apparatus at these ends) until it becomes clear that in every case

$$h : i :: j : k.$$

Call the *resistances* of the parts h, i, j, and k, respectively, H, I, J, and K. Then, from the known relation between resistance and length, § 422, we have

$$H : I :: J : K, \quad \text{and} \quad h : i :: J : K,$$

whence

$$\frac{J}{K} = \frac{h}{i}.$$

This shows that, if we know the number of ohms in either J or K, and know the ratio of the two *lengths* h and i, we can at once find the number of ohms in the other resistance, K or J. This is the theory of the Wheatstone-*bridge** method of measuring resistances.

In the further use of the bridge the copper wire from a to b is removed.

EXERCISE 56.

MEASUREMENT OF RESISTANCE WITH THE WHEATSTONE BRIDGE.

Apparatus: A Daniell cell. A Wheatstone bridge (No. 106). An astatic galvanometer (No. 107). The 20-m. copper coil (No. 103c). A box of resistance-coils (No. 108), with values given in ohms, or at least some known resistance of about 5 ohms.

Introduce the copper coil, the resistance of which is to be measured, as J, between the binding-posts d and e in Fig. 300. Introduce a known resistance as K, between the posts f and g, in the same figure. Connect one wire from the galvanometer to the binding-post m. Connect the other galvanometer-wire to the piece that makes sliding contact with the straight G. s. wire, setting the contact at first at the middle point of this wire.

Attach one of the cell-wires to the binding-post a. *Touch* the other cell-wire for an instant to the post b, and note the effect upon the galvanometer. If the needle does not move, it is probable that the contact of the slider with the G. s. wire is not good. If the needle does move, observe the direction and magnitude of its motion.

* Wheatstone was the inventor of the method. The wire leading across from p to p' was originally called a bridge. The main apparatus is now called a bridge.

Move the sliding contact to some new position, and again note the effect upon the galvanometer when the cell wire touches b for an instant. Experiment in this way till some point p is found which gives no perceptible current when contact is made and continued for a second or two at b. (*Prolonged* contact at b will allow the copper coil to

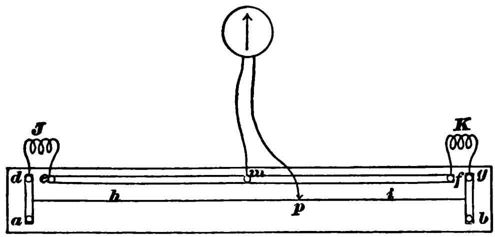

FIG. 300.

be heated by the current and suffer change of resistance.) It is best, after a little experimenting, to introduce such a resistance at K that the equilibrium point p will be not far from the middle of the wire.

The distances h and i are now read. Observing that the resistances of the thick copper strips of the bridge are very small compared with J and K, we have

$$\frac{J}{K} = \frac{h}{i}, \quad \text{or} \quad J = K \times \frac{h}{i}.$$

The resistance of the copper wire is thus found in ohms. To find its specific resistance we must take account of its *length*, 20 m., and its diameter, which may be found by the gauge.

426. Temperature-coefficient of Resistance.—The electrical resistance of most conductors changes with change of temperature. In most metals, and alloys of metals thus far tested, the resistance increases with rise of temperature. The exact relation between the two changes through a great range of temperature is probably very complicated, but for many purposes it is near enough to the truth to assume that the resistance increases *in proportion* to the increase of temperature. This may be put into mathematical form thus:

$$R_2 = R_1 + \gamma R_1(t_2 - t_1) = R_1(1 + \gamma(t_2 - t_1)),$$

where R_1 = the resistance at temperature t_1,
and R_2 = " " " " t_2.

The factor γ, which is different for different metals, is called the *temperature-coefficient* of resistance. Solving for γ, we have

$$\gamma = (R_2 - R_1) \div R_1(t_2 - t_1).$$

PROBLEMS.

(1) If the resistance of a certain G. s. wire is 15 ohms at 0° C., what is its resistance at 70° C., the temperature-coefficient being .00044?

(2) If the resistance of a certain silver wire is 20 ohms at 0° C., and 25.7 ohms at 75° C., what is the temperature-coefficient?

EXERCISE 57.

CHANGE OF RESISTANCE WITH CHANGE OF TEMPERATURE.

Apparatus: A Daniell cell. A Wheatstone bridge. An astatic galvanometer. A known resistance of 1 or 2 ohms. A "temperature-coil" (No. 109). A vessel of ice-water. The boiler (No. 80), without top.

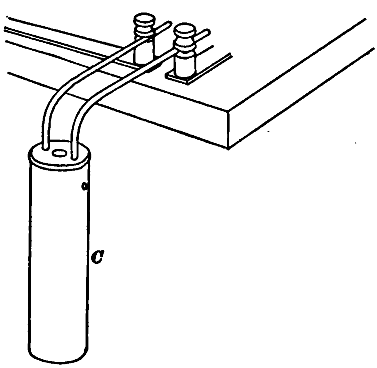

Fig. 301.

The unknown resistance J (see preceding Exercise) is now the "temperature-coil," which is attached to the Wheatstone bridge in the manner indicated by Fig. 301.

Measure carefully the resistance of this coil while it is in ice-water. Measure its resistance again with equal care while it is in the boiling water.

Calculate the temperature-coefficient of resistance.

427. Battery Resistance.—From what we have already learned concerning resistance it is possible to make a shrewd guess as to the effect, upon battery resistance, of changes in size or distance of plates, or combination in series or in multiple. Nevertheless, the following Exercise, in which these matters are touched upon, will not be unprofitable.

EXERCISE 58.

BATTERY RESISTANCE.

Apparatus: Two similar Daniell cells. A galvanoscope. A commutator. A resistance-box.

Allow the pores of the cups to fill with liquid (see Exercise 54) before the cups are placed in the jars.

Variation of Effective Plate-area.—Put one of the cells in circuit, through the commutator, with the 5-turn section of the galvanoscope.

Keeping the porous cup in contact with one side of the jar, and the copper and zinc as near together as may be (see Fig. 302), make and record the reading, then reverse the commutator and read and record again.

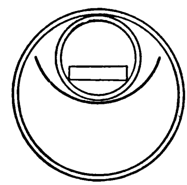

FIG. 302.

Without changing the distance between the copper and the zinc, raise both of them till they dip about 1 cm. only below the surface of the liquids. While they are in this position, read, reverse, and read again.

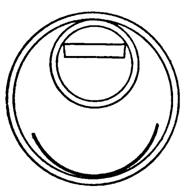

FIG. 303.

Variation of Distance between Plates.—Keeping the porous cup in contact with the side of the jar, put the plates as far apart as may be (see Fig. 303), and while they are in this position read, reverse, and read again.

Cells in Parallel.—Join the two cells in parallel, zinc to zinc and copper to copper (see Fig. 304), and put them thus joined into the circuit, through the commutator and the resistance-box, with the 5-turn section of the galvanoscope.

Adjust the resistance-box so that none of its coils shall be in the circuit; read, reverse, and read again.

Introduce a resistance of 5 or more ohms, read, etc.

Cells in Series.—Join the two cells in series, other arrangements being as before.

Take readings with no coils of the box in circuit.

Take readings with the same coils in circuit that were used with the cells in parallel.

Under what conditions does the multiple, or parallel, arrangement of the cells give a stronger current than the series arrangement?

Under what conditions does the series arrangement give a stronger current than the multiple arrangement?

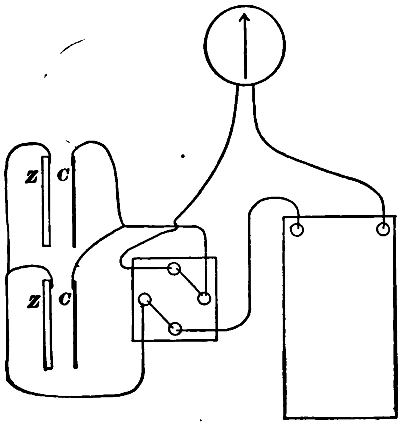

FIG. 304.

428. Resistance of Cells in Combination; Resistance of the Whole Circuit.—The fact is, that *The resistance of n similar cells in series is n times as great as that of one of the cells alone*, and

The resistance of n similar cells in multiple is equal to $\frac{1}{n}$ *that of one cell alone.*

If the resistance of a battery is called R_b, and that of the part of the circuit external to the battery is called R_e, Ohm's law (§ 419) takes the form

$$C = \frac{E}{R_e + R_b}.$$

Observe that if R_e is very large in comparison with R_b—one hundred times as large, for instance,—doubling or trebling R_b will make but little difference in the value of C.

But if R_e is not large compared with R_b, doubling or trebling the latter will make a great difference in the value of C.

Recall the fact (§ 414) that combining cells in *series* makes the e. m. f., E, of the battery proportional to the number of cells, while combining them in multiple leaves the e. m. f. of the battery equal to that of one cell.

Accordingly, when R_b is very small compared with R_e, the current C will be greatest when the cells are joined in series. See Exercise 58.

If R_b is large compared with R_e, it may be of advantage to combine the cells in multiple, sacrificing E, to some extent, for the sake of reducing greatly the resistance of the circuit. See Exercise 58.

When one has a given number of similar cells to be arranged at will for the purpose of sending the maximum current through a certain known external resistance, the proper rule to follow (of which no proof will be here given) is this: *Join the cells in such a way as to make the resistance of the battery equal, as nearly as may be, to the resistance of the external part of the circuit.*

This rule will sometimes require all the cells to be arranged in series, which will make the battery resistance a maximum, but it must not be supposed that this arrangement is adopted *for the sake* of making the battery resistance large. It is adopted for the sake of making the *electromotive force* large, and *in spite of* the fact that it increases the battery resistance.

To illustrate the working of the rule, let us suppose that we have to arrange 12 cells, each having an electromotive force of 1 volt and a resistance of 1 ohm, in such a way as to send the strongest current through an external resistance of 3 ohms. With the cells all in series we have

$$C = \frac{E}{R_e + R_b} = \frac{12}{3 + 12} = \frac{4}{5} \text{ (ampere)};$$

NP 504

with the cells 2 abreast and 6 in series,

$$C = \frac{6}{3 + 3} = 1 \text{ (ampere)};$$

with 3 abreast and 4 in series,

$$C = \frac{4}{3 + \frac{4}{3}} = \frac{12}{13} \text{ (ampere)};$$

with 4 abreast and 3 in series,

$$C = \frac{3}{3 + \frac{3}{4}} = \frac{4}{5} \text{ (ampere)};$$

with 6 abreast and 2 in series,

$$C = \frac{2}{3 + \frac{1}{3}} = \frac{3}{5} \text{ (ampere)};$$

with all the cells abreast,

$$C = \frac{1}{3 + \frac{1}{12}} = \frac{12}{37} \text{ (ampere)}.$$

We have in this discussion assumed the resistance of a cell to be a fixed quantity. The fact is, however, that the resistance of a cell, like its electromotive force, is somewhat variable, being dependent upon the strength of current and certain other conditions.

PROBLEMS.

(1) If the resistance of a single galvanic cell is 0.8 ohm, how great is the resistance of 4 such cells joined "in series"?

(2) What would be the resistance of 4 of these cells joined abreast, or in multiple arc?

(3) What would be the resistance of the same number of these cells joined in a square, two cells wide and two cells long?

(4) If each of these cells had an electromotive force of 1.8 volts, what would be the electromotive force of each of the combinations mentioned in the three preceding problems?

(5) If each combination in turn were used in a circuit having an external (outside the battery) resistance of 100 ohms, how great would the current be in each case?

(6) How great would the current be in each case if the external resistance were 1 ohm?

(7) How great would the current be in each case if the external resistance were 0.1 ohm?

(8) If 6 cells, each having an e. m. f. of 2 volts and a resistance of 0.5 ohm, were joined in series in opposition to a series of 12 cells, each having an e. m. f. of 1.5 volts and a resistance of 1 ohm, the external resistance being 2 ohms, which set of cells would prevail, and how great would the current be? (The e. m. f. of the whole is the difference of the e. m. f.'s of the two sets of cells. The total resistance is the same as if all the cells were directed the same way.)

(9) If an arc-lamp has a resistance of 5 ohms and requires a current of 10 amperes, what e. m. f. does it require?

(10) If a sixteen-candle power incandescent electric lamp requires a current of 0.5 ampere and an e. m. f. of 100 volts, what is its resistance?

(11) If the e. m. f. available for an electric circuit is 2000 volts, and if the resistance of a single arc-lamp is 5 ohms, how many such lamps placed in series would reduce the current to 10 amperes, the proper strength for such lamps?

(12) If an incandescent lamp has a resistance of 200 ohms and requires a current of 0.5 ampere, how many cells connected in series, each having an e. m. f. of 1 volt and a resistance of 0.5 ohm, would be required to operate it?

(13) If an arc-lamp having a resistance of 5 ohms requires a current of 10 amperes, would 1000 cells like those of the preceding problem. arranged 100 in series and 10 abreast, be more or less than sufficient to operate it?

CHAPTER XXXIV.

ELECTROMAGNETISM AND INDUCED CURRENTS; IMPORTANT APPLICATIONS OF ELECTRICITY AND MAGNETISM.

Electromagnetism.

429. Discovery.—Oersted of Copenhagen (1777–1851) discovered the directive action of the electric current upon the magnetic needle in 1820. In the same year Arago, a Frenchman (1786–1853), and Davy (§ 285) discovered that an electric current can produce magnetization in soft iron. These discoveries have been followed up by a multitude of experimenters, and now *electromagnets*, that is, masses of iron or steel made magnetic by the passage of an electric current through a wire coiled about them, are familiar objects to nearly everybody.

EXPERIMENT.

Connect the poles of a galvanic cell by means of a piece of uncovered copper wire and dip one part of this wire into a mass of iron filings. It was in this way that Arago made his discovery.

Show the magnetizing action of the current by passing it through a coil of wire surrounding a core of soft iron.

430. Electromagnetic Telegraph.—The essentials of a telegraphic outfit are very simple.

The *battery* may be of one or more galvanic cells, according to the resistance of the circuit.

The *sounder* (Fig. 305) consists of an electromagnet, m, a lever l, pivoted at p and carrying a piece of soft iron i, which iron, attracted by the electromagnet, is drawn down-

ward, stretching the spring *s*, and bumping the end of the lever against the lower stop. It is this bumping which gives the familiar click of the instrument in operation and justifies its name.

Two clicks coming close together are called a *dot*, two clicks about twice as far apart are called a *dash*. Various

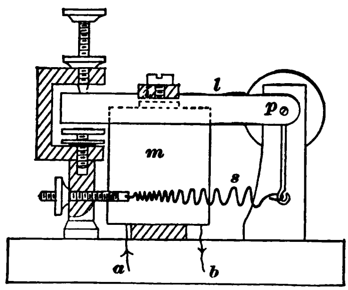

FIG. 305.

groups of dots and dashes indicate the letters of the alphabet. In the following table a dot is indicated thus, -, and a dash thus, —. The table is called Morse's alphabet, from its inventor, an American (1791–1872), who was one of the foremost leaders in the development of telegraphy.

A - —	J - — — —	S - - -
B — - - -	K — - —	T —
C — - — -	L - — - -	U - - —
D —- - -	M — —	V - - - —
E -	N —- -	W - — —
F - - — -	O — — -—	X — - - —
G — — -	P - — — -	Y — - — —
H - - - -	Q — — - - -.	Z — — - -
I - -	R - — -	

This alphabet is still the international code.

The alphabet now used in this country differs from that of Morse in the following cases:

C - - -	O - -	X - — - -
F - — -	P - - - - -	Y - - - -
J — - — -	Q - - — -	Z - - - -
L ——	R - - -	

The *key* (Fig. 306) consists of a lever pivoted at *p*, subject to a spring *c*, operated by the hand pressing down the knob *k* against the stop *m*, which operation closes the cir-

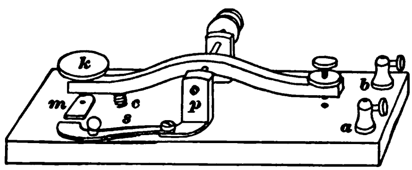

Fig. 306.

cuit. The action of the spring makes it possible to work the lever up and down rapidly.

When not in use the key is *shunted*, that is, *side-tracked*, by a metal bar *s*, called a *switch*, connecting *m* and *p*.

The arrangement of a short telegraphic line is indicated in Fig. 307, where the left-end switch is open and the right-end switch is closed. The massage now enters at the left.

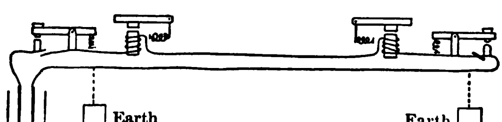

Fig. 307.

On permanent lines the return wire is dispensed with, connection being made with the earth at each end of the line, as the dotted lines of the figure indicate.

When a line is very long the battery at one end may not be sufficient to work the sounder effectually at the other end. In such a case it is made to operate an electromagnet

called a *relay*, slight pulsations of which bring into action another battery at the distant station, and the current from this second battery works the sounder.

EXERCISE 59.

TELEGRAPHIC SOUNDER AND KEY.

Put together, "assemble," the parts of a very simple telegraphic key and sounder, the latter delicate enough to be operated by the current from a single Daniell cell.

The key may be something like Fig. 308, where *b* is a strip of

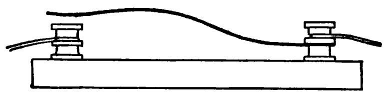

FIG. 308.

spring-brass, and the sounder may be correspondingly simple.

431. The Telephone.—The necessary apparatus of a telephone line is as simple as that of a telegraph line, but its adjustment is far more delicate and difficult.

In Fig. 309 *m* is a mouthpiece behind which, held rather loosely, is a diaphragm, *D*, of thin soft iron. Beyond the

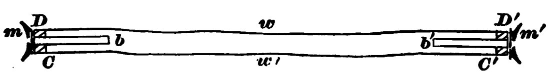

FIG. 309.

diaphragm is a steel magnet *b*, on the nearer end of which is a coil of fine wire *C*. At the other end of the line are the counterparts *m'*, *D'*, etc.

Each of the diaphragms lies all the time in a magnetic field. Any movement of the diaphragm *D*, for instance, makes a slight change in the strength or distribution of the lines of magnetic force in and about the end of the magnet *b*. This change *induces* (§ 437) a current of electricity in the coil *C*, and this current, passing also through *C'*, produces a slight magnetic change in or about the end of *b'*, and so causes a movement of the iron diaphragm *D'*.

The chief new wonder revealed by the telephone was, that movements so slight as those of the second diaphragm could produce audible sounds, and that these sounds could simulate and reproduce human speech.

The arrangement of apparatus shown in Fig. 309 can be used for short distances only. It consists of two Bell *receivers*, which are used also as *transmitters*. For long lines a Blake *transmitter* is used in conjunction with a battery and an induction-coil (§ 441).

The plan of the Blake transmitter is shown in Fig. 310, where *m* is the mouthpiece, *D* the metal diaphragm, between which and the soft carbon "button" *C* there is a

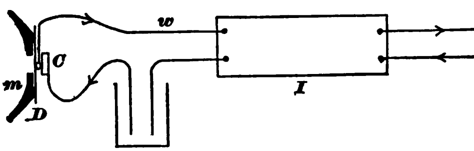

FIG. 310.

bit of platinum suspended in metallic connection with the wire *w*. *I* is an induction-coil. When the apparatus is not in use a small current of electricity runs from the battery through *C* to the platinum, which presses *C* lightly, then through *w* to the primary coil of *I*. The secondary of *I* is in connection, by means of two wires or one wire and the earth, with the distant telephone.

Speech directed against *D* varies many times a second the pressure of *D* against the platinum and of the platinum against *C*. This variation of pressure changes the resistance of the circuit, which resistance is largely at the point of contact with *C*, and so varies the strength of the current sent through the primary of the induction-coil. Accordingly, induction-currents, corresponding to the vibrations of *D*, are sent to the receiving telephone, which reproduces in speech what is spoken into the transmitter.

432. Definition of Electric Motors.—An electric motor is a machine for producing mechanical energy, or work, from the energy, or work, of electric currents.

The possibility of such a machine is shown by the cases already described in this book of magnets moved by electric currents. In fact, every galvanometer is in some sense an electric motor; but in the ordinary sense of the term an electric motor is a contrivance by means of which continuous rotation of a coil of wire carrying an electric current, or of a combination of magnets and coils, is maintained by current action.

The fact that this motion, which always accomplishes some mechanical work, requires the *expenditure* of electric energy and tends to weaken the electric current that maintains it, will be discussed later (§ 439).

Certain experiments will presently be described, which illustrate the development of electric motors from the stage of mere scientific interest to that of commercial importance.

433. An Elementary Motor.—Fig. 311 represents a coil of wire, C, mounted in such a way as to revolve freely between the poles, N and S, of a horseshoe magnet. The ends of the coil are soldered to plates of metal m and m', which are fastened to the upright axis a, but are insulated from it, that is, separated from it by some non-conductor of electricity. The arch at the top of the figure is of brass, and supports two binding-posts, p and p', which are insulated from it. From p and p' elastic strips of metal, s and s', reach to the plates m and m'.

A current of electricity entering at p will pass by s to m, then through c to m', and out by way of s' and p'. The current used with this apparatus may be supplied by one or more Daniell cells.

The line of the magnetic force at the centre of a coil due

to a current in the coil itself is (see Exercise 53) at right angles with the plane of the windings, and so directed that, to an observer looking *forward* along it, the current circu-

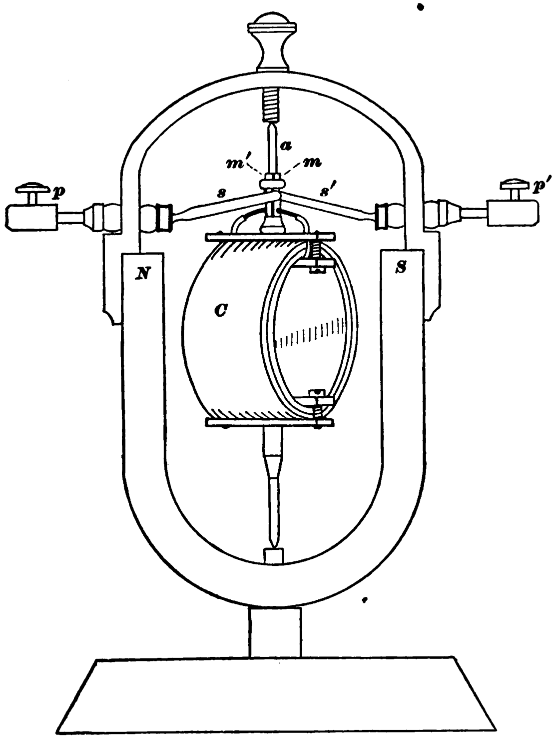

FIG. 311.

lates *clockwise* in the coil. This line of magnetic force will now be called the *axial magnetic line of the coil.*

The lines of magnetic force of the *magnet* are directed from *N* to *S*.

EXPERIMENT I.

Fasten back the springs *s* and *s'* so that they shall not touch *m* and *m'*. Setting the coil in various positions, touch the battery

wires to *m* and *m′*, and continue the experiments until the following facts are illustrated :

1st. *That there are two positions of the coil from which the current does not turn it, namely, those in which the axial magnetic line of the coil is parallel to the line NS.* These two positions of the coil may be called its dead-points.

2d. *That, if the coil is in any other position than one of its dead-points when the current is sent through it, the coil tends to turn in that direction which will bring it, by the shortest course, into the position where its axial magnetic line is in the* SAME DIRECTION *as the line NS, the line of the magnet's force.*

The coil can be made to rotate past its *dead-points* by its own momentum, and, if the contacts of the springs *s* and *s′* against *m* and *m′* are so contrived as to send the current in through *m* and *m′* alternately, the changes taking place at the dead-points, the coil can be kept in continuous rotation in one direction.

EXPERIMENT 2.

Release the springs *s* and *s′*, so that they will bear lightly against the plates *m* and *m′* respectively, and so adjust them that each spring will change from one plate to the other when the coil is at either dead-point.

Connect the wire from the copper end of the battery with *p*, the other wire with *p′*, and observe the effect. Then reverse the connections at *p* and *p′*, and again note the effect.

434. Electromagnetic Fields and Armatures.—The apparatus just used is an electric motor, with a *magnetic field* produced and maintained by a *permanent* magnet, and a revolving part, or "*armature*," containing no iron. The direction of rotation of such a motor depends upon the direction of the electric current.

As electromagnets (§ 429) are more powerful than permanent magnets, it is an obvious improvement to use an electromagnet to furnish the field, and this is done in all large motors. It is less obvious, but equally true, that there is a great advantage in furnishing the armature with

an iron *core* upon which the coil is wound, and this is usually done.

These changes give power, and they bring another advantage, namely, that the armature revolves always in the same direction without regard to the direction of the current. This fact can be shown by means of a simple form

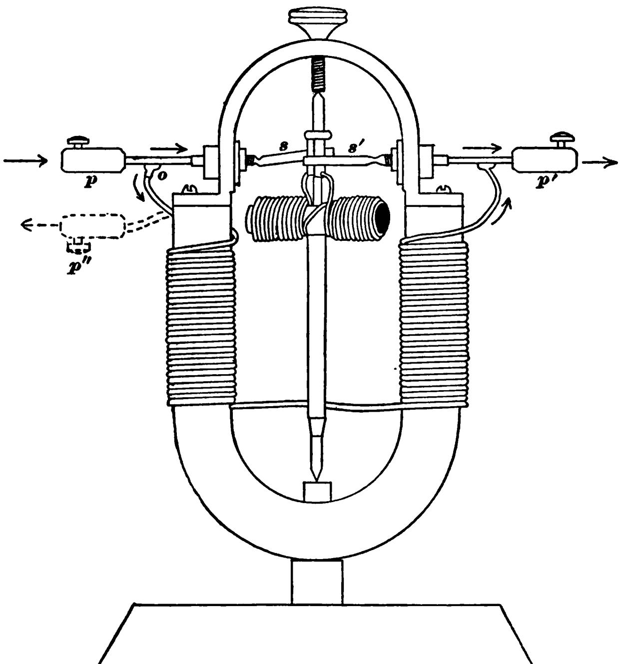

FIG. 312.

of apparatus like that of Fig. 312. In this apparatus the current entering at p divides at the point o, part going through the windings of the field-magnet to p', where it joins the other part, which has passed through the windings of the armature.

A conductor that leads off from the main circuit and returns to it farther on is called a *shunt*, and a motor like that of Fig. 312, in which the current is thus divided, is called "shunt-wound" motor. Motors for business purposes are usually shunt-wound.

It is easy to modify the connections of Fig. 312 (see dotted lines) so as to make the whole current go through the windings of the field-magnet and the windings of the armature. The motor is then said to be "series-wound."

The armature of the series-motor, like that of the shunt-motor, revolves in the same direction whether the current enters at p or at p', the fact being that reversing the current reverses the direction of the magnetic lines of the field and reverses the magnetic direction of the armature at the same time. If only one of these conditions were reversed the direction of rotation would be reversed.

EXPERIMENT.

Illustrate by means of Apparatus No. CXXIV the facts stated in § 434.

435. Ring Armatures and Drum Armatures.—The two plates m and m' on the axis of revolution in Fig. 311, by means of which, when in revolution, the current in the coil is reversed at every passage of a "dead-point," make what is called the *commutator* of the motor. Motors are usually made with a considerable number of coils, or "sections," set at different angles around the axis of revolution of the armature, and the commutator consists of a number of bars, one for each "section" of the windings, placed upon the axis.

Fig. 313 shows an armature with four sections, S_1, S_2, S_3, S_4, and four commutator-bars, or plates, b_1, b_2, b_3, b_4. The sections are wound on a ring of soft iron, and each section is connected directly by wires with two of the commutator-bars. Two conductors, called the "*brushes*,"

corresponding to the springs s and s' of Fig. 311, lead the current to the commutator, and take it away after it has traversed the sections of the armature. If the current is just now entering at the bar b_2, by way of the upper brush, it divides there, part going through sections S_1 and S_4, the other through sections S_2 and S_3, and these parts, uniting

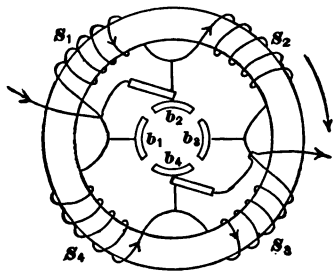

FIG. 313.

at the bar b_4, where the second brush touches, go out as one current.

If now the magnetic lines of the field in which the armature revolves are horizontal and extend from left to right, it will be seen, from what is said in § 433, that all the sections tend to rotate in the same direction, that is, clockwise.

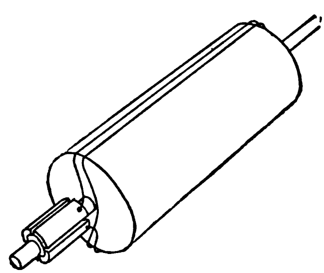

FIG. 314.

But the only way the sections have of rotating is to turn the whole armature about its axis, and this they do.

The form of armature shown in Fig. 313 is called a *ring* armature. The sections are, one may say, wound on the *rim* of a wheel, the soft iron ring.

Another form of armature has no ring, but rather a *hub* of soft iron upon which the sections are wound, as in Fig. 314. This form is called a *drum* armature. The principle of the two forms is the same.

EXPERIMENT.

Show in operation upon the lecture-table a small motor of industrial form.

EXERCISE 60.

ELECTRIC MOTOR.

Put together, "assemble," the parts of a small electric motor, to be driven by a single Daniell cell.

Induced Currents.

436. Number of Lines of Magnetic Force through a Circuit.—We have heretofore used the term *line of magnetic force* to indicate merely a line along which a compass will lead in a magnetic field, and, as the number of possible positions of the compass in any given cross-section of the field is countless, the number of such lines of force in any magnetic field, or even any small portion of it, is countless.

If, however, one undertakes to make a diagram of a magnetic field, as in Exercises 50 and 53, drawing in a certain number of continuous lines of force, one will find that these lines lie closest together in those parts of the diagram where the strength of the magnetic field is greatest. In fact, it is very convenient, and, so, very common, to speak of a magnetic field of unit strength as having one line of magnetic force per unit area, 1 sq. cm., of cross-section—to speak of a magnetic field of strength h as having h lines of magnetic force per unit area of cross-section, etc.

If, therefore, a loop of wire, inclosing an area of A sq. cm. is placed in a magnetic field of strength h, with its plane at *right angles* with the direction of the magnetic force, it is customary to say that the number of lines of magnetic force embraced by this coil is $A \times h$.

If the coil is placed with its plane *parallel to* the lines of force, it embraces none of the lines.

If the coil is so placed that its plane is neither at right angles with nor parallel to the lines of force, the number of lines embraced by it is something between 0 and $A \times h$.

Faraday invented the quantitative method of treating lines of magnetic force, and it is of very great convenience in summarizing the laws of induction of electric currents which he discovered.

437. Induced Currents.—Faraday discovered in 1831–32 that *when the number of lines of magnetic force embraced by a closed conducting circuit is increased or diminished, a current of electricity is set up in the circuit, which current is proportional to the rate at which the change is occurring, and lasts only so long as the change is in progress. A current produced in this way is called an* INDUCED CURRENT. *The direction of the induced current depends upon the direction, increase or decrease, of the change in the number of lines of force embraced.*

If there is already a current flowing in the circuit, the induced current is combined with the other, causing sometimes an increase and sometimes a decrease of the total current.

EXPERIMENT.

(1) Connect the binding-posts p and p' of the apparatus shown in Fig. 311 with the terminals of the astatic galvanometer. Then whirl the coil quickly about on its axis, first in one direction and then in the other, noting the effect upon the galvanometer.

Observe the direction of the induced current in each case, and compare it with the direction of the current which, if sent through the

armature from a battery, would cause rotation of the armature in the same direction.

(2) Try similar experiments with the apparatus of Fig. 312. (In this case it may, for success, be necessary to magnetize the iron of the "field" magnet independently, by holding the pole of a permanent magnet near one end of it, or by sending a current of electricity through a considerable number of extra turns of wire, wound upon it but not in circuit with the rest of the windings. It will hardly be practicable, in driving this apparatus by hand, to compare the direction of the induced current with that of a *current* which would drive the armature in the same direction, for the induced current may not be strong enough to affect the field-magnet perceptibly.)

438. Dynamos; Relation between Dynamos and Motors. —We have now seen a number of machines which, driven by the power of an electric current, produce mechanical power. When used in this way they are called *electric motors* (§ 432).

The experiments of the preceding Article illustrate the fact that when such machines are driven by mechanical power they produce a current of electricity. When used in this way they are called *dynamos*.

The terms *field*, *armature*, *commutator*, *brushes*, *series-wound*, *shunt-wound*, etc., have the same meaning when applied to dynamos as when applied to motors. This fact, however, is to be noticed, that in the shunt-motor the current from the outside line is divided between the field-magnet coils and the armature, while in the shunt-dynamos the current from the armature is divided between the outside line and the windings of the field-magnet.

439. Energy Absorbed in Motors and Dynamos.—When we see that a motor, driven by a current, yields mechanical energy, we may be sure that it absorbs electrical energy, since, on the whole, no machine can give out more energy than it takes in (§ 271); but it is not at first obvious how the current going through a motor in operation is doing

therein any more work than it would do if the motor were at rest.

The first experiments of § 437, however, illustrate the fact that when the armature of a motor is driven by *mechanical means*, a current is *induced* in it, opposite in direction to the current which would drive the armature in its present direction of revolution, the direction of the *field* being unchanged. So, when a motor is driven in the ordinary way, by a current, the revolution tends to set up in the armature a current opposite in direction to that actually flowing through it. In other words, it is harder to maintain the given current in the revolving armature than it would be to maintain it in the same armature at rest. This opposition is analogous *in effect* to polarization in a battery.

This opposing force is called *counter-electromotive force*, and it is in overcoming this curious opposition that electrical energy is turned into mechanical energy. All the rest of the electrical energy spent in the motor goes to *heat* its windings and its cores, and is worse than wasted.

In the best motors the counter-e.m.f. is nearly as great as the e.m.f. applied to the armature from without, so that the current which passes through the latter is only a small part of what would pass through if the armature were at rest. Indeed, most large motors are used in such a way that if the armature could be suddenly stopped, without reduction of the e.m.f. applied from the external line, the windings of the armature, if not especially protected, would be ruined almost instantly by the too-powerful current which would flow through them. This damage is guarded against by making the armature current pass through a short piece of easily melted wire called a *fuse*, which melts before the armature suffers harm.

The law that *every induced current of electricity is in*

such a direction as to oppose the change which produces it is called, from its discoverer, the law of *Lenz.* It follows from the law of conservation of energy (§ 271); for if a certain change, motion of an armature, for instance, caused a current tending to *maintain* that motion, we should have *perpetual motion*, that is, a continual supply of energy without corresponding expenditure of energy.

In the case of the dynamo, it is not at first obvious that the mechanical power which drives it is doing anything more than to overcome the resistance of mechanical friction. But, in fact, the mechanical power has to supply the energy of the electrical current as well. The following experiment will illustrate this fact.

EXPERIMENT.

Disconnect the field-magnet coils from those of the armature in a small industrial motor or dynamo, and send a strong independent current through the coils of the field-magnet.

Leaving the *brushes* unconnected by any conductor outside the armature, set the armature into *very rapid motion* by means of a cord wound around the axle, or otherwise; and then, leaving it to itself, see how many seconds it takes to come to rest. In this case the armature is like a galvanic cell in open circuit. It is sending out no current, doing no external work and very little internal.

Connect the brushes by means of a short wire, and again, after setting the armature in motion as before, see how long it takes to come to rest. In this case there is a considerable current both inside and outside the dynamo, and consequent expenditure of energy.

A motor absorbs electrical energy and gives out a partial equivalent in mechanical work; not a full equivalent, for some energy is wasted in heating the apparatus.

A dynamo absorbs mechanical energy and gives out a partial equivalent in electrical work; not a full equivalent, for some energy is, in this case also, wasted in heating the apparatus.

Electricity has worked many wonders within the last century, but it has not created one foot-pound of energy. It is a carrier, not a producer.

EXERCISE 61.

THE DYNAMO.

Put together the parts of a dynamo which, driven by the motor of Exercise 60, will give a current capable of affecting, at least, the astatic galvanometer.

440. Electric Lamps.—These are of two classes, incandescent lamps, so called, and arc-lamps. They are nearly always maintained by currents from dynamos.

The *incandescent lamp* consists of a thread, or *fibre*, of carbon, heated red-hot or white-hot by the passage of an electric current, and kept in a vacuum to prevent its burning out, which would occur instantly if it were exposed to the air.

On account of its thinness, and the high electrical resistance of carbon, the fibre is made incandescent by a rather weak current, weaker than the current sometimes used in the Exercises of this book. It is an interesting fact that the resistance of carbon diminishes with rise of temperature. The fibres are made by charring slender strips of vegetable matter, bamboo, for example, apart from the air.

The *arc-lamp* consists of two sticks of carbon, usually held and controlled by some machinery, separated by a short air-space, across which a current of electricity flows with a brilliant light. This light in the "arc" is due to the intense heating of the air and particles of carbon by the passage of the current.

The resistance of the heated air-gap, partially filled with flying particles of carbon, is only a few ohms, about 5, so that a moderate difference of potential, about 50 volts, between the ends of the arc, suffices to maintain it, the current being some 10 amperes. If the air were cold and there were no carbon-dust in it, many thousands of volts would be required to send a spark across it. Accordingly, when an arc-lamp is to be lighted, the carbons are made to touch each other for a moment, until the current, passing

through the point of contact, heats it to incandescence. Then the carbons are separated and the arc is established. The touching and separation of the carbons is accomplished automatically by the machinery of the lamp.

441. The Induction-coil: Description.—A so-called *induction-coil*, or *Ruhmkorff coil*, is a contrivance for producing transient but rapidly repeated induced currents of electricity.

The general plan of the apparatus is shown in Fig. 315. The part i is a core of straight soft-iron wires, around which

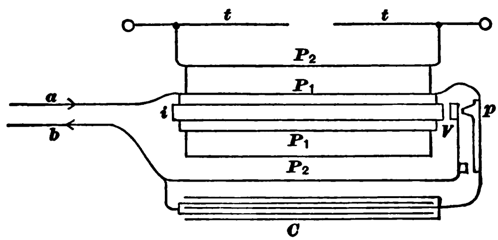

FIG. 315.

is a coil, P_1P_1, called the *primary coil*, of comparatively coarse wire; outside the primary, and insulated from it, is a *secondary coil*, P_2P_2, consisting of many turns of very fine carefully covered wire, of which coil t and t are the terminals.

The main current enters the primary by a and after traversing the windings comes out at the farther end, where it passes by the stop p through the spring, or vibrator, V, at the top of which is a piece of soft iron, back to the wire b by which it leaves the apparatus.

This current magnetizes the core i, which attracts away from p the soft iron at the end of V and so breaks the circuit. This interruption of the current causes the core to lose its magnetism, and this change induces a strong though momentary electromotive force in the secondary, sending a spark across a considerable air-space between t and t.

The core having lost its magnetism, *V* swings back to its normal position, thus closing the circuit, and the whole operation is repeated with a rapidity which depends greatly upon the character of the vibrator.

At the point of rupture, between *V* and *p*, a spark occurs, and this spark, by making a heated conducting *arc* (see § 440) across the air, may cause the primary current to die out so slowly that no strong induction effects are produced. This difficulty is removed, to a certain extent, by connecting *V* and *p* with the opposite sides of a *condenser* (§ 395), *C*, which is usually placed in a wooden case beneath the coil. When the break occurs, there is a flow of electricity for an instant into the condenser, which lessens the flow across the air from *p* to *V*, the final result of the rather complex operation being a more sudden demagnetization of the core and a more powerful effect in the secondary.

442. Uses of the Induction-coil.—Evidently the terminals of the secondary may be made of such length that the spark between them will occur a long distance away from the main apparatus. This fact makes it possible to use the induction-coil for lighting gas-jets in places not easily accessible, and for exploding charges of powder, etc., in mines.

The coil is used to some extent in the practice of medicine. Any one who connects the terminals of a small induction-coil, in operation, with his bare hands will experience a very peculiar and powerful stimulus of the nerves in his hands and arms. Such a stimulus is considered beneficial in certain forms of nervous debility. The experiment of connecting the terminals by means of the hands should not be tried with a powerful coil.

The most conspicuous use of the induction-coil at present is to send momentary currents of electricity through so-called *vacuum-tubes.*

443. Vacuum-tubes.—The electrical resistance of gases at ordinary temperatures and at atmospheric pressure is very great, but electricity passes with comparative readiness through gases in a certain state of rarefaction. If the rarefaction is carried beyond a certain point, which is different for different gases, the resistance begins to increase and finally it becomes much greater than that of the gas under atmospheric pressure. This fact leads us to the belief that the "ether" (§ 345) of an absolute vacuum is a non-conductor of electricity, in the usual sense of the word conductor; although it may be that electricity can be *forced* through it by means sufficiently violent.

A *vacuum-tube* (see Fig. 316) is usually a glass tube containing some highly rarefied gas, penetrated by two or more platinum wires, called electrodes, which are sealed into the

FIG. 316.

wall of the tube. When the secondary terminals of an induction-coil in operation are connected with the electrodes of such a tube, transient currents of electricity pass through the gaseous space, producing curious and often very beautiful effects.

Vacuum-tubes in which the glass itself has very complicated forms, and which are used largely to produce beautiful color effects, are called *Geissler* tubes, after a celebrated maker of such apparatus. Tubes using still higher vacua, provided with electrodes terminating in disks or portions of spheres, and often with much other internal furniture of metal, mica, etc., are commonly called Crookes tubes, after an English investigator, whose experiments made such tubes widely known.

444. Cathode Rays.—It was noticed long ago that the *cathode* of a vacuum-tube, the electrode by which the cur-

rent leaves the tube, shows phenomena of peculiar interest. In highly rarefied tubes, like those of Crookes, some kind of influence appears to proceed in straight lines from the cathode; for certain effects are observed at spots within the unobstructed reach of such lines and not at all or very little elsewhere. Thus Fig. 317 shows a shadow cast by a screen, *S*, placed in the course of rays from the cathode, *C*, within a Crookes tube.

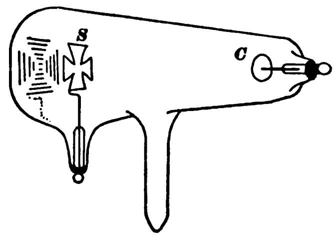

Fig. 317.

Crookes believed, and some others have been inclined to believe with him, that there is an actual projection of some material from the cathode to the points affected. Others have believed this projection to be only apparent or, if real, accidental. Whatever the true theory of the effect is, the term *cathode rays* has come into use to express the propagation of some *influence* from the *cathode*.

About the year 1892 Hertz (§ 446) and Lenard found that cathode rays could penetrate, or at least produce an effect through, opaque matter, such as thin films of metal.

445. Roentgen Rays.—In the latter part of 1895, Professor Roentgen of Würzburg discovered that from a vacuum-tube in which the cathode rays are active some influence goes out in straight lines which can penetrate through many opaque bodies—wood and flesh, for example—and produce photographic effects beyond.

It appears that the Roentgen rays are not the same as cathode rays, but are produced at, and given out from, any solid body, such as glass or metal, upon which the cathode rays strike.

Roentgen rays falling directly upon the eye do not give the sensation of light. But they can produce photographic

effects upon ordinary photographic plates, and fluorescent* effects upon layers of various substances, and in this indirect way they serve the eye.

The fact that gives to the Roentgen rays a peculiar and even a weird interest is this, that they penetrate flesh much more readily than bone, and as they go always in straight, or nearly straight, lines, they produce by photography or fluorescence well-defined shadows of the skeletons of living bodies.

There has been much discussion as to whether the Roentgen rays are of the same nature as light-waves. The fact that they are reflected and refracted very little, if at all, made people cast about for some theory to explain them as something different from ether-vibrations, but it now seems probable that they are really very short waves—of the same nature as light-waves, but much shorter. The uncertainty as to their nature caused Roentgen to call them *X-rays*, and by this name they have become widely known.

Roentgen rays have been produced with very great success from a Crookes tube agitated by the discharge from a Holtz electrical machine (§ 399), one pole of the machine being connected directly with one electrode of the tube, while a spark-gap is left between the other pole and the other electrode.

A common form of Crookes tube for the production of Roentgen rays is shown in Fig. 318, where c is the cathode, a is the anode, and P is the body, a plate of metal, which, receiving the cathode rays, gives out the Roentgen rays.

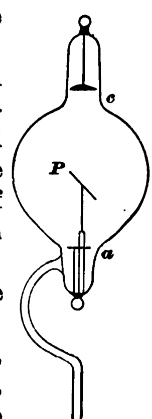

FIG. 318.

* Certain bodies receiving radiations give out, without being hot, radiations of a different wave-length from any of those falling upon them. This effect is called *fluorescence*, fluor-spar being a substance that shows it.

The Electromagnetic Theory of Light.

446. The Experiments of Hertz.—The wave-theory of light, and so the existence of the luminiferous ether, was proved early in this century by Young and Fresnel.

The experiments of Faraday led him to believe electrical and magnetic forces, "lines of force," to be transmitted by some medium, probably the luminiferous ether. He expressed this opinion in 1851.

Maxwell, a close student of Faraday's works, developed Faraday's idea mathematically, and arrived at certain definite conclusions, which could be tested by experiment. Some of them were soon tested, and the results did much to confirm the theory; but a real demonstration, capable of making an impression on the public, was not accomplished until 1887, when Hertz, a young German, showed that electromagnetic waves are sent out from the secondary of an induction-coil, and travel for considerable distances through the surrounding space.

The instrument used by Hertz to detect his waves at a distance from their source he called an electrical *resonator*. It was a frame of wire of such dimensions and shape that electrical oscillations within it had the same time of vibration as the waves the instrument was intended to detect. The analogy between the use of this instrument and that of the acoustic resonators of Helmholtz (§ 369) is obvious.

The waves discovered by Hertz were some meters long, whereas the waves which affect our eyes as light are only a few hundred-thousandths of a centimeter long. His waves were of electrical origin, and were traced through space by their electrical effects. They were beyond question electromagnetic waves, yet they were reflected like light-waves, they were refracted like light-waves, and they traveled with about the same velocity as light-waves. There could be no doubt that the two kinds of waves are essentially the same.

MICHAEL FARADAY

Thus was completed one of the greatest intellectual triumphs of the century, the proof of the electromagnetic nature of light.

QUESTIONS AND PROBLEMS.

(1) A magnetic pole of 20 units strength is placed at a distance of 10 cm. from a similar pole of 50 units strength. Find the force between them.

(2) A magnetic pole of 4-units strength is placed in a field of .5 unit strength. What will be the force exerted upon the magnetic pole?

(3) What length of wire .3 mm. in diameter will have the same resistance as 15 m. of wire 2 mm. in diameter?

(4) A wire 3 ft. long has a diameter of .64 cm. What must be the diameter of a wire 25 ft. long if it offers the same resistance?

(5) If the resistance of a German-silver wire 1 m. long and 1 mm. in diameter is .27 ohms, what length of German-silver wire 0.35 mm. in diameter will it take to make a resistance-coil of 5 ohms?

(6) If a foot of iron wire .05 in. in diameter has a resistance of .24 ohms, what is the resistance of 10 miles of iron telegraph-wire having a diameter of .25 in.?

(7) If a piece of silver wire 15 ft. long weighs 36 grains, and its resistance is 1.5 ohms, what will be the resistance of a wire 40 ft. long which weighs 25 grains?

(8) A wire 1 meter long and .5 mm. in diameter has a resistance of 8 ohms. Calculate the specific resistance.

(9) Two points in a circuit are joined in multiple arc by two wires whose resistances are 50 and 80 ohms respectively. Find the resistance of the circuit between these two points.

(10) Three wires whose resistances are 50, 60, and 20 ohms respectively are joined in multiple arc. Find their joint resistance.

(11) Three wires whose resistances are 3, 8, and 11 ohms respectively are joined in multiple arc. What is the resistance of the combination?

(12) The resistance of a current between two points is 40 ohms, but on adding another wire it falls to 35 ohms. What is the resistance of the wire?

(13) The resistance between two points is found to be 100 ohms. This must be reduced to one half this amount. How many wires, each having a resistance of 1000 ohms, must be added in multiple?

(14) Two points are connected by a wire 1.5 mm. in diameter. How much greater or less will the resistance be between these points than it would be if they were connected by two wires, in multiple, one of which had a diameter of 1.25 mm., and the other of .25 mm.?

(15) A battery which is producing a current of 1 ampere delivers this current through two wires, side by side, having a resistance of 80 and 50 ohms respectively. What is the strength of the current in each wire?

(16) How strong a current in a Daniel cell will be required to cause the copper plate to gain 5 gm. per hour?

(17) A battery of 20 cells is connected up in series with an external resistance of 100 ohms. The e. m. f. of each cell being 1.5 volts and its internal resistance 4 ohms, what is the current-strength?

(18) A battery of 30 cells is connected up in series with an external resistance of 200 ohms. If the e. m. f. of each cell is 1.8 volts, and its internal resistance 7.5 ohms, what is the current-strength?

(19) If the cells in the previous example had been arranged 3 abreast and 10 in series, what would have been the current-strength?

(20) What is the ratio of the currents produced by a cell of e. m. f. 1.4 volts and an internal resistance of 4 ohms, if its poles are first connected by a wire of .6 ohms resistance and afterward by a wire of 60 ohms resistance?

(21) How many cells in series, each having an e. m. f. of 2 volts and an internal resistance of 1.5 ohms, will produce 1 ampere current through an external resistance of 7.5 ohms?

(22) A battery of 20 cells, each having an e. m. f. of 1.5 volts and an internal resistance of 5 ohms, sends the strongest possible current through an external resistance of 3000 ohms. Find the strength of the current.

(23) A battery of 6 cells, each having an e. m. f. of 1.8 volts and an internal resistance of 5 ohms, is arranged in series and delivers its current through 3 wires, in multiple, of 100, 80, and 60 ohms, respectively. How much is the current in each of these wires?

(24) What arrangement will cause a battery of 10 cells, each having an e. m. f. of 1 volt and an internal resistance of 5 ohms, to send the greatest possible current through a resistance of 2 ohms?

(25) There are 24 cells, each having an e. m. f. of 1.5 volts and an internal resistance of 6 ohms. How should these cells be arranged

to give the strongest possible current through an external resistance of 15 ohms?

(26) The e. m. f. of a Bunsen cell is 1.85 volts and its internal resistance 1.5 ohms. How many of these cells, arranged in the best manner possible, will be needed to send a current of not less than $\frac{1}{4}$ ampere through an external resistance of 20 ohms?

(27) A battery consists of 10 Daniel cells, each having an e. m. f. of 1.08 volts and an internal resistance of 2 ohms. What is the greatest current this battery can produce through an external resistance of 4 ohms?

(28) Given 16 cells, each with an internal resistance of 1 ohm and an e. m. f. of 2 volts, working through an external resistance of 8 ohms, find the best arrangement.

(29) How can 24 cells of 1.6 volts e. m. f. and 2 ohms resistance be best arranged to overcome an external resistance of 10 ohms?

(30) How much will the current be that is obtained in examples 28 and 29?

(31) Two lamps arranged in multiple arc, each having a hot resistance of 30 ohms, are connected with a dynamo whose e. m. f. is 50 volts and internal resistance 3 ohms. What is the strength of the current going through each lamp?

(32) Fifty incandescent lamps arranged in multiple arc, each of which has a resistance of 40 ohms and requires a current of 1 ampere, are lighted by a dynamo which has an internal resistance of .5 ohm. What is the e. m. f. of the dynamo?

(33) How many lamps arranged in multiple arc can be lighted by a dynamo whose e. m. f. is 110 volts and resistance 2 ohms, if each lamp offers a resistance of 24 ohms and requires a current of 1 ampere?

(34) How many lamps, each requiring 50 volts and a .6-ampere current, could be lighted by a machine capable of doing 1 horse-power of external work?

(35) It is desired to use a 50-volt dynamo to light a 40-volt 10-ampere arc light. What additional resistance ought to be put into the circuit?

(36) If a lamp requires 8 watts to run it, how many lamps could be run with 5 horse-power?

(37) A lamp has a hot resistance of 16 ohms and requires a current of 2 amperes. What horse-power must be used to run it?

(38) How many watts are used in lighting a 16-candle-power lamp requiring an e. m. f. of 100 volts and a current of .5 ampere?

APPENDIX IV.

LIST OF APPARATUS FOR THE "EXERCISES," CONTINUED FROM PAGE 173.

Most of the articles in this list should be furnished to every student who is to perform all of the Exercises. But

No. 54 will serve for 6 members of the class.

Nos. 61, 62, 63	will	each	serve	for	3	members	of	the	class.
Nos. 69, 70, 71, 72, 73	"	"	"	"	2	"	"	"	"
No. 74	"	"	"	"	4	"	"	"	"
Nos. 76, 77, 78	"	"	"	"	3	"	"	"	"
No. 79	"	"	"	"	4	"	"	"	"
Nos. 93, 94, 95, 96	"	"	"	"	2	"	"	"	"
No. 68	"	"	"	"	the school.				

Several articles in the list are identical with articles mentioned under different numbers in Appendix III.

No. 50. A 10-kgm. or 30-lb. straight spring-balance.

No. 51. A guard of wood or metal to bestride the bar of No. 50 and prevent the violent recoil in Exercise 26. See Fig. 124. The guard should be of such a length as not to allow more than 1 cm. of recoil, and therefore, if wires of very different strengths are used, a guard suitable for each should be supplied.

No. 52. A wooden cylinder about 2.5 cm. in diameter and about 3 cm. long, perforated and cut in such a way that it can be slipped on to the hook of the balance (No. 50), but will not turn around on the hook. See Fig. 124.

No. 53. A wooden cylinder to be conveniently fastened upright by means of a screw to the top of the table. See Fig. 124.

No. 54. Screw calipers reading to 0.001 cm.

No. 55 A. A rod of clear, straight-grained white pine about 102 cm. long and 1.3 cm. square. Much care should be taken to make the width and thickness exact after the rods are thoroughly dried. See Exercise 29.

No. 55 B. A rod twice as wide, but similar in all other respects to 55 A.

No. 56. A set of three hard-wood prisms, each about 3 cm. long

and 2 cm. wide, the ends shaped as in Fig. 319. The height from base to apex on the end should be the same for all. These prisms are intended to support the rod and the index used in Exercises 28 and 29.

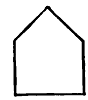

FIG. 319.

No. 57. A strip of wood about 32 cm. long, 0.5 cm. wide, and 0.2 mm. thick, to serve as an index in Exercises 28 and 29.

No. 58. A 10-cm. scale divided to mm., attached to a base-block so as to stand upright, for use in Exercises 28 and 29.

No. 59. A metal pan about 12 cm. in width, with looped strings attached, to carry the weights used in Exercises 28 and 29.

No. 60. Set of iron weights, 100, 200, 300, 500, 1000 gm., for use in Exercises 28 and 29. Corresponding weights marked in ounces (see No. 19) may be used in place of gram-weights.

No. 61. A rod of clear straight-grained ash 1 m. long and 1 cm. square in cross-section, one end of which is fitted firmly into the middle of a cross-bar about 32 cm. long, 1 cm. thick, and 3 cm. wide (at the middle). Near each end of the cross-bar is a peg to take the loop of a string, the pegs being 30 cm. apart. Projecting through the cross-bar, in line with the axis of the rod, is a round nail or brad 2 cm. long. (In place of the cross-bar a circle of wood may be used, as in Fig. 128.)

A cleat about 4 cm. long and 2 cm. square, having on one side a notch 1 cm. wide and 0.5 cm. deep, goes with the rod for attaching it to the horizontal bar above the table-top. Cleat not shown in Fig. 128.

No. 62. A rod like No. 61, but 2 cm. square in cross-section and provided with a correspondingly heavy cross-bar and cleat.

No. 63. A sheet of cardboard or metal upon which is traced one-eighth part of the circumference of a circle of 15.5-cm. radius, the arc being divided into degrees and half-degrees. This is for use in the twisting experiments with Nos. 61 and 62.

No. 64. Two small tumblers. See Fig. 141.

No. 65. Tubes and pinch-cock for the Exercise on Balancing Columns. See Fig. 141.

No. 66. Wooden support for No. 65, with meter-rod. See Fig. 141.

No. 67. (See No. VI.) Glass tube for Boyle's law (see Fig. 144), about 0.7 cm. in diameter inside; the closed arm about 30 cm., the other about 110 cm. long. The short arm of this tube should be of very uniform bore.

No. 68. An inexpensive barometer, the readings of which should

not differ more than 0.3 cm. from those of a standard instrument. (See Exercise 33.)

No. 69. A 2-liter glass bottle, provided with a perforated rubber stopper through which extends a short piece of glass tubing connected with a thick, soft, rubber tube about 15 cm. long, carrying a strong pinch-cock. (This piece of apparatus is for Exercise 34 on the Density of Air (see Fig. 146). The bottle and the tubing must be strong enough to be in no danger of collapsing when all the air is removed from it. Red antimony-rubber is recommended for the stopper and tube, as it is very pliable.)

No. 70. Glass U-tube each arm of which is about 1 m. long and 0.5 cm. in diameter inside. For pressure-gauge in Exercise 34. (See Fig. 146.)

No. 71. (See No. XVII.) Platform-balance weighing from 1 kgm. to 0.1 gm., provided with a set of brass weights.

No. 72. (See No. V.) Air-pump for both exhaustion and compression, with simple base for attachment to floor or wall. The capacity should be not less than 100 cu. cm. per stroke. (See Fig. 146.)

No. 73. A metal Y tube with attached rubber tubes, for connecting Nos. 69, 70, and 72, as in Fig. 146.

No. 74. A smooth flat board, protected from warping by cross-pieces at the ends, upon which a square 30 cm. on the side is laid off and divided into squares each 5 cm. on the side, the lines being made with a pencil or a knife, a hole about 0.3 cm. in diameter being drilled nearly through the board at every crossing (see Fig. 320); the under side of the board is covered with a thick, smooth sheet of tin. Several iron pegs fitting well holes in the board. Three smooth round marbles equal in size, or, better, three steel bicycle-balls to support the board in a horizontal position and give it freedom of motion.

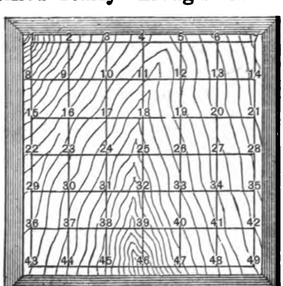

FIG. 320.

For certain uses (see p. 258) the board should be provided with a bar, marked off in centimeters and millimeters, extending across its top and raised a few millimeters from the surface.

It is quite as well to make the board in 2-inch squares as in 5-cm. squares.

No. 75. A bed for No. 50, to hold the balance flat on its back and

to raise the horizontal string leading from the hook a very little above the surface of the board No. 74, when this is resting upon the supporting balls. (See Fig. 321.)

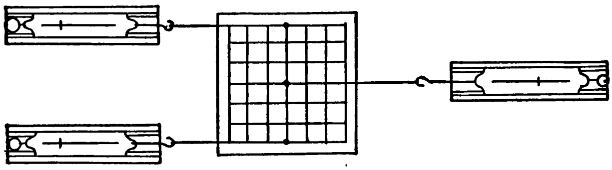

FIG. 321.

No. 76. Two strong brass carriages (see Fig. 322) each large enough to hold about 1 kgm. of iron, and sufficiently well made to continue in motion after being gently started down an incline of 1 in 50. To each carriage is attached a small, thin rubber tube about 50 cm. long and of such quality that it will bear stretching to twice its original length. For use in Exercise 36.

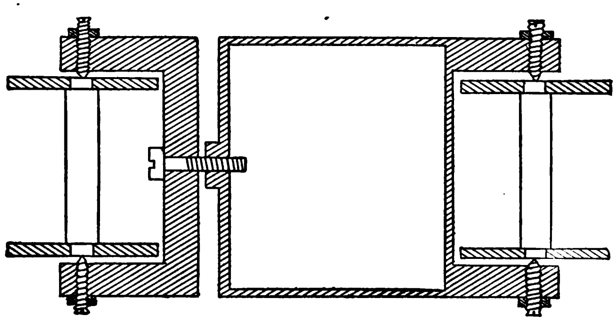

FIG. 322.

No. 77. Two smooth straight planks, each about 120 cm. long and 15 cm. wide, to serve as tracks for No. 76 (see Fig. 185). Each plank should have at one end an elevating-screw capable of raising that end 2 cm. higher than the other; and along all four edges of one side cleats rising about 1.5 cm. above the surface. The clear space between the side cleats should be as much as 11 cm. wide.

No. 78. Cast-iron weight or weights amounting to about 1 kgm., of such dimensions as to be carried securely in one of the carriages No. 76.

No. 79. The whole apparatus used for Exercises 37 and 38 on Action and Reaction, as follows:

Two ivory balls, one of about 50 gm., the other three or four times as heavy, each provided with a hook for suspension.

A board about 20 cm. long, 6 cm. wide, and 1 cm. thick, beveled along one edge (from which the pendulum-balls are to hang), having three fine slots in the acute edge of the bevel, No. 1 near one end, No. 2 separated from No. 1 by a distance equal to the sum of the radii of the balls, No. 3 about 0.3 cm. farther from No. 1 (see Fig. 192).

Several yards of small, uncovered, copper wire, strong enough to bear twice the weight of the large ball. In use the wires are attached to the pegs or tacks in the supporting board and hang down through the notches, Nos. 1 and 3 when putty is in use (Fig. 195), Nos. 1 and 2 at other times.

A base-board 1 m. long with graduated raised bar along its top, carrying two detents for holding the balls in place before release. (See Fig. 194.) The detents should be attached at any part of the bar, and the line of sight through the slots of the two uprights (see Fig. 193) should be parallel to the bar. (The base-board here described is long enough if the suspension is not longer than 2 m. If the suspension is much longer than this, which is desirable, two base-boards can be used.)

No. 80. A cylindrical sheet-copper vessel about 15 cm. tall and 10 cm. in diameter, supported on three legs, which are kept from spreading by a circular plate, or a ring, of sheet copper connecting them all at the bottom. These legs should raise the vessel about 20 cm. from the table upon which they stand. This vessel is to be used as a boiler, and has, leading out about 2 cm. from the top, a side tube through which the steam may be carried off when the top of the vessel is closed. A conical tube of sheet-copper about 30 cm. tall is made to fit the top of the cylindrical vessel internally, as the cover of a tin pail fits. It must fit well, for the junction should be, as nearly as practicable, steam-tight. The top of the cylinder should therefore not be wired, but should be left flexible. The open top of the conical tube is about 2.5 cm. in diameter and of such a shape as to be readily closed by a cork stopper. A side tube leads out from the cone about 2 cm. from the top. Through this tube, the other apertures being closed, steam escapes when the apparatus is used in testing the boiling-point of a thermometer.

A small mercury-gauge accompanies this vessel and may be attached to the lower outlet in testing the effect of pressure upon boiling temperature in Exercise 39. (See Fig. 209.)

No. 81. A sheet-copper dipper about 10 cm. deep and 4 cm. wide encircled, about 2 cm. from the top, by a flat flange of sheet-copper about 4 cm. wide. See Exercise 43 (Fig. 219).

No. 82. An inexpensive paper-scale centigrade thermometer graduated a few degrees below 0° and above 100° C.

No. 83. A Bunsen burner.

No. 84. A rod or tube of brass about 5 mm. in diameter and about 60 cm. long, to one end of which a very short tip of pointed steel wire is soldered. For use in Exercise 40.

No. 85. A tube of sheet-iron, tinned or "galvanized," open and slightly flared at both ends; about 2.5 cm. in diameter and a few mm. shorter than No. 84, which is to be heated within it by the action of steam. (See Fig. 211.)

The ends are provided with very short cork stoppers, *flush* with the ends of the tube, which are perforated so that the brass rod (No. 84), when placed within the tube, extends slightly through the stopper at each end.

No. 86. A wooden rack for holding No. 85 in position when in use. (See Figs 211 and 212.)

No. 87. A glass tube containing dry air retained by a column of mercury for use in Exercises 41 and 42.

Take a glass tube 50 cm. long, about 0.5 cm. outside and about 0.15 cm. inside, in which a column of mercury about 10 cm. long does not vary more than 1% of its length when measured in different positions. Attach to this tube another, of similar dimensions but not necessarily regular in size of bore, by means of a short thick-walled piece of antimony-rubber tubing. Heat the calibrated tube, about 3 cm. from the free end, with a small flame, and draw out the tube at this point till it is so small that it can be readily melted off when the proper time comes.

To fill this tube with dry air follow the indications of Fig. 323, placing the tube to be filled inside a sheet-iron heating-tube similar to but shorter than No. 85, the narrowed part *n* being just outside the heater. Steam is sent through the heating-tube, and at the same time an air-pump connected with *o* draws air in at *i*, through the drying-bottle containing strong sulphuric acid and glass beads, through the heated glass tube, and out through the mercury in the bottle at the other end. After five minutes of this operation it may be assumed that the glass tube is dry. The current of air is then stopped and the tube is melted off and sealed at *n*. The pump is then worked again to draw out a part of the air in the tube, so that

mercury may take its place. The degree of exhaustion attained is indicated by the gauge connected at *g*. When enough air has been drawn out, disconnect the pump and let full atmospheric pressure act upon the mercury in the bottle, forcing it up into the tube. The confined dry-air column in the tube should finally reach about two-

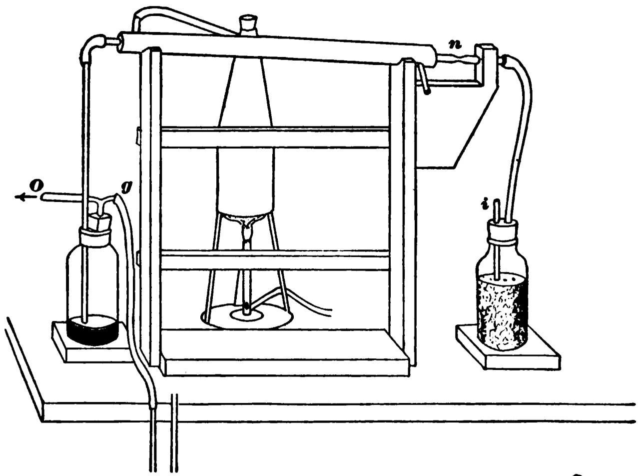

FIG. 323.

thirds of the way from the sealed end to the rubber joint, when both arms of the tube are horizontal at ordinary temperature and pressure.

No. 88. A "galvanized iron" tray about 60 cm. long, 15 cm. wide and 5 cm. deep, having a tubulure at one end. This tray is to hold ice-water or snow for cooling the air-column, the tube containing which is thrust through a cork in the tubulure. If the tray has two tubulures at each end, it can be used to cool four air-tubes at once. (See Fig. 217.)

No. 89. A calorimeter about 600 cu. cm. capacity. The thin nickel-plated brass vessels, larger at top than at bottom, sold at hardware stores as "liquor-shakers," serve exceedingly well for this purpose.

No. 90. Half a kilogram of lead shot.

No. 91. A strong canvas bag 30 cm. long and 20 cm. wide for pounding ice.

No. 92. A glass trap for catching the water formed by condensation in the conducting-tube in Exercise 46. (See Fig. 232.)

No. 93. A small spy-glass.

No. 94. Glass tube about 2.5 cm. in diameter and about 1.2 m. long, provided with a piston consisting of a cork stopper and a rod of wood or metal. For use in Exercise 48.

No. 95. Tuning-fork of about 256 double vibrations per second.

No. 96. Apparatus for determining the number of vibrations per second of No. 95, by means of a tracing on smoked glass. (See Fig. 259.) The pendulum should make three or four single beats a second.

No. 97. A straight bar magnet about 15 cm. long and 1 cm. square in cross-section. For Exercise 50.

No. 98. A small magnetic compass with needle 2 or 3 cm. long.

No. 99. A small copper-zinc cell for one fluid, consisting of a small glass jar, or tumbler, and two thin strips, one of zinc and one of copper, each about 10. cm. long and 1 cm. wide, each provided with a copper wire, about No. 22, some 30 cm. long. A block of wood about 3 cm. wide, fitted to the top of the jar, carries two spring clamps for holding the metal strips.

No. 100. A galvanoscope, with coil about 15 cm. in diameter, wound with 15 turns of wire, about No. 20, in such a way that 5, 10, or 15 turns may be used at will. This instrument is not to be treated as a tangent galvanometer, and the compass upon it is small. See Fig. 281.

No. 101. The solid parts of a small Daniell cell, the copper and zinc plates provided with wires, about No. 20, some 30 cm. long.

No. 102. A simple commutator for electric currents. (See Fig. 287.)

No. 103a. Six wooden spools (Fig. 324, *half size*) each wound with uncovered No. 30 German-silver wire, two of them carrying 200 cm. each, one carrying 160 cm., one 120 cm., one 80 cm., one 40 cm.

No. 103b. A similar spool with 200 cm. of No. 28 German-silver wire.

No. 103c. A similar spool with 2000 cm. of No. 30 covered copper wire.

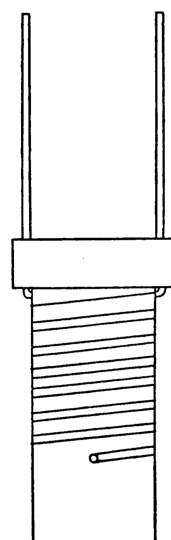

Fig. 324.

(All of the G. s. wires of Nos. 103*a* and 103*b* should be of the same quality, all of the No. 30 being, if possible, from the same piece.

Every spool should be *soaked* in melted paraffin before the wire is laid on it. Every wire should be wound in such a way as to

have no magnetic effect. Every spool should be *dipped*, not soaked, in melted paraffin after it is wound.

At some point on each spool the wire should be left so exposed that its diameter can be measured by means of screw calipers. For this purpose a bit may be left projecting beyond the point of soldering to the heavy copper terminals. (I am indebted to Mr. C. C. Hyde, at one time my laboratory assistant, for the replacement of a cumbrous "resistance-rack" by these coils.—E. H. H.)

No. 104. A pair of double binding-posts for use with the spools of No. 103.

No. 105. A pair of triple binding-posts for introducing two of the spools of No. 103*a* into the circuit in parallel. (Not *necessary*.)

No. 106. A simple form of Wheatstone's bridge, shown in Fig. 300, with thick connecting strips of brass or copper of very small resistance in comparison with the thin German-silver wire, *hi*, one meter long, which is stretched along a meter-rod (not shown), and is neatly soldered to the strips at each end. At *a*, *b*, *d*, *e*, *f*, *g*, and *m* are binding-posts. The post *a* has two nuts, and under the lower nut is a washer, the edge of which comes just over the inner edge of the metal strip beneath. The post *b* is similarly provided. This makes it possible to stretch a thin wire from *a* to *b* and fasten it securely beneath the washers, so that just one meter of its length will be exposed, as in Fig. 298. Battery connections can be made at *a* and *b* without disturbing this wire, which is, however, not kept in place during the ordinary use of the apparatus. The gaps *de* and *fg* should be of such a width as to receive conveniently the terminals of the resistance-coils of No. 103.

A suitable slide is provided for making contact with the wire *hi*.

No. 107. An inexpensive astatic galvanometer, that shown in Fig.

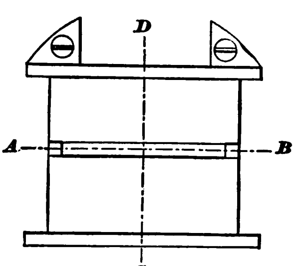

FIG. 325.

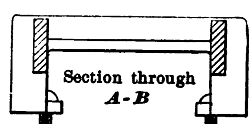

FIG. 326.

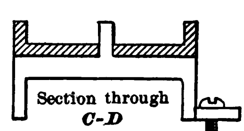

FIG. 327.

299, for instance. The small cost of this form is partly due to the fact that the metal frame on which the wire is wound is in such shape that it can be cast. Fig. 325 shows a top view of this frame before winding. Figs. 326 and 327 show sections. All these figures are one-half size.

No. 108. A resistance-box with a range from 1 ohm to 10 ohms or higher.

No. 109. A temperature-coil, for showing change of resistance with change of temperature (see Fig. 328.) This consists of fine copper wire, uncovered, wound in grooves in a hollow hard-rubber cylinder. The terminals are convenient for inserting the coil in one of the gaps of the Wheatstone bridge (No. 106.) (See Fig. 301.)

In use the coil, so far as the windings extend, is immersed in water. A hole in the plug at the top of the cylinder gives admission to a thermometer.

No. 110. The parts of a telegraphic sounder and key, the simpler the better, to meet the requirements of Exercise 59.

No. 111. The parts of a small electric motor, to meet the requirements of Exercise 60.

No. 112. The parts of a small dynamo, to meet the requirements of Exercise 61.

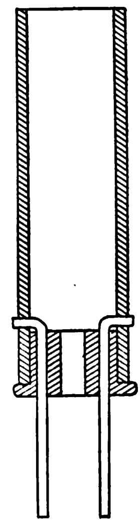

FIG. 328.

LIST OF APPARATUS FOR THE "EXPERIMENTS," CONTINUED FROM PAGE 177.

The articles marked thus: * may be dispensed with if economy is an important consideration.

No. L. Four wire frames and a clay pipe for experiments on surface tension of liquid films. Figs. 130–132.

No. LI. Mariotte's bottle, Fig. 135.

No. LII. Brass cylinder 14 cm. in diameter and 7 cm. long with well fitting piston. This is a part of Gage's "Seven-in-One" apparatus, Fig. 137.

No. LIII. Double cone on track consisting of two inclined and non-parallel rails. The double cone appears to roll up hill. Exp. 1, § 236.

No. LIV. Wooden cylinder about 10 cm. long and 5 cm. in diameter

heavily loaded on one side internally, for eccentric motion in rolling along a horizontal surface. Exp. 2, § 236.

No. LV. Apparatus for testing effect of gravity on a body having horizontal motion. (See Fig. 190.)

No. LVI. Apparatus for showing reaction of a stream of water escaping from a vessel. (See Fig. 196.) The cylinder is about 30 cm. tall and 5 cm. in diameter.

No. LVII. Three rods, one of copper, one of iron, and one of glass. (See § 283.) In place of these may be used any common form of "Ingenhaus's apparatus" for showing differences of thermal conductivity.

No. LVIII. Half a dozen assorted test-tubes with a supporting rack.

No. LIX. A rod of brass and wood about 20 cm. long and 1.5 cm. in diameter. (See Fig. 202.) The wood should be especially well seasoned, so that it will not in drying become smaller than the rod.

No. LX. A sheet of "touch-paper" made by soaking blotting-paper in a strong aqueous solution of saltpetre and then drying.

No. LXI. Tubular glass rectangle for showing convection currents in water. The internal diameter of the tube should be about 1 cm., and the dimensions of the rectangle 30 by 20 cm. Fig. 204.

No. LXII. Glass bulb 4 or 5 cm. in diameter, with a straight tube about 20 cm. long and 3 or 4 mm. inside diameter. (See § 288, Exp. 1.)

No. LXIII. Copper or brass ball and ring about 3 cm. in diameter. (See Fig. 205.)

No. LXIV. Two glass flasks of the same size, about 500 cu. cm. capacity, each provided with a perforated rubber stopper and a narrow glass tube. (See Fig. 206.) The tubes should be alike. (See Exp. 2, § 289.)

No. LXV. Compound metal bar for showing unequal expansion. (See Fig. 207.)

* **No. LXVI.** A "metallic" thermometer, with open or glass-covered back to show the mechanism.

* **No. LXVII.** A clock balance-wheel, showing device for self-adjustment to temperature-changes. (See Fig. 214.)

No. LXVIII. "Trevelyan's rocker" and a massive flat ring of lead. (See Fig. 215.)

* **No. LXIX.** An air-thermometer suitably mounted. (See Fig. 218.)

No. LXX. Set of six glass beakers, two of 1 liter capacity, two of 500 cu. cm., and two of 100 cu. cm.

No. LXXI. A round-bottomed glass flask of about 500 cu. cm. capacity, having a branch tube at the neck, equipped as in Fig. 223,

for illustrating the nature of boiling. This flask should be strong enough to bear complete exhaustion of the air.

No. LXXII. An iron ring-stand, having two rings and two arms provided with clamps.

No. LXXIII. Glass tube about 80 cm. long and 0.8 cm. in diameter, sealed at one end. (See Fig. 227.)

No. LXXIV. An iron mercury-well about 90 cm. deep, preferably one with a glass basin at the top. (See Fig. 227.)

No. LXXV. Two thin watch-glasses.

No. LXXVI. A *cryophorus* (see Fig. 231). The whole amount of water within should be too little to half-fill either bulb, otherwise the apparatus is likely to be broken when freezing occurs.

No. LXXVII. A "fire-syringe" and 5 gm. of good tinder. (See § 335.)

No. LXXVIII. A small "differential thermometer," consisting of two bulbs partly filled with some volatile colored liquid, connected by a slender stem bent twice at right angles (Fig. 236). This instrument must be of such size and supported in such a way that it can be conveniently used under the bell-jar, which may be furnished with No. V. (See § 335, Exp. 2.)

* **No. LXXIX.** Cubical brass box, about 8 cm. deep, having one of its vertical sides brightly polished, one coated with some varnish capable of bearing a pretty high temperature, one painted a dead white, one painted dead black. (See. Fig. 247.)

* **No. LXXX.** Thermopile. (See Fig. 247.)

* **No. LXXXI.** Reflecting galvanometer, with curved mirror or with lens for projection (see Fig. 329), for use with No. LXXX. The light should enter at the side and be reflected to the scale *SS*.

* **No. LXXXII.** Welsbach burner (*w*, Fig. 329), for use with No. LXXXI. The chimney is surrounded by a metal shield having a hole about 0.7 cm. in diameter at the height of the brightest part of the mantle. It is well to have a vertical wire fastened across this hole.

* **No. LXXXIII.** Scale (*SS*, Fig. 329) not less than 1 m. long, in divisions of about 2.5 cm., on tracing-cloth, properly supported, for use with No. LXXXI.

No. LXXXIV. Pair of spherical metallic mirrors as much as 30 cm. in diameter, suitable for experiment on the reflection of obscure radiations and sound-waves. (See Fig. 255.)

No. LXXXV. An alarm-bell small enough to go under the bell-jar which accompanies No. V. (See § 355.)

No. LXXXVI. Rotating apparatus suitable for carrying Crova's disk, etc.

No. LXXXVII. Crova's disk. (Fig. 251.) See directions in Mayer's Sound.

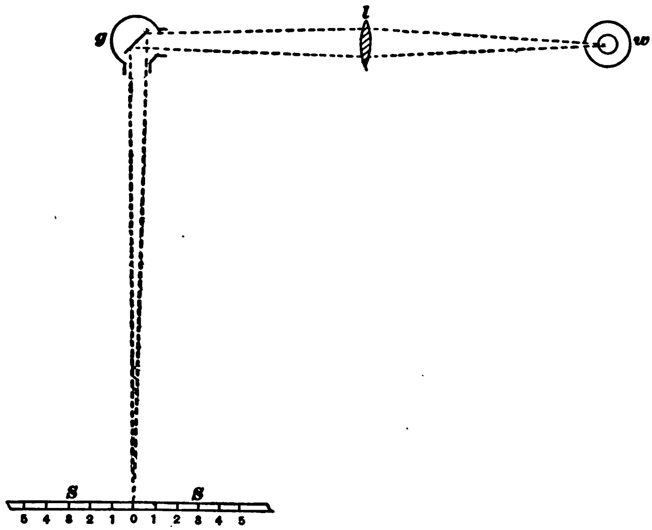

FIG. 329.

No. LXXXVIII. An ear-trumpet.

No. LXXXIX. Apparatus for showing that a musical sound is produced by regularly timed impulses, and that the same number of impulses per second not regularly timed do not produce a musical sound. This may be a disk, with holes to be blown through or with projecting pegs to strike a card, revolved on the rotating apparatus No. LXXXVI. (See Fig. 256.)

No. XC. Straight piece of clock-spring, about 50 cm. long. (See § 363.)

No. XCI. Two unmounted tuning-forks, one considerably higher, and the other, if practicable, considerably lower, in pitch than No. 95. A single fork of adjustable pitch may take the place of several ordinary forks. (See § 365.)

No. XCII. Two tuning-forks of the same pitch, mounted upon sounding-boxes. (See Fig. 258.)

No. XCIII. A bass-viol bow for setting tuning-forks into action.

No. XCIV. A sonometer of simple form. (See Exp. 1, § 370.)

No. XCV. Two plates of brass, one square and one round, each about 15 cm. in diameter, for showing Chladni figures with sand. (See Fig. 264.) Each plate mounted ready for use.

No. XCVI. Organ-pipe about 2.5 cm. in diameter inside and about 75 cm. long with side openings and stops for illustrating the position of nodes. (See § 373.)

No. XCVII. A dozen very soft iron nails.

No. XVCIII. A light, well-pivoted magnetic needle 12 or 15 cm. long. (See Fig. 268.)

No. XCIX. Simple dipping-needle. (See Fig. 267.)

No. C. Bar of very soft iron about 50 cm. long and 1.5 cm. in diameter. (See § 378.)

No. CI. A dozen bars of thin hard steel like hack-saw blades. (See § 381.)

No. CII. Smooth rod of vulcanized india-rubber or gutta-percha about 30 cm. long and 2 cm. in diameter. (See § 386.)

No. CIII. A smooth glass tube about as large as the gutta-percha rod. (See § 303.)

No. CIV. Spool of fine silk thread and one of fine cotton thread.

No. CV. One dozen pith balls.

CVI. A catskin.

CVII. A piece of good silk as large as a catskin.

CVIII. A gold-leaf electroscope, having at the top a flat metal plate replaceable by a ball which is furnished.

CIX. An electrophorus 20 or 30 cm. in diameter.

CX. Hollow metal sphere mounted on an insulating stand, accompanied by a metal ball about 1.5 cm. in diameter, mounted on an insulating handle about 15 cm. long. (See Fig. 273.)

CXI. A dissecting Leyden jar, that is, a jar the coatings of which may be easily put on or off. (See Fig. 274.) The coatings should not be painted or varnished.

CXII. A Leyden-jar discharger, with glass or hard-rubber handle.

CXIII. A simple form of "induction" electrical machine, Voss, Toepler-Holtz, etc. (See § 399.)

CXIV. A brass chain, a meter or more long, for connecting one rod of the electrical machine with the outer coating of a Leyden jar.

CXV. A wire cage, with meshes about 1 cm. square or finer, large

enough to cover the gold-leaf electroscope, and a metal plate or sheet of wire netting to set this cage on. (See Fig. 278.)

No. CXVI. A battery of fifty very small zinc-copper cells, without porous cups, connected in series. The cells are filled with ordinary water when in use, for charging an electroscope. (See § 402.)

No. CXVII. A dozen sheets of thin paper, about 20 cm. square, which have been soaked in melted paraffin. (See § 402.)

No. CXVIII. A lecture-table galvanometer, having oblong horizontal windings, within which is placed a magnet pivoted like a dipping-needle. An index at right angles with the magnet projects from the top of the coil and moves, in a vertical circle, along a graduated arc. (See Fig. 283.)

No. CXIX. Apparatus for the electrolysis of water and collection of the component gases. (See Fig. 286.)

No. CXX. A Daniell cell much smaller than No. 101.

No. CXXI. An electromagnet showing a marked effect with current from a Daniell cell. (See § 429.)

No. CXXII A telegraphic sounder and key.

No. CXXIII. Electric motor with permanent magnet for the field and coreless armature, to be driven by one or two Daniell cells. (See Fig. 311.)

No. CXXIV. Electric motor with electromagnetic field and armature, the latter having a straight soft-iron core; capable of being easily changed from shunt to series arrangement. To be driven by one or two Daniell cells. (See Fig. 312.)

No. CXXV. Small fan-motor.

No. CXXVI. A Ruhmkorff coil capable of giving a spark 2 or 3 cm. long in air and giving good results with vacuum-tubes.

No. CXXVII. A set of four Geissler tubes.

* **No. CXXVIII.** A Roentgen-ray outfit, battery, induction-coil (No. CXXV), Crookes tube, and fluoroscope.

APPENDIX V.

A FEW EQUIVALENTS IN THE ENGLISH AND METRIC SYSTEMS.

1 meter = 1.0936 yards.
1 " = 3.2809 feet.
1 " = 39.3705 inches.
1 kilometer = 0.6214 mile.
1 gram = 15.4323 grains = 0.0353 ounce.
1 kilogram = 2.2046 pounds avoirdupois.

1 yard = 0.9144 meter.
1 foot = 0.3048 "
1 inch = 0.0254 "
1 mile = 1.6093 kilometers.
1 pound avoirdupois = 0.4536 kilogram.
1 ounce = 28.35 grams.

The following are approximate equivalents:

1 decimeter = 4 inches.
1 meter = 1.1 yards.
1 kilometer = $\frac{5}{8}$ of a mile.
1 kilogram = $2\frac{1}{5}$ pounds.

APPENDIX VI.

TABLE OF PHYSICAL CONSTANTS.

Name.	Breaking-strength, kgm. per sq. cm.	Density, grams per cu. cm.	Mean Coefficient of Cubical Expansion from 0° to 100° C.	Mean Specific Heat between 0° and 100° C.	Melting-point, Centigrade.	Latent Heat of Melting, gm-deg. C.	Boiling-point, Centigrade.	Latent Heat of Evaporation, gm-deg. C.	Electrical Resistance, silver the standard.
Copper........	4100	8.9	.000051	.093	1100	80?			1.07
Gold..........	3000	19.3	.000044	.032	1100				1.36
Iron..........	6100	7.8	.000036	.113	1600?	35?			6.4
Lead..........	100 to 300	11.3	.000088		330	5.6	1500		12.3
Platinum....	3100	21.5	.000027	.032	1900?	27			6.4
Silver.........	3100	10.5	.000058	.056	1000	21			1.0
Tin...........	200 to 410	7.2	.000069	.055	230	14	1500		7.1
Zinc..........	200 to 510	7.0	.000088	.093	420	28	1000		3.55
Brass:									
(cast).......	1200	8.3	.00005+	.094−	900?				6.4 to 8.2
(hard-drawn)	4100 to 9200	8.5	.000057	.09+	"				
Steel:									
(cast).......	8200 to 10200	7.8+	.000034	.12	1400?				
(wire).....	10200 to 345,000	7.9?	.000037	.12?					
Glass:									
(crown).....	300 to 600		.000025	.19	400				Very large
(flint).......		3.5−	.000025	.19?					Very large

APPENDIX VII.

VAPOR-PRESSURES AT VARIOUS TEMPERATURES EXPRESSED IN MEGADYNES* PER SQUARE CENTIMETER.

Temperature Centigrade.	Alcohol.	Bisulphide of Carbon.	Ether.	Mercury.	Water.
0°	.017	.17	.25	.0000	.006
10°	.032	.27	.38	.0000	.012
20°	·059	.40	.58	.0000	.023
30°	.10+	.58	.85	.0000	.042
40°	.18	.82	1.2	.0000	.073
50°	.29	1.14	1.7	.0000	.123
60°	.47	1.6	2.3	.0001	.198
70°	.72	2.1	3.1	·0001	.310
80°	1.08	2.7	4.0	.0002	.472
90°	1.6	3.5	5.2	.0004	.701
100°	2.3	4.4	6.6	.0006	1.014
110°	3.2	5.5	8.3	.0010	1.44
150°	9.8	12.		.0045	4.8
190°				.018	12.6

* The megadyne is 1.000,000 dynes. To find the pressure in centimeters of mercury column multiply the pressure in megadynes per square centimeters by 75.

APPENDIX VIII.

VALUE IN MILLIMETERS OF BROWN & SHARPE WIRE-GAUGE NUMBERS.

Number.	Diameter mm.	Number.	Diameter mm.
1	7.348	17	1.150
2	6.544	18	1.024
3	5.827	19	0.912
4	5.189	20	0.812
5	4.621	21	0.723
6	4.115	22	0.644
7	3.656	23	0.573
8	3.264	24	0.511
9	2.906	25	0.455
10	2.582	26	0.405
11	2.305	27	0.361
12	2.053	28	0.321
13	1.828	29	0.286
14	1.628	30	0.255
15	1.459	31	0.227
16	1.291	32	0.202

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